Nonequilibrium Dynamics
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DYNAMICS OF RELATIVISTIC HEAVY ION COLLISIONS AND THE QUARK GLUON PLASMA

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Outline

1. Introduction
2. Physics of the quark gluon plasma
3. Relativistic heavy ion collisions
    - Elliptic flow
    - “perfect fluidity”
    - Higher harmonics
4. Some topics in relativistic hydrodynamics

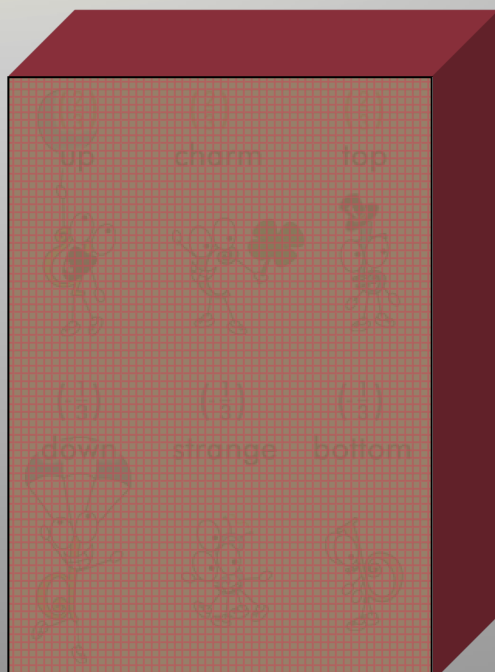
Introduction

- “Condensed” matter physics for elementary particles
- Heavy ion collisions as a playground of non-equilibrium physics in relativistic system
- Recent findings of the quark gluon plasma and related topics

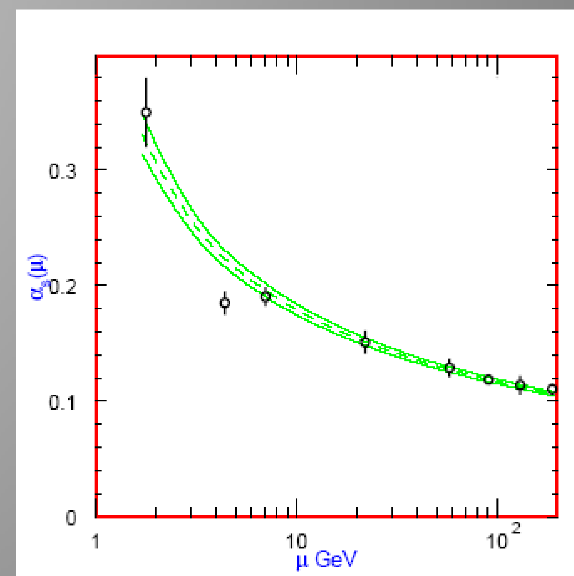
PHYSICS OF THE QUARK GLUON PLASMA

Two Aspects of Quantum ChromoDynamics

QCD: Theory of strong interaction



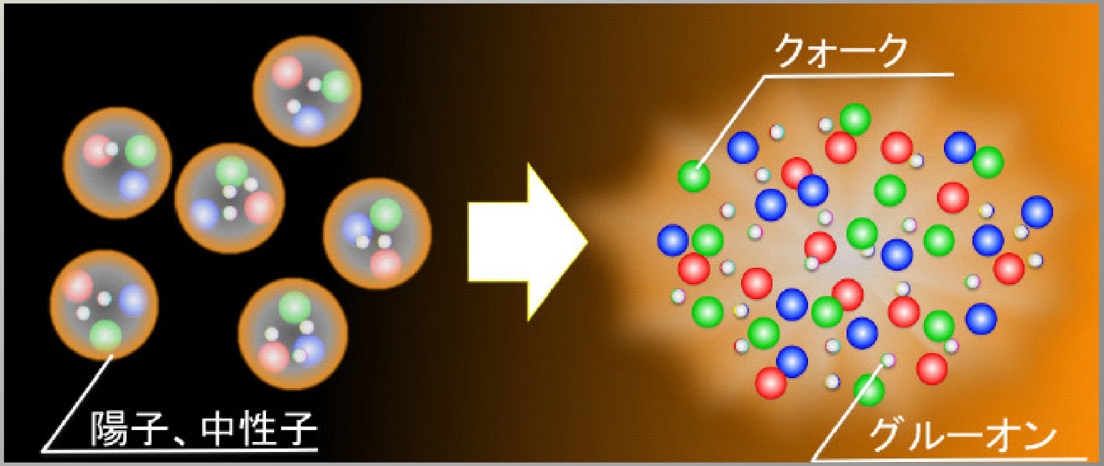
Confinement of color charges



coupling vs. energy scale
Asymptotic free of QCD

What is the Quark Gluon Plasma?

