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NUMERICAL SIMULATIONS OF ACTIVE PHENOMENA IN THE SOLAR ATMOSPHERE

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1. Introduction

The recent development of electronic computers have enabled us to solve nonlinear, nonsteady hydrodynamic or magnetohydrodynamic equations numerically. However, at present there does not exist such an almighty numerical scheme for solving these partial differential equations, as the Runge-Kutta scheme for the ordinary differential equations. Therefore in applicating numerical (magneto-) hydrodynamics to solar active phenomena, one is faced with many problems, e.g. selection of scheme, numerical instability, boundary condition and etc. Within a framework of the finite difference scheme, we first give a brief summary of these problems appearing in numerical (magneto-) hydrodynamics (Section 2). In spite of the immaturity of numerical (magneto-) hydrodynamics, applications to solar active phenomena have been done increasingly during the recent ten years and have developed solar physics. We next review the recent development of numerical solar (magneto-) hydrodynamics, especially applications to shock waves and jet phenomena (Section 3). Finally the limitations and the possibilities of numerical solar (magneto-) hydrodynamics in future are briefly discussed (Section 4).

2. Numerical (Magneto-) Hydrodynamics

2.1. Definition

We will define 'Numerical (Magneto-) Hydrodynamics' as the numerical simulation of (magneto-) hydrodynamical phenomena which is represented by the equations of conservations of mass, momentum, energy and magnetic flux, including the approximations of the incompressibility, the specified temperature and the nonmagnetic hydrodynamics.

2.2. Numerical Scheme

The schemes used in Numerical (Magneto-) Hydrodynamics are classified by their various characters, i.e. whether the grid is referred to space (Euler) or mass (Lagrange); whether the space derivative is the past (explicit) or the present (implicit), that is, in the explicit case $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \rightarrow u_i^{n+1} - u_i^n = (u_{i+1}^n - u_{i-1}^n)/$ $(2 \Delta x/ \Delta t)$, while in the implicit case $u_i^{n+1} - u_i^n = (u_{i+1}^{n+1} - u_{i-1}^{n+1})/(2 \Delta x/ \Delta t)$; what method is used to solve the original partial differential equations numerically, e.g. the finite difference method, the characteristic method and the finite element method. The most frequently used method is the finite difference method (FDM). This method is based on the direct approximations of the original partial differential equations to the difference equations, i.e. the finite difference approximations (Richtmyer and Morton, 1967). The characteristic method (CM) is also frequently used. In this method, the characteristic differential equations derived from the original partial differential equations are solved numerically (Hoskin, 1964; Richardson, 1964; Sauerwein, 1966). The finite element method (FEM) has not yet been used in solar physics (and probably in astrophysics) except for one case (Sakurai, 1976). This method is based on a variational teqnique known as Ritz's method (Strang and Fix, 1973). Although there are some methods other than these (e.g. Beam Scheme; see also Leibacher and Stein, 1975), almost all the methods used in solar physics can be classified into these three (see Table I).

The finite difference methods are also classified into many different schemes, i.e. the Lax-Wendroff (LW) scheme (Richtmyer and Morton, 1967), modified Lax-Wendroff (MLW) scheme (Rubin and Burstein, 1967), leapfrog scheme (see Roche, 1972), Flux Corrected Transport (FCT) scheme (Boris and Book, 1973; Book, Boris and Hain, 1975; Boris and Book, 1976) and etc. These schemes belong to the explicit and Eulerian ones in the finite difference methods.

