

Magnetothermal Instability in the Solar Atmosphere

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We discussed an application of the magnetothermal instability (MTI) to the solar atmosphere. This instability proposed by Balbus (2000) occurs in weakly collisionless plasmas where non-isotropic thermal conduction plays a role in a magnetized atmosphere. Suppose a stratified atmosphere under the gravity with horizontal magnetic fields. When the outward temperature gradient is negative even though it is sub-adiabatic, this instability is triggered. By the given perturbation to the magnetic field lines, the outer gas is heated by the conduction, becomes lighter, and obtains the buoyancy. As it moves outward, this cycle is repeated and suffers from the positive feed-back leading to the instability. The time scale of the maximum growth is given as approximately $\sqrt{H/g}$ where H is the scale height, and g is the gravity. The magnetic field must be weak enough since its tension force contributes as a restoring force.

The solar corona is a dilute hot atmosphere where the thermal conduction is non-isotropic. The MTI is possible to work in the upper corona around a few solar radii above the photosphere where the temperature is decreasing outward and the scale height is about one solar radius. The condition for weak horizontal magnetic field might be satisfied above a closed loop in the lower corona. If the MTI is effective in such regions, it might contribute to generate the waves or perturbations in the solar wind.

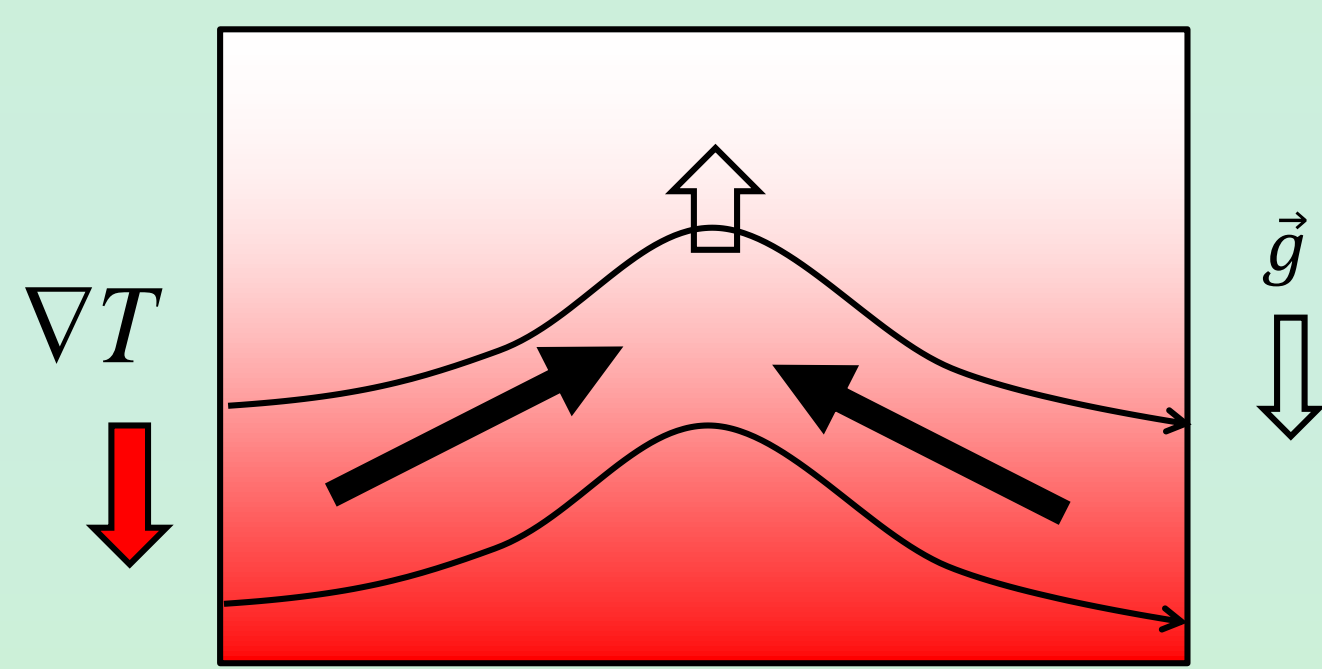
1. Introduction

Magnetothermal Instability: Balbus (2000)