Hadron Gas ➡ Quark Gluon Plasma (QGP)



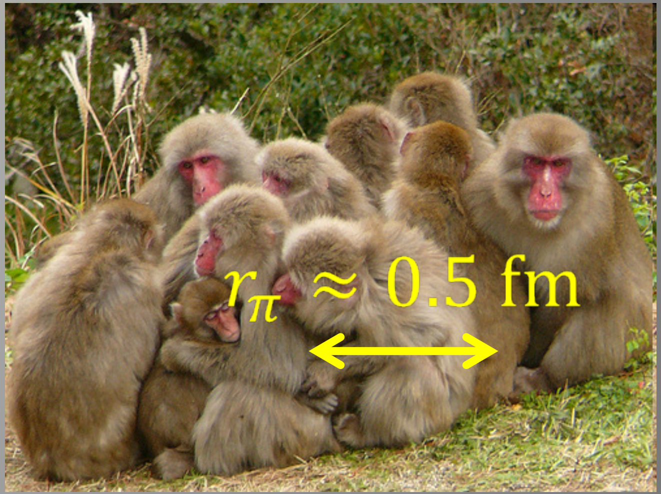
Low ← Temperature → High

Degree of freedom: Quarks (matter) and gluons (gauge)
Mechanics: Quantum ChromoDynamics (QCD)

➡ Novel matter under extreme conditions

How High?

Suppose "transition" happens when a pion (the lightest hadron) gas is close-packed,



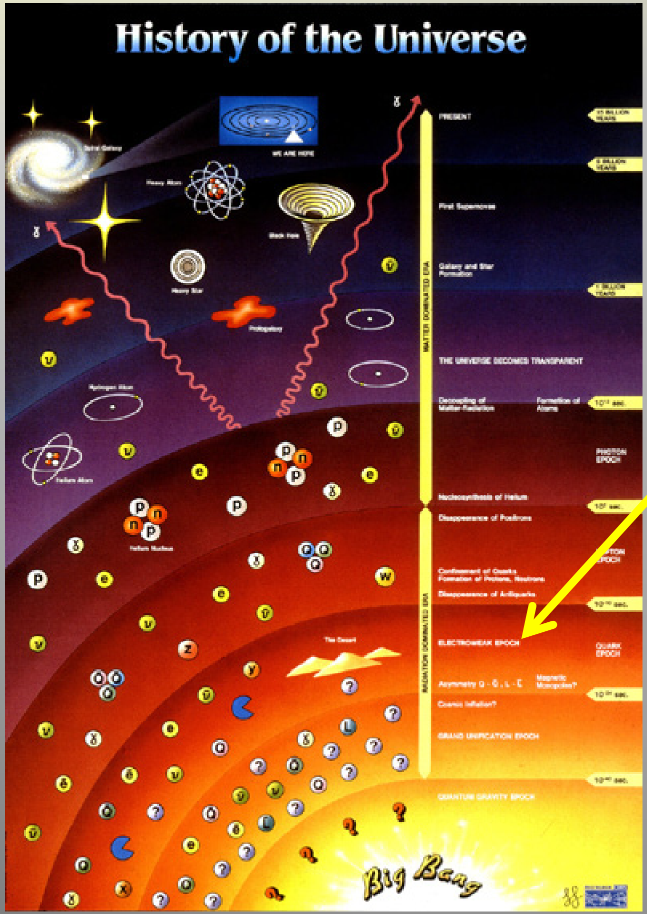
$$n_\pi \approx \frac{1}{\frac{4\pi r_\pi^{\,3}}{3}} \approx 3.6T^3$$

$$\Rightarrow \quad T_C \approx 100 \text{ MeV} \approx 10^{12} \text{ K}$$

Note 1: T inside the Sun ~ 10^7 K

Note 2: T_c from the 1st principle calculation ~2×10^{12} K

Where/When *was* the QGP?