2.3. Problems in Numerical Simulations

First of all, it must be emphasized that there is no almighty scheme, while there are many numerical schemes as shown in Section 2.2. Therefore we are faced with the problem of the selection of the scheme. The usual selection rule is as follows; (1) accuracy, (2) stability, (3) small computational time and (4) easiness of programing. The finite difference method satisfies this selection rule to a certain extent and has been used frequently. Therefore in later part of this section we will discuss about the problems appearing in the finite difference methods

The most serious problem in the finite difference methods is that of the numerical instability. For example, in the Lax-Wendroff type scheme which is most frequently used, the numerical oscillations appears around the shock and the contact discontinuity. In some circumstances the amplitudes of these oscillations grow without limit, that is, the instability occurs. In the case of the shock wave these oscillations can be removed by an introduction of the artificial viscosity (Richtmyer and Morton, 1967), but the oscillations around the contact discontinuity cannot be removed. FCT scheme significantly improved these behavors around the shock and the contact discontinuity (Boris and Book, 1973; Weber, 1978). As for the more detailed comparison of many FDM schemes, the reader is reffered to the review paper by Sod (1978).

The numerical instability is also produced by the reflection of the wave at the open boundary. Even if the instability does not occur, the reflection wave may affect the inner region of the numerical simulation. Therefore the boundary condition is very important (even for the characteristic method and the finite element method).

Mathematically a part of this problem is reduced to the mixed initial and boundary value problem. That is, the boundary condition must be compatible with the inner region. In the case of the one dimensional hyperbolic partial differential equations (e.g. compressible, inviscid and adiabatic MHD equations), this problem can be solved by the characteristic relations (Courant and Hilbert, 1962). Chu and Sereny (1974) applied these relations to the finite difference method. Nakagawa and Steinolfson (1976) introduced these relations to astrophysics and called them the compatibility relations. In the case of the two or three dimensional hyperbolic partial differential equations, however, the mixed problem has not yet been resolved mathematically (Nogi, 1979), although the characteristic relations can be formally constructed (e.g. Jeffrey and Taniuti, 1964; Richardson, 1964; Sauerwein, 1966; Shibata, 1979). The difficulty in multi-dimensional problem is originated from the fact that the chracteristic curve becomes the characteristic surface. From the same reason and the complexity of the formal characteristic relations, the multi-dimensional characteristic method (Richardson, 1964; Sauerwein, 1966) is not frequently used.

It should be noted that even the compatibility relations in one dimension cannot cmpletely remove the reflection of the wave at the open boundary. Hedstrom (1979) investigated a nonreflecting boundary condition for one dimensional hyperbolic equations. Rudy and Strikwerda (1980) also studied it for Navier-Stokes equations (see also Sundstrom, 1975; Orlansky, 1976; Gustafsson and Kreiss, 1979).

3. Applications to Solar Active Phenomena

In spite of the immaturity of Numerical (Magneto-) Hydrodynamics as shown in the previous section, many simulations have been performed about solar active phenomena during the recent ten years. These simulations can be classified into following categories;

(1) sunspot and related phenomena---interaction between magnetic field and convection (2MHD) etc.

(2) acoustic waves and shocks in the photosphere and the chromosphere (1HD)

(3) jet phenomena---spicules and surges (1HD)

(4) flare---thermal evolution (1HD) etc.

(5) coronal disturbances---MHD waves in the corona (2MHD) etc.

(6) others

(Some contents of them are summarized in Table I. Note that Table I does not include the numerical simulations of the dynamical phenomena associated with the solar wind because too many simulations have been performed in this field.) Since there is no space to review the whole contents of these simulations, we will confine ourselves to categories (2) and (3) in the following subsections.

3.1. Acoustic Waves and Shocks in the Photosphere and the Chromosphere

Although the title of this subsection does not seem to be related to active phenomena, the waves and the shocks are fundamentally important for understanding of the physics of active phenomena (see Section 3.2). Therefore we will present a simple review of the simulations in this category.

The first numerical simulation in this category was performed by Stein and Schwartz (1972). They computed the one dimensional vertical propagation of the single acoustic pulse including effects of radiation and ionization approximately, and compared the results with weak shock theory. Figure 1 shows a typical example of the acoustic wave propagation in the photosphere and the chromosphere, which is reproduced from the paper of Stein and Schwartz (1972). This figure includes almost all the fundamental physics of one dimensional propagation of acoustic waves and shocks in a stratified atmosphere. First, one sees in this figure that there is the formation of the shock wave from the finite amplitude acoustic wave. This is the well-known nonlinear effect. Second, the growth of the amplitude of the acoustic waves and shocks is found from this figure. This is also the well-known property of acoustic wave propagation in a gravitationally stratified isothermal atmosphere. (Note that in the solar photosphere and the chromosphere the temperature can be assumed to be constant within an error of factor two.) Third, the most interesting result in Figure 1 is the matter ejection by the passage of the shock wave in the upper chromosphere. Since the code used by Stein and Schwartz is the Lagrangian code, one will see in this figure that the matter