In the initial state, magnetohydrostatic equilibrium with sub-adiabatic stratification. The temperature is larger at the bottom. The weak magnetic field is horizontally imposed. When perturbation is given to lift up the central part of the field, the apex is cooler than the feet, resulting in a heat transport by the conduction along the field lines, making the apex buoyant. As the apex goes up, the field line becomes more vertical leading to the increase of the heat flux due to the increase in temperature gradient, and more heat transport, so there is a runaway.

Purpose of this study

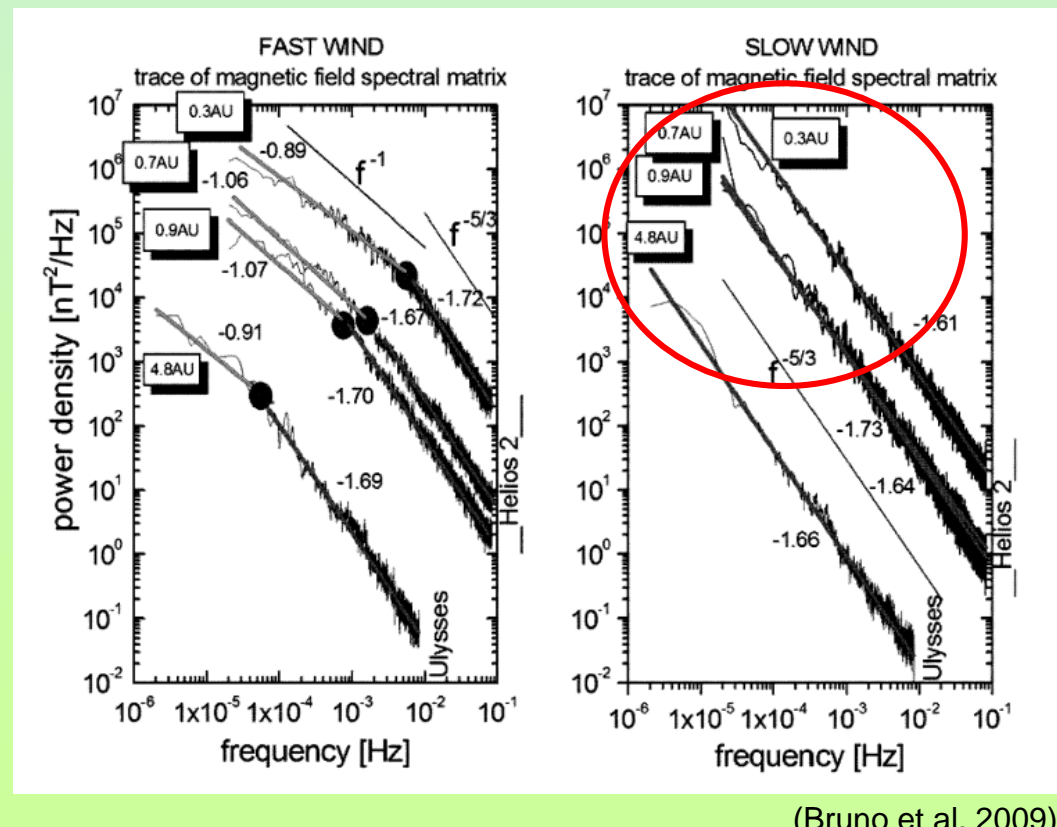
Are the MTI applicable to the solar atmosphere?
If yes, what is the consequence?
We applied the linear analysis results.