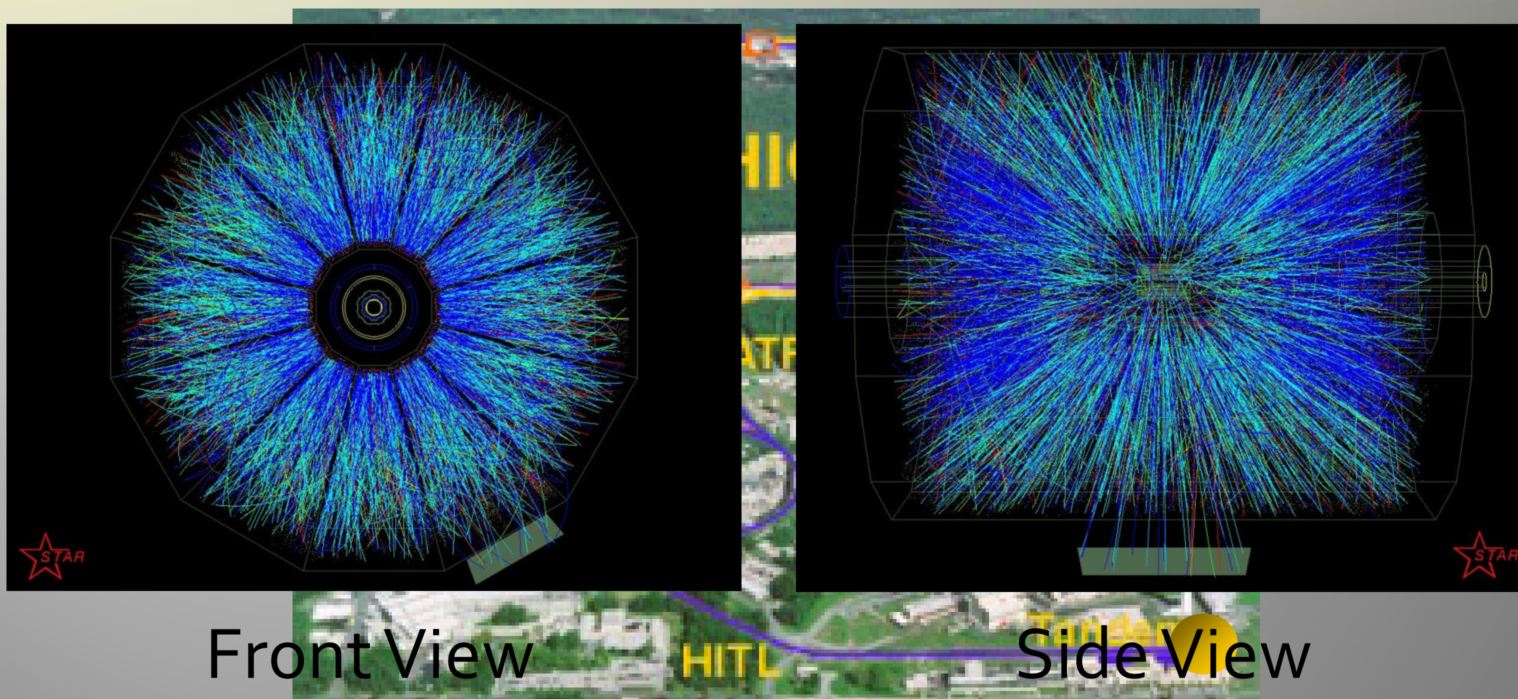
History of the Universe
~ History of form of matter

Micro seconds after Big Bang

Our Universe is filled
with the QGP!