Table I Numerical Solar (Magneto-) Hydrodynamics

author (year)	dimension[1] HD or MHD	geometry[2]	energy[3] eq.	scheme[4]	level[5]	fundamental physics; application
[sunspot and related phenomena]						
Altschuler et al. (1968a,b)	2MHD(I)	r-z	T=const	?	ph	motion of ring current; Evershed motion, surge
Meyer et al. (1974)	2MHD(B)	x-y	cond.	Eu.Ex.	conv	interaction between magnetic field and convection, decay of sunspot, formation of intense fluxtube, running pennumbral waves
Peckover, Weiss (1978)	2MHD(B)	x-y	T=given	Eu.Ex.	conv	
Galloway (1978) Galloway, Moore (1979)	2MHD(B)	r-z	cond.	Eu.Ex.	conv	
Schüßler (1979a)	2MHD	x-y	T=const	FCT	conv	rising motion of horizontal flux tube due to magnetic buoyancy
Schüßler (1979b)	2MHD	x-y	T=given	FCT	conv	nonlinear dynamo
Cloutman (1979)	2HD	x-y	cond.	Eu.Im.	conv	convection; granulation
Shibata (1980)	1HD	along closed field (S≠const)	T=given	MLW	ph-ch	downflow in a rising flux tube; birth of sunspot
[acoustic waves and shocks in the photosphere and the chromosphere]						
Stein, Schwartz (1972, 1973)	1HD	r	rad.	Lag.Ex.	ph-ch	shock; heating of chromosphere
Kneer, Nakagawa (1976)	1HD	x	rad.	Eu.Im.	ph-ch	shock; radiative hydrodynamics; heating of chromosphere
Ulmschneider et al. (1977) Kalkofen, Ulmschneider (1977) Ulmschneider, Kalkofen (1977) Hammer, Ulmschneider (1978) Ulmschneider et al. (1978)	1HD	x	rad.	CM	ph-ch	radiative damping of acoustic waves, shock; heating of chromosphere
Gouttebroze, Leibacher (1980)	1HD	?	?	?	ph-ch	waves, oscillations; formation of Mg II K and Ca II K lines

Table I (continued)

author (year)	dimension HD or MHD	geometry	energy eq.	scheme	level	fundamental physics; application
[jet phenomena]						
Bessey, Kuperus (1970)	1HD	x	heating source fixed to mass	Lag.Ex.	ph-ch	shock, thermally driven gas motion
Steinolfson et al. (1979)	1HD	r	ad.	MLW	ch-co	mass ejection by the pressure gradient force; surge
Suematsu et al. (1980) } Shibata et al. (1980) }	1HD	x	ad.	MLW	ph-co	mass ejection by the shock wave and the pressure gradient force; spicule, surge
[flare]						
Hirayama, Endler (1975)	1HD	x	cond.	?	ch-co	conduction front, flare
Kostyuk, Pikelner (1975)	1HD	x	cond.	Lag.Im.	ch-co }	conduction front, downward directed shock, heating by high energy electrons; flare
Kostyuk (1976)	1HD	along closed field (S=const)	cond.	Lag.Im.	ch-co }	
Craig, McClymont (1976)	1HD	x	cond.rad.	Lag.Im.	co(pr)	conduction front, shock; heated flare filament
Henoux, Nakagawa (1978)	1HD	x	rad.	Lag.Im.	ph-ch	chromospheric response by irradiation of soft x-ray; flare
Antiochos, Krall (1979)	1HD	along closed field ($S\neq$const)	cond.rad.	Lag.Im.	tr-co	conduction front, evapolation of chromospheric matter; flare
Nagai (1979)	1HD	along closed field (S=const)	cond.rad.	Eu.Im.	ph-co	conduction front, evapolation, downward directed shock; flare
Krall et al. (1980)	1HD	along closed field ($S\neq$const)	cond.rad.	Lag.Im.	tr-co	conduction front, evapolation; long decay X-ray event
Smith, Auer (1980)	1HD			FCT		