(Okoshi & Saito w/ Shiota picture from Shibata & Ohyama 2004)

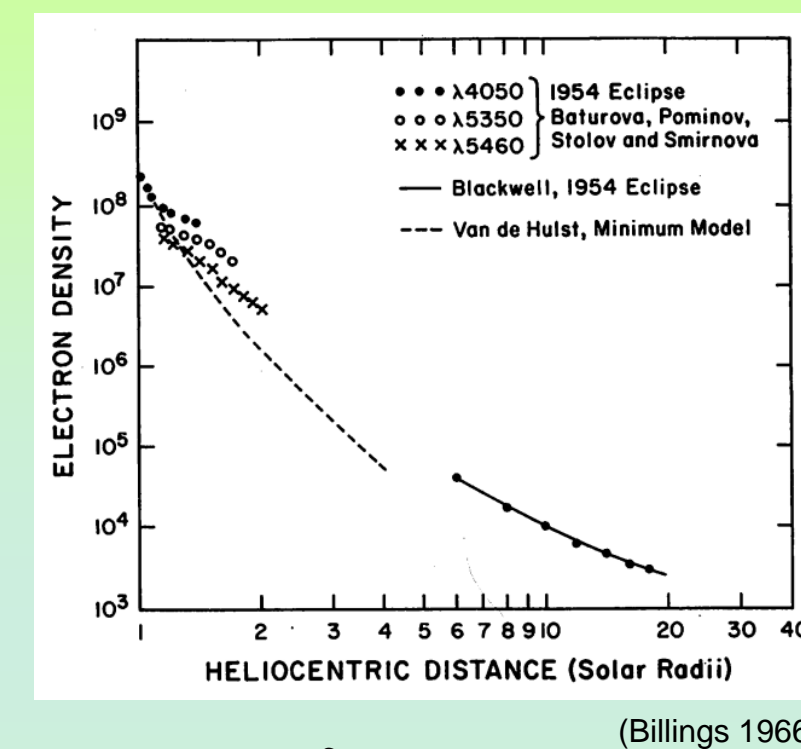
Solar wind turbulence

By the measurements of in-situ observations of spacecrafts, it is known that there exists a turbulence in the solar wind plasmas. It is composed of a superposition of Alfvén waves. The source of the waves is believed to be the solar lower atmosphere. However, the detailed mechanism for the generation is not understood yet.

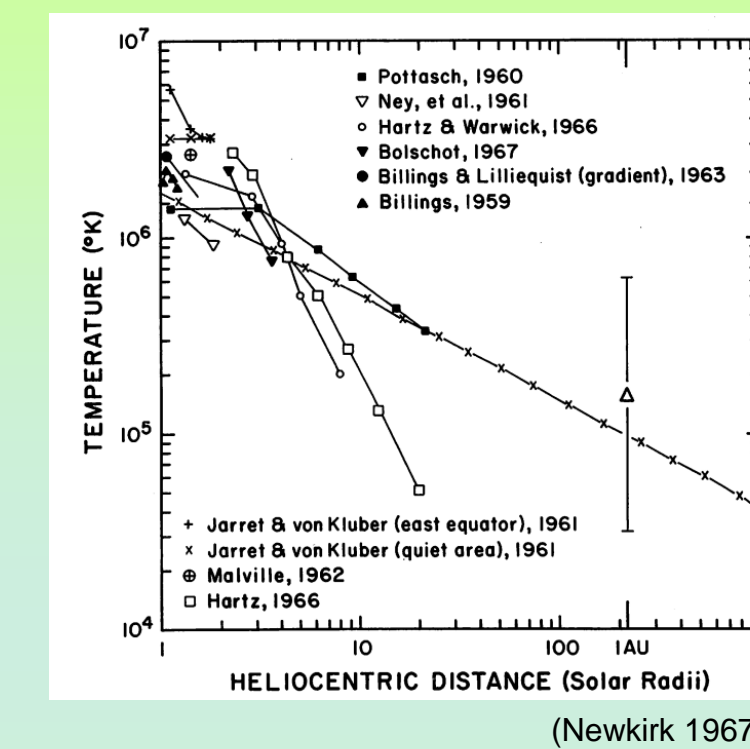


(Bruno et al. 2009)