RELATIVISTIC HEAVY ION COLLISIONS

The QGP on the Earth

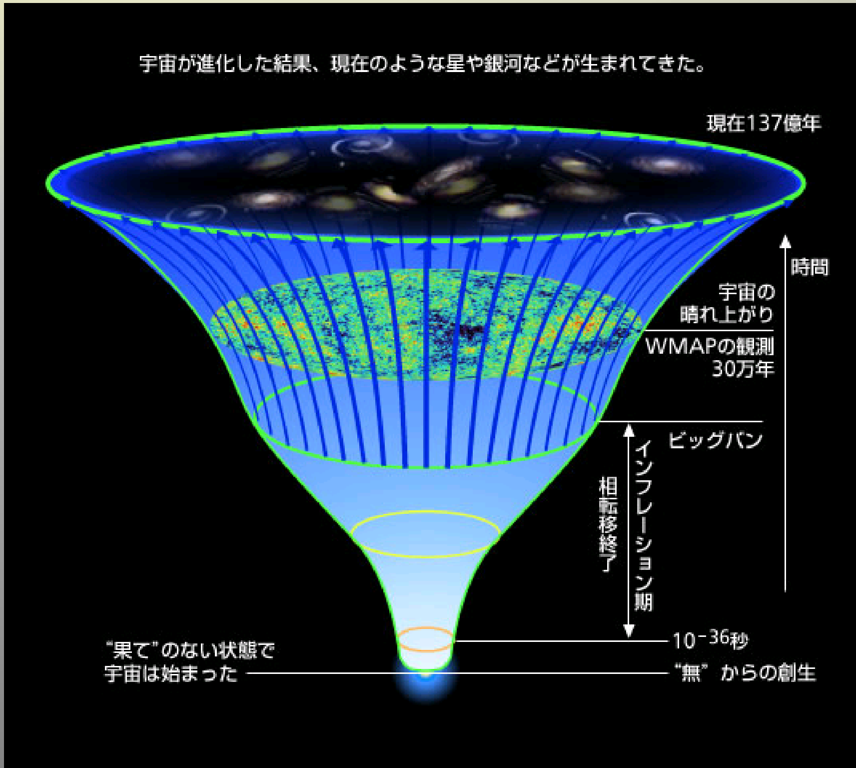


Front View Side View

Relativistic heavy ion collisions

Turn kinetic energy ($v > 0.99c$) into thermal energy

Big Bang vs. Little Bang



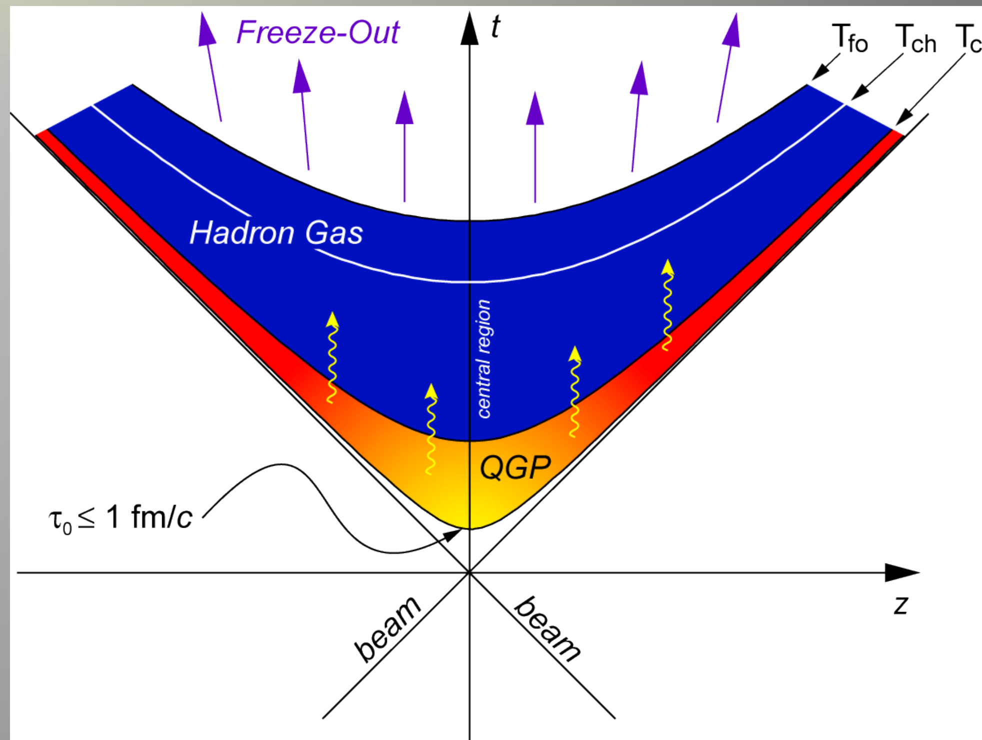


Figure adapted from http://www-utap.phys.s.u-tokyo.ac.jp/~sato/index-j.htm

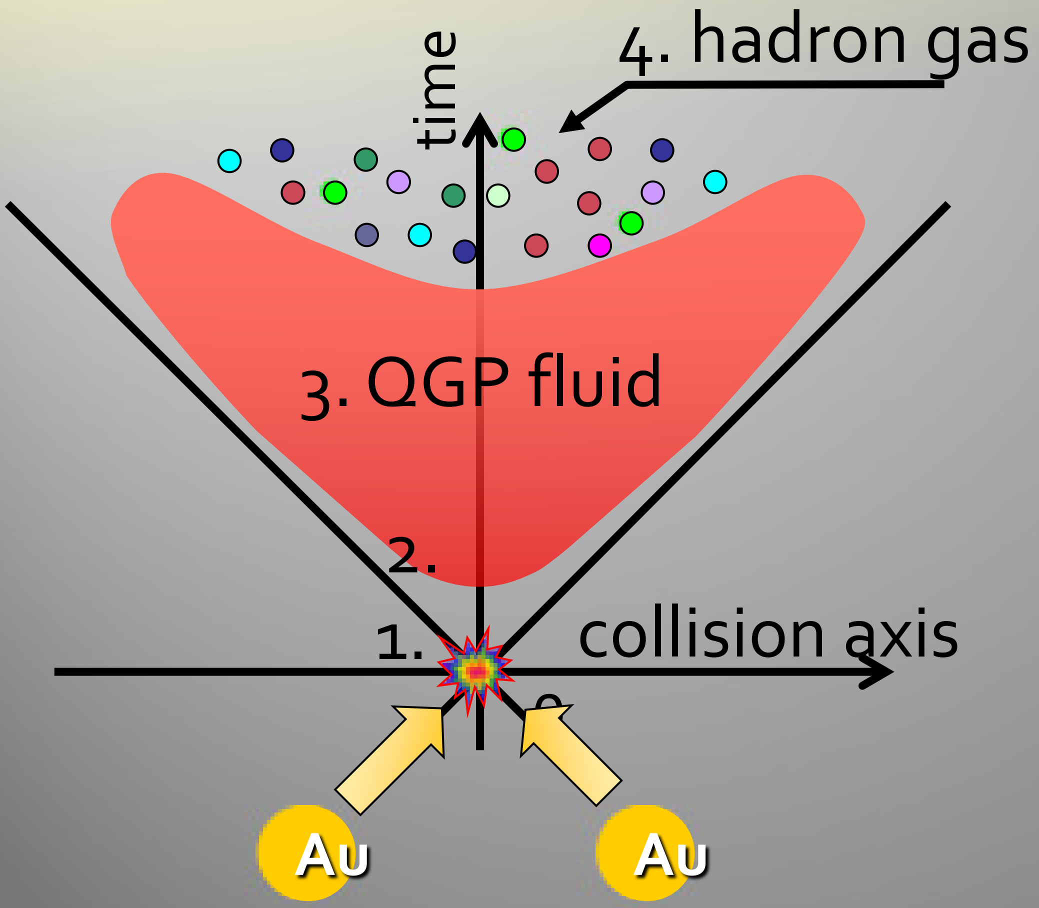
Big Bang vs. Little Bang (contd.)

	Big Bang	Little Bang
Time scale	10^{-5} sec >> m.f.p./c	10^{-23} sec ~ m.f.p./c
Expansion rate	10^{5-6}/sec	10^{22-23}/sec
Spectrum	Red shift (CMB)	Blue shift (hadrons)

Non-trivial issue on thermal equilibration

m.f.p. = Mean Free Path

Non-Equilibrium Aspects of Relativistic Heavy Ion Collisions



4. Kinetic approach for relativistic gases

3. Dissipative relativistic fluids

2. Local equilibration

1. Entropy production