Table I (continued)

author (year)	dimension HD or MHD	geometry	energy eq.	scheme	level	fundamental physics; application
[flare (continued)]						
Nakagawa et al. (1976)	2MHD	x-y	ad.	MLW	co	coronal response by emerging flux; infall impact theory of flare
Sokolov et al. (1977) } Sokolov, Kosovichev (1978) }	1MHD	x	joule heating	?	ph ch	instability due to local joule overheating; flare
Zeitsev et al. (1978)	1MHD	x		?	co	turbulent shock wave across magnetic field; moving cloud
[coronal disturbances]						
Nakagawa et al. (1975)	1HD	r	ad.	MLW	co }	acoustic shock, coronal response by mass ejection; coronal transients
Wu et al. (1975)	1HD	r	friction	MLW	co }	
Steinolfson, Nakagawa (1976,1977)	1HD	r	ad.	MLW	co }	
Smith et al. (1977)	2MHD	x-y	ad.	MLW	co }	MHD wave (fast mode), coronal response by flare or mass ejection; coronal transients
Nakagawa et al. (1978) } Wu et al. (1978) }	2MHD	r-ϕ(eq)	ad.	MLW	co }	
Steinolfson et al. (1978) } Dryer et al. (1979) }	2MHD	r-θ(mer)	ad.	MLW	co }	
[others]						
Browne, Bessey (1973)	1HD	x	rad.cond.	LW	ch-co	dynamical model of corona
Hildner (1974)	2MHD	x-y	rad.	LW	co	thermal instability; prominence formation
Kopp, Pneuman (1976)	1HD	along open & closed field ($S \neq$ const)	T=const	CM	co	flow in moving magnetic flux tube; formation of loop prominence (post flare loop)

Table I (continued)

author (year)	dimension HD or MHD	geometry	energy eq.	scheme	level	fundamental physics; application
[others]						
Sakurai (1976)	3MHD		ad.	FEM	co(pr)	screw-mode instability of magnetic flux tube; eruptive prominence
Weber (1979)	2MHD	x-y	ad.	FCT	co	vertical neutral sheet; coronal streamer
Antiochos (1980)	1HD	along closed field ($S \neq$ const)	cond.rad.	Lag.Im.	co	thermal instability; formation of loop prominence (post flare loop)

1; e.g. 2MHD=2dimensional MHD, I=incompressible fluid, B=Boussinesq fluid

2; r-z=cylindrical geometry, x-y=cartesian, r=spherical, x=cartesian, S=cross section, eq=equatorial plane, mer=meridional plane

3; cond.=thermal conduction, rad.=radiation loss, ad.=adiabatic

4; Eu.=Eulerian, Lag.=Lagrangian, Ex.=Explicit, Im.=Implicit
FCT=Flux Corrected Transport Scheme, LW=Lax Wendroff scheme, MLW=Modified Lax Wendroff scheme, CM=Characteristic Method, FEM=Finite Element Method

5; ph=photosphere, ch=chromosphere, co=corona, tr=transition region, conv=covection zone, pr=prominence which is initially located at the height of about 2000 km is ejected by the shock wave up to the height of more than 5000 km. This matter ejection is fundamentally the same as the spicule model of Suematsu, Shibata, Nishikawa and Kitai (1980). Finally, there is another interesting thing in this figure, i.e. a standing wave wake left behind the propagating acoustic wave (Lamb, 1932), which oscillates at the acoustic cut off frequency ($1/\tau_a = c/(4\pi H) \approx 1/200\ s^{-1}$, where c is the sound velocity and H is the scale height). This is due to the dispersive effect of a gravitationally stratified atmosphere (i.e. the dispersion relation for an infinitesimal amplitude wave is that $\omega^2 = k^2c^2 + \omega_a^2$, where $\omega_a = \frac{2\pi}{\tau_a}$). This standing wave wake also becomes the shock wave at greater heights because of the nonlinear effect, and this recurrent shock waves produce the recurrent matter ejections. These results are also shown in the spicule model of Suematsu et al. (1980).