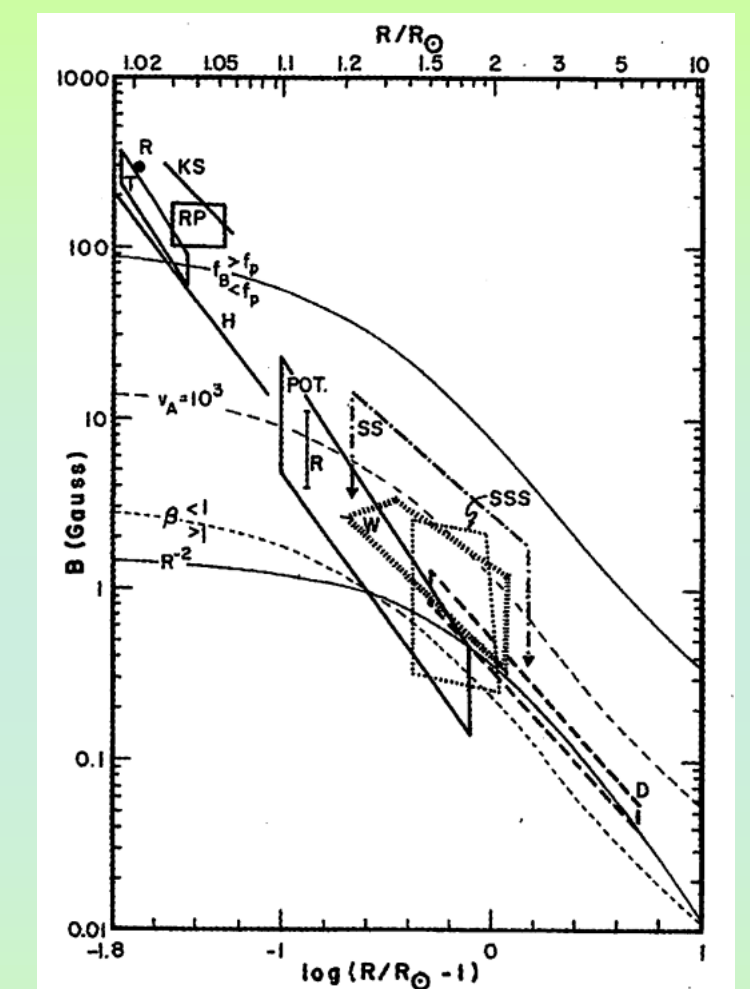
Solar Atmosphere



(Billings 1966)



(Newkirk 1967)



(Dulk & McLean 1978)

$$\frac{n_e}{n_{e0}} \approx \left(\frac{r}{R_s}\right)^{\delta_n} \delta_n \approx -4, n_{e0} \approx 10^8 \text{ cm}^{-3}$$

$$\frac{T}{T_0} \approx \left(\frac{r}{R_s}\right)^{\delta_T} \delta_T \approx -0.5, T_0 \approx 2 \text{ MK}$$

$$\bar{s} \equiv \frac{s}{R} = \delta_s \ln \frac{r}{R_s} \quad \delta_s = \frac{\delta_r}{\gamma - 1} - \delta_n \approx 3.2 \quad \frac{ds}{dr} > 0, \text{ i.e. stable for thermal convection}$$

$$\frac{g}{g_0} \approx \left(\frac{r}{R_s}\right)^{-2} \quad g_0 = 2.7 \times 10^4 \text{ cm}^2/\text{s}$$

