How to Determine the Physical Parameters that Govern Wave Dissipation Scales

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1. Motivation - Wave Heating

AIM: To devise a technique for the determination of physical parameters that govern wave dissipation time/spatial scales.

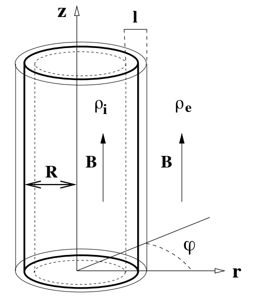
WHY: To assess and quantify the possible role of MHD wave heating in the solar atmosphere.

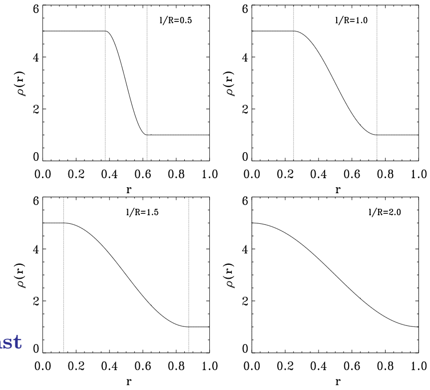
HYPOTHESIS: Observed transverse oscillations show evidence for in situ damping. If interpreted as MHD kink waves, resonant damping offers a plausible explanation. In that case, damping vs. dissipation scales are governed by the cross-field density structuring.

THIS WORK: We present a method to combine wave observations and theory to determine the cross-field density structuring of solar atmospheric waveguides.

2. Cross-Field Density Structuring

PHYSICAL MODEL: Classic one-dimensional waveguide with cross-field density variation over a length-scale l.





Relevant parameters
$\zeta=\rho_i/\rho_e$ density contrast
l/R transverse scale

3. Why/How Relevant

Wave damping and dissipation governed by different time/spatial scales for wave energy transfer → phase mixing → resistive diffusion. Cross-field density determines:

⋆ Resonant damping time/spatial scales

$\tau_{\rm damping}$ and $L_{\rm damping}$ are functions of $(l/R,\ \rho_i/\rho_e)$

⋆ Creation of small scales by phase mixing $L_{\rm pm}=2\pi/(t\|\omega'_A\|)$

⋆ How fast energy is transferred to small length-scales

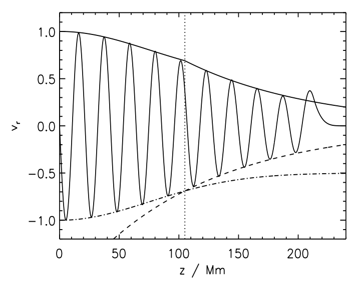
⋆ Onset of resistive diffusion, important when $l_{\rm ra}\sim(R_m\|\omega'_A\|)^{-1/3}$. This scale is reached in a time $t_{\rm ra}\sim R_{\rm m}^{1/3}\|\omega'_A\|^{-2/3}$.

⋆ Energy carried by the wave and fraction of that energy that can be converted into heat

4. Damped Propagating Transverse Waves

Theory predicts the existence of two damping regimes for propagating/standing transverse waves damped in space/time (Pascoe et al. 2013; Ruderman & Terradas 2013).

Radial velocity amplitude along waveguide - spatial damping



Gaussian Regime

$$\frac{L_g}{\lambda}=\left(\frac{2}{\pi}\right)\left(\frac{R}{l}\right)^{1/2}\left(\frac{\zeta+1}{\zeta-1}\right)$$

Exponential Regime

$$\frac{L_d}{\lambda}=\left(\frac{2}{\pi}\right)^2\left(\frac{R}{l}\right)\left(\frac{\zeta+1}{\zeta-1}\right)$$

Height of Change of Regime

$$h=\frac{L_g^2}{L_d}=\lambda\left(\frac{\zeta+1}{\zeta-1}\right)$$

5. Inversion Method - Bayesian Approach

BAYES' THEOREM: $$p(\boldsymbol{\theta}|d)=\frac{p(d|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(d)}$$

$p(\boldsymbol{\theta}|d)$: posterior; $p(d|\boldsymbol{\theta})$: likelihood function; $p(\boldsymbol{\theta})$: prior; $p(d)$: evidence

State of knowledge on model parameters $\boldsymbol{\theta}$ is a combination of what is known a priori independently of the data, $p(\boldsymbol{\theta})$, and the likelihood of obtaining a data realization actually observed as a function of the parameter vector, $p(d|\boldsymbol{\theta})$.

PARAMETER INFERENCE:
How each parameter is constrained by data:
Marginal Posteriors

$$p(\theta_i|d)=\int p(\boldsymbol{\theta}|d)d\theta_1\dots d\theta_{i-1}d\theta_{i+1}\dots d\theta_N$$

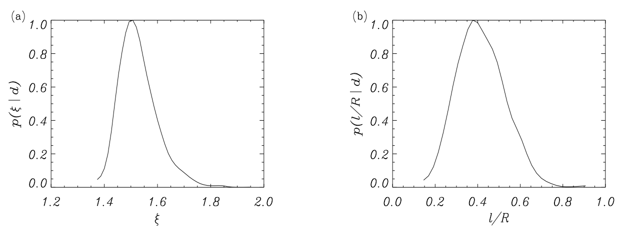
$\theta=(\zeta,\ l/R)$ and $d=(L_{\rm g},\ h)$

6. Inversion Result

Bayesian inversion of cross-field density structuring.
Observations: gaussian/exponential damping length scales.
Unknowns: density contrast ζ, transverse scale l/R.

Arregui, Asensio Ramos, & Pascoe (2013, ApJL 769, L34)

MARGINAL POSTERIORS



$L_g/\lambda=5.0\pm0.1$, $h/\lambda=4.9\pm0.1$ ⟶ $\zeta=1.52^{+0.08}_{-0.05}$, $l/R=0.41^{+0.12}_{-0.10}$

Summary

⋆ Determination of the cross-field density structuring is crucial to assess/quantify role of waves in heating processes.

⋆ The existence of two damping regimes enables to constrain the transverse density structuring in oscillating waveguides.

⋆ Our Bayesian inference tool ensures a consistent inversion, with correct propagation of uncertainty.