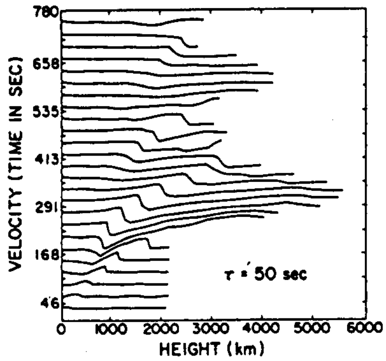

Fig. 1. Typical example of the acoustic wave propagation in the solar atmosphere, reproduced from Stein and Schwartz (1972).

After the computations of Stein and Schwartz (1972, 1973), many simulations have been performed in relation to the acoustic wave propagation in the solar atmosphere, improving the numerical scheme and the treatment of the radiation process (Kneer and Nakagawa, 1976; Ulmschneider et al., 1977; Kalkofen and Ulmschneider, 1977; Ulmschneider and Kalkofen, 1977; Hammer and Ulmschneider, 1978; Ulmschneider et al., 1978). However, the fundamental physics of the acoustic wave (and shock) propagation in the chromosphere is not significantly altered by these improvements because the radiative relaxation time is very large ($\gtrsim$ 100 sec) in this region (e.g. Giovanelli, 1978).

3.2. Jet Phenomena (Spicules and Surges)

Until quite recently, there was no time dependent jet model which agrees well with observations, although some simulations have been performed in order to account for surges (Altschuler et al., 1968) and spicules (Bessey and Kuperus, 1970).

Recently, however, the numerical simulations of the jet phenomena significantly developed. Steinolfson et al. (1979) presented the time dependent surge models for the first time, by performing one dimensional hydrodynamic simulations on the assumption of adiabtic motion (Figure 2). In their models surges are constructed by the pressure gradient force which is the consequence of a sudden increase in pressure at the top of the chromosphere. On the other hand, Suematsu et al. (1980) presented the nonsteady spicule model for the first time, by using the similar method and assumptions to those of Steinolfson et al. (1979). In Suematsu et al.'s model, as described in Section 3.1, the jets are produced by the shock wave which is originated from a sudden appearance of the bright point in the photosphere or the low chromosphere. Therefore formation mechanism of the jet in the spicule model of Suematsu et al. (1980) is essentially different from that in the surge model of Steinolfson et al. (1979). Shibata et al. (1980) paid attention to this difference and computed various jet models whose roots

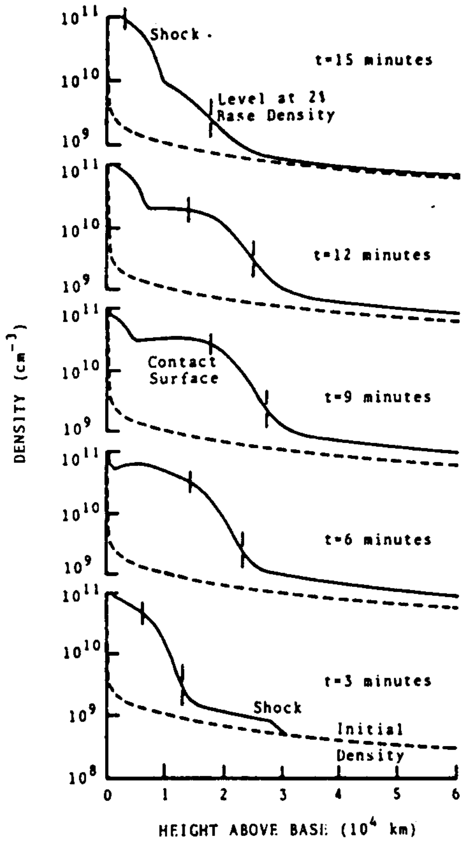

Fig. 2. Surge model of Steinolfson et al. (1979)