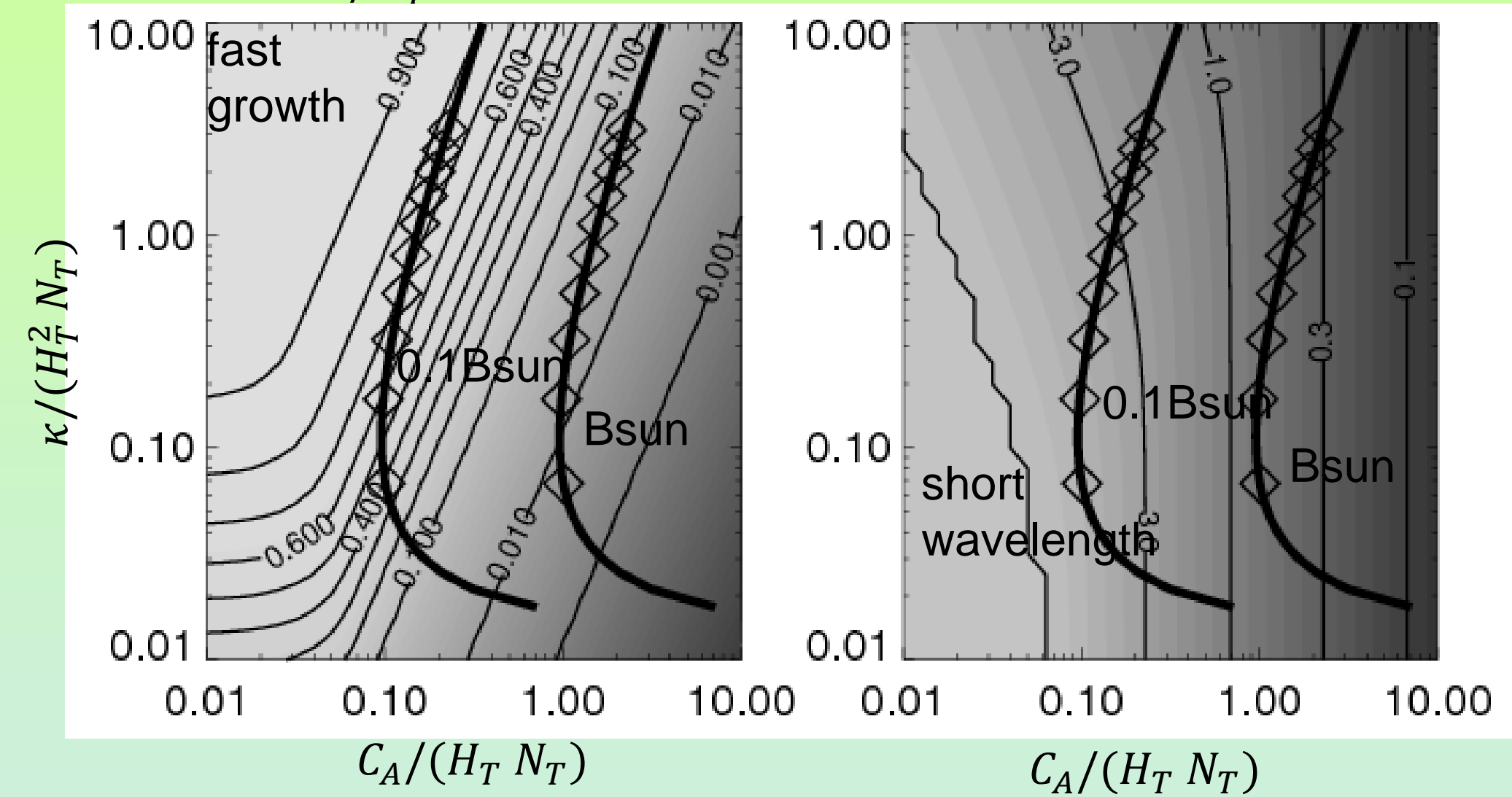
$$\kappa = \kappa_0 \left(\frac{r}{R_s}\right)^{5/2} \left(\frac{n_e}{n_{e0}}\right)^{-1} = \kappa_0 \left(\frac{r}{R_s}\right)^{\delta_\kappa} \quad \kappa_0 = 1.2 \times 10^{17} \text{ cm}^2/\text{s} \quad \delta_\kappa = \frac{5}{2}\delta_r - \delta_n \approx 2.8$$

$$\frac{B}{B_0} = \left(\frac{r}{R_s} - 1\right)^{-1.5} \quad B_0 = 0.5 \text{ G}$$