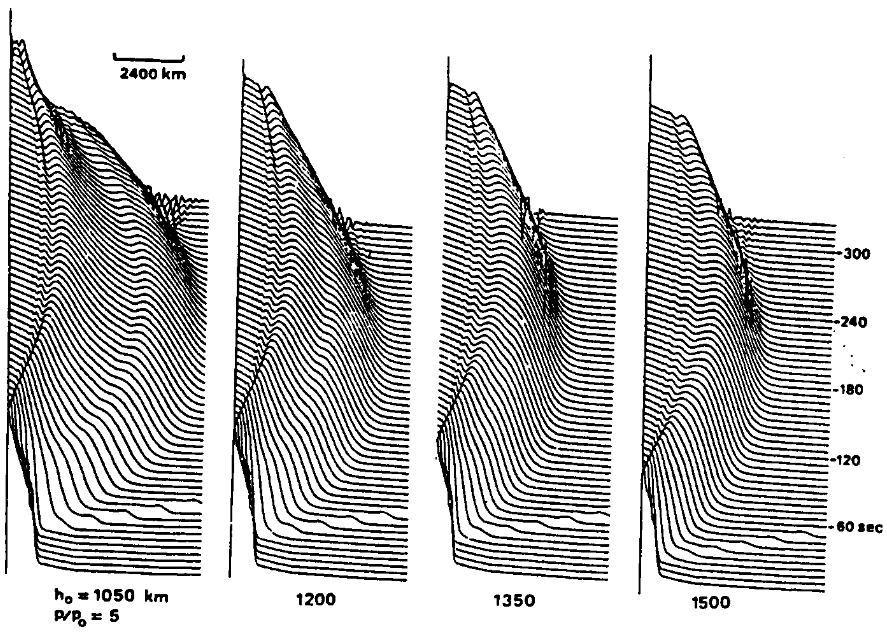

Fig. 3. Various jet models whose roots are located at various heights (h_0) in the chromosphere. The figures show the density.

(bright points or explosions) are located at various heights in the chromosphere, by performing one dimensional hydrodynamic simulations. The results are summarized as follows. The jets can be generally classified into two types. If the sudden pressure enhancements (bright points or explosions) occur below the <u>middle chromosphere</u>, the jets are constructed by the <u>shock wave</u>. Otherwise the jets are constructed by the <u>pressure gradient force</u>.

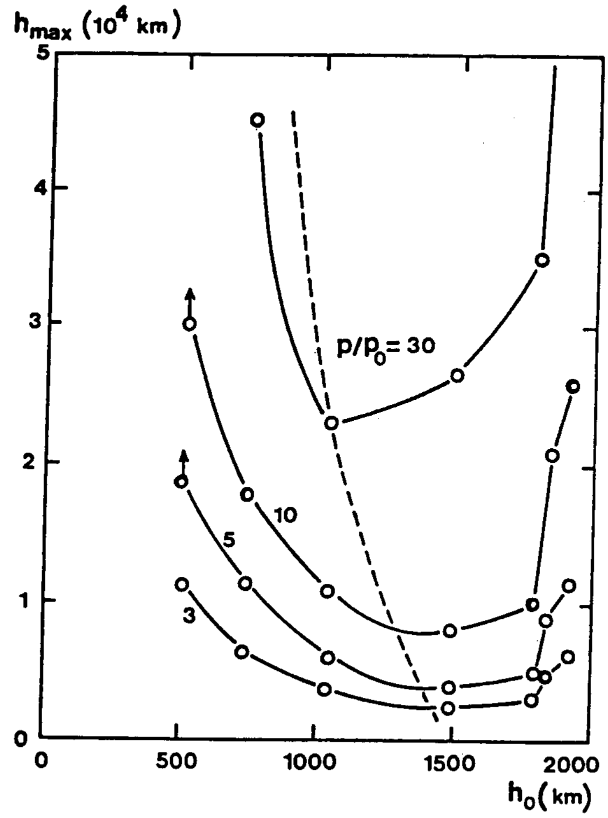

Fig. 4. Maximum heights of various jet models. The dashed curve denotes the critical heights (h_c) which separates two types of jet.

Figure 3 shows the example of the simulation results of Shibata et al. (1980). The h_0 is the height of the base of the model atmosphere, which is measured from $\tau_{5000} = 1$. The p/p_0 is the strength of the pressure increase at h_0. From this figure we see that there are two contact surfaces in the case of $h_0 = 1050$ km. The lower contact surface is ejected by the pressure gradient force, and the upper one is ejected by the shock wave. Since the upper contact surface corresponds to the top of the jet, it can be said that the jets in this type are produced by the shock wave. On the other hand, in the case of $h_0 = 1500$ km there can be seen only one contact surface, which is ejected by the pressure gradient force. The critical height (h_c), which separates two types, is about 1400 km for $p/p_0 = 5$. Figure 4 shows the maximum height of the jet as a function of h_0 for $p/p_0 = 3$, 5, 10 and 30. It should be noted that the local minimum (dashed line) corresponds to the critical height, i.e. in the right hand side of the dashed line the jets are produced by the pressure gradient force, and in the left hand side the jets are produced by the shock wave. The behavior of h_{max} curve can be qualitatively understood as follows. For $h_0 > h_c$, the jet materials are accelerated by the pressure gradient force per unit mass ($\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$) near the base. Since the density (ρ) decreases with h_0, $\frac{dv}{dt}$ increases with h_0. Hence h_{max} increases with h_0. For $h_0 < h_c$, the jet matters are accelerated by the shock wave. The strength of the shock wave decreases with h_0 for fixed p/p_0, because the growth of the shock wave decreases with decreasing the region of the shock propagation in the chromosphere (i.e. $h_{tr} - h_0$ decreases with h_0, where h_{tr} is the height of transition region). The h_{max} decreases with decreasing the shock strength. Thus h_{max} decreases with increasing h_0. It must be emphasized that the critical height ranges from 1000 km to 1500 km for $3 \lesssim p/p_0 \lesssim 30$. Hence, it is concluded that if the bright point (or explosion) occurs below 1000 km (middle chromosphere) the jets are constructed by the shock wave. This means that small surges associated with Ellerman bombs (Bruzek, 1974) are produced by the shock wave, because Ellerman bombs take place in the low chromosphere (Roy and Leparskas, 1973).

Finally we will mention that these two types of jet can be understood by the simple hydrodynamical principle. Figure 5 (a) and (b) represents the schematic