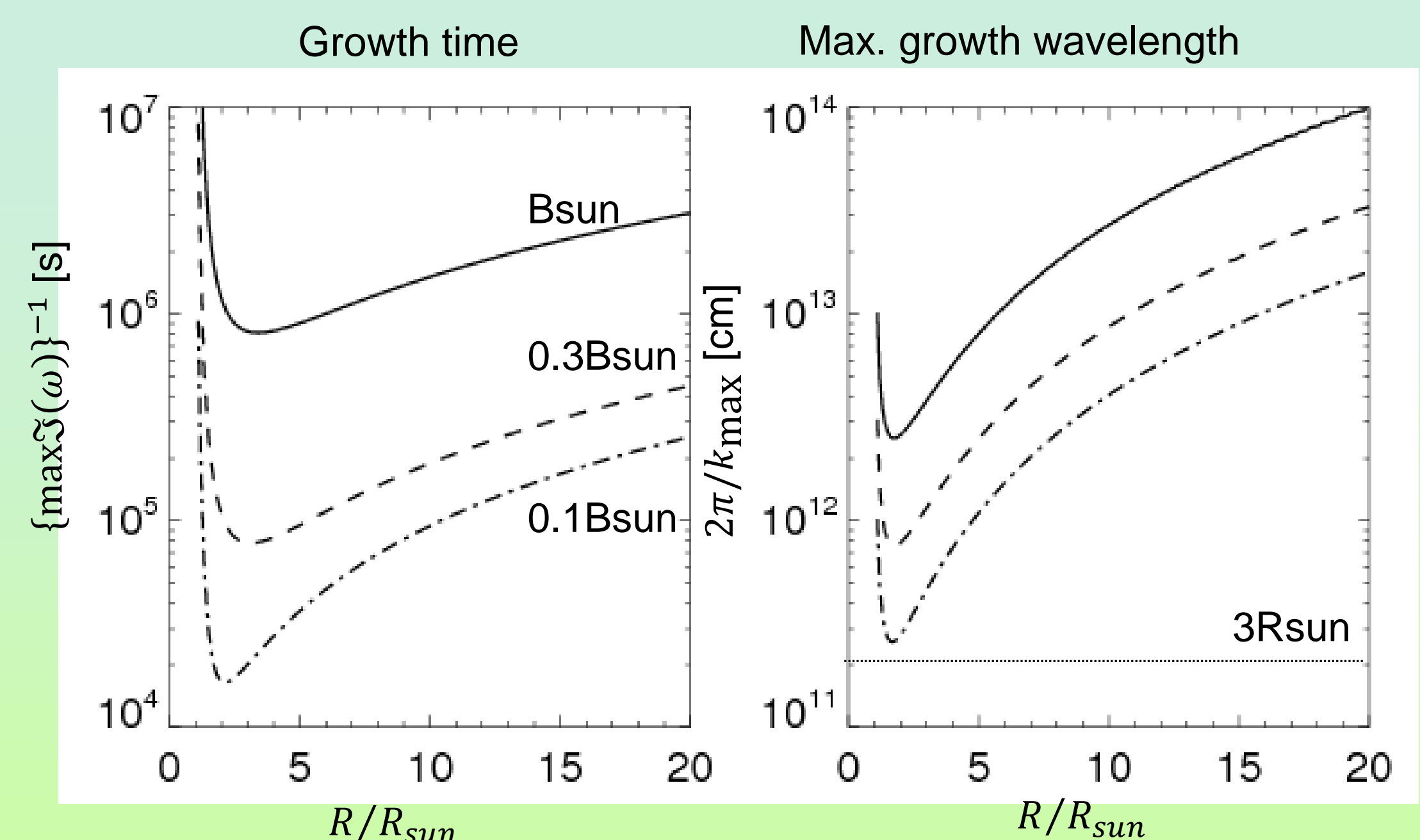
3. Results

Growth rate & wavelength of the fastest growing mode

$$\sin \theta = 1, N^2/N_T^2 = 2.6$$



- For smaller C_A , i.e. weaker B ,
 - growth rate \uparrow , wavenumber \uparrow
- For larger κ ,
 - growth rate \uparrow , wavenumber \downarrow



- The fastest growth occurs around $r=3R_{sun}$.
- The growth of MTI is very slow ($>$ several 10 days) and in too large wavelength ($\sim 30 R_{sun}$) for typical solar B strength.
- If B is weak ($\sim 0.1 B_{sun}$), the growth is possible. Growth time is $\sim 10^4$ sec, and wavelength is $\sim 3 R_{sun}$.

2. Dispersion relation of MTI

According to Quataert (2008), for perturbation as $\exp[-i\omega t + ikx]$, the dispersion relation of magnetothermal instability is

$$0 = \omega \tilde{\omega}^2 + i\omega_c \tilde{\omega}^2 - N^2 \omega \sin^2 \theta + i\omega_c N_T^2 \sin^2 \theta$$

where

$$\tilde{\omega}^2 = \omega^2 - C_A^2 (\vec{k} \cdot \vec{b})^2, \quad \omega_c = \frac{2}{5} \tilde{\kappa} (\vec{k} \cdot \vec{b})^2, \quad N^2 = \frac{\gamma-1}{\gamma} g \frac{ds}{dz}, \quad N_T^2 = -g \frac{d \ln T}{dz}$$

$\tilde{\kappa} = \kappa / (nk_B)$: geometrical thermal conductivity [cm^2/s]