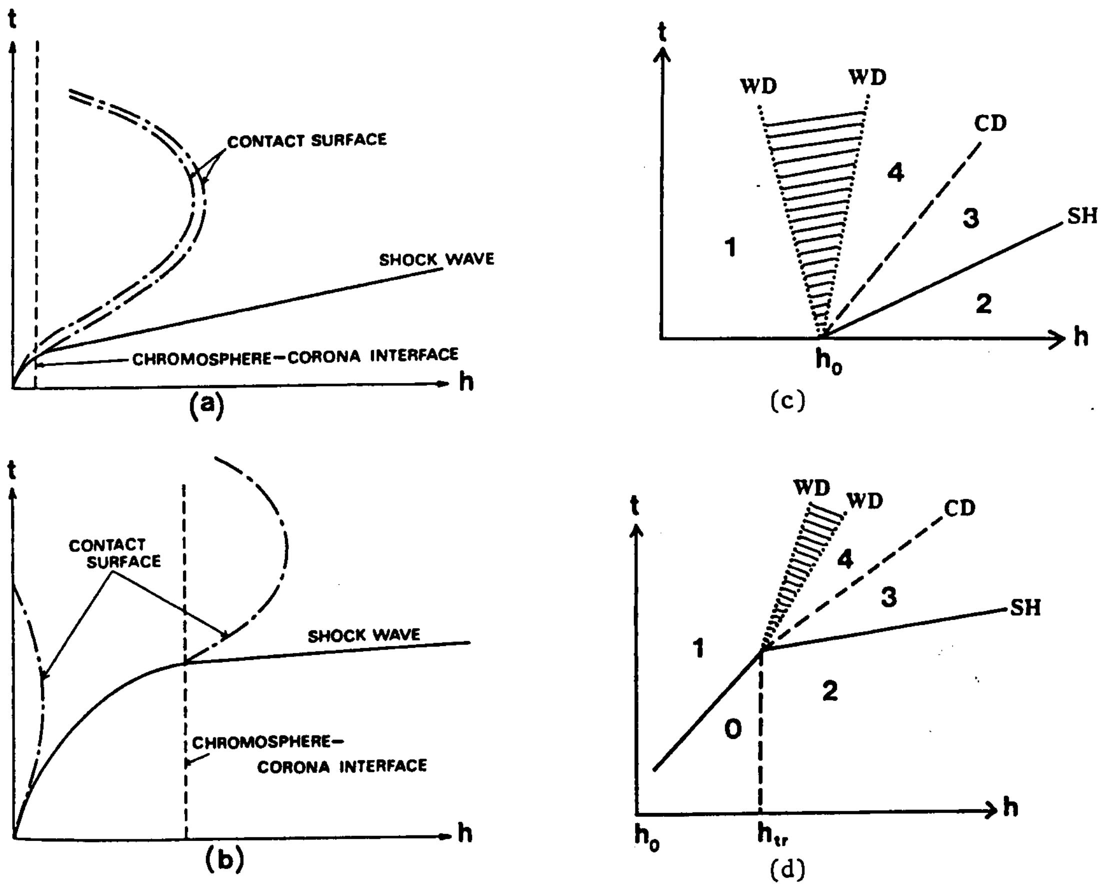

Fig. 5. Schematic diagram of the propagation of the contact surface and the shock wave for both types ((a) and (b)) and the idealized model of the initial discontinuity for both types ((c) and (d)).

diagram of the propagation of the contact surface and the shock wave for both types. An essential hydrodynamical structure of the jet in Figure 5 (a) is the same as that of the shock tube (Figure 5 (c)) if the gravity is neglected. On the other hand in the case of Figure 5 (b) the shock wave which has propagated through the chromosphere plays an important role for the formation of jet, and the final ejection of matter is determined from the collision of the shock wave with the chromosphere-corona interface (a kind of contact surface) as has been already pointed out by Osterbrock in 1961 (see Figure 5 (d)). Although the simulations are highly nonlinear and complex, the initial velocity of the shock propagation and the jet (contact surface) can be quantitatively explained by the idealized model of initial discontinuity in Figure 5 (c) and (d) (Shibata et al., 1980).

4. Prospects for Numerical Solar (Magneto-) Hydrodynamics

We will comment on the future possibility of numerical solar MHD from the view point of the computer's capacity. For example, let us consider whether or not 3MHD simulations of surges with similar method to that of Shibata et al. (1980) are possible. Since 1HD simulations (for the case of 200 grid points and 4000-8000 time steps) required 1-2 min by using Facom M200 at the Data Processing Center of Kyoto University, which is probably one of the most rapid cmputers in the world, 3MHD simulations ($200 \times 200 \times 200 \times$ factor) require more than 4×10^4 min ($\gtrsim 670$hr). It is apparent that 3MHD simulations of surges are impossible at present and even in the near future. (These 3MHD simulations are also inhibited by the storage capacity of the computer.) However 2MHD simulations ($\gtrsim 200$ min) are not impossible. (The intrinsic difficulty in the numerical solar MHD comes from the smallness of the scale height H in the photosphere and the chromosphere, e.g. $H \approx 150$ km at the level of T = 5000 K. As numerical computations require that Δz (grid size) $\lesssim H/10$ because of accuracy and stability, the vertical grid number becomes very large.)

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