$$\kappa = \kappa_0 T^{5/2}$$

$\vec{b} = \vec{B}/B$: unit vector in B-field direction

$C_A = B/\sqrt{4\pi\rho}$: Alfvén speed

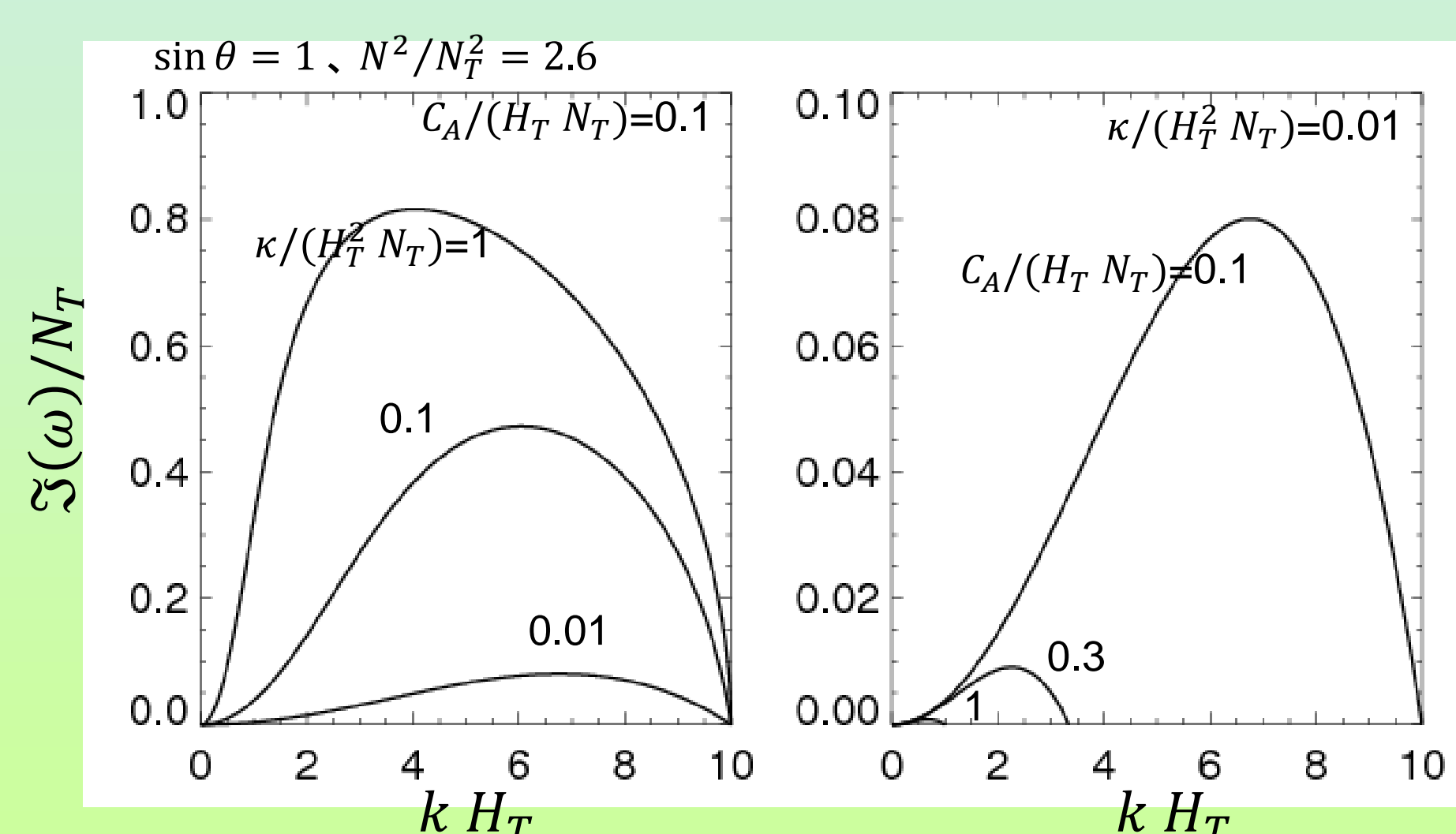
$\bar{s} = s/(k_B/m_p)$: entropy (normalized)

$$\sin^2 \theta = k_{\perp}^2/k^2$$

When conductivity is strong $\omega_c = \frac{2}{5} \tilde{\kappa} (\vec{k} \cdot \vec{b})^2 \gg \omega$, and B-field is weak $\omega^2 \gg C_A^2 (\vec{k} \cdot \vec{b})^2$,
 $\omega^2 \approx -N_T^2 \sin^2 \theta$

where

$$H_T = -\left(\frac{d \ln T}{dz}\right)^{-1}, \quad N_T = \sqrt{g/H_T}$$



4. Summary

We applied the magnetothermal instability (MTI; Balbus 2000, Quataert 2008) to the solar atmosphere for the explanation of the generation of low frequency ($< 10\text{-}4$ Hz) Alfvén waves as a source of MHD turbulence in the slow solar wind in mind.

- The growth rate is maximum around $3R_{sun}$.
- The growth of MTI is very slow ($>$ several 10 days) and in too large wavelength ($\sim 30 R_{sun}$) for typical solar B strength.
- If B is weak ($\sim 0.1 B_{sun}$), the growth is possible. Growth time is $\sim 10^4$ sec, and wavelength is $\sim 3 R_{sun}$.

References

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Bruno et al., 2009, Earth Moon Planet, 104, 101; Dulk & McLean, 1978, Solar Phys., 57, 279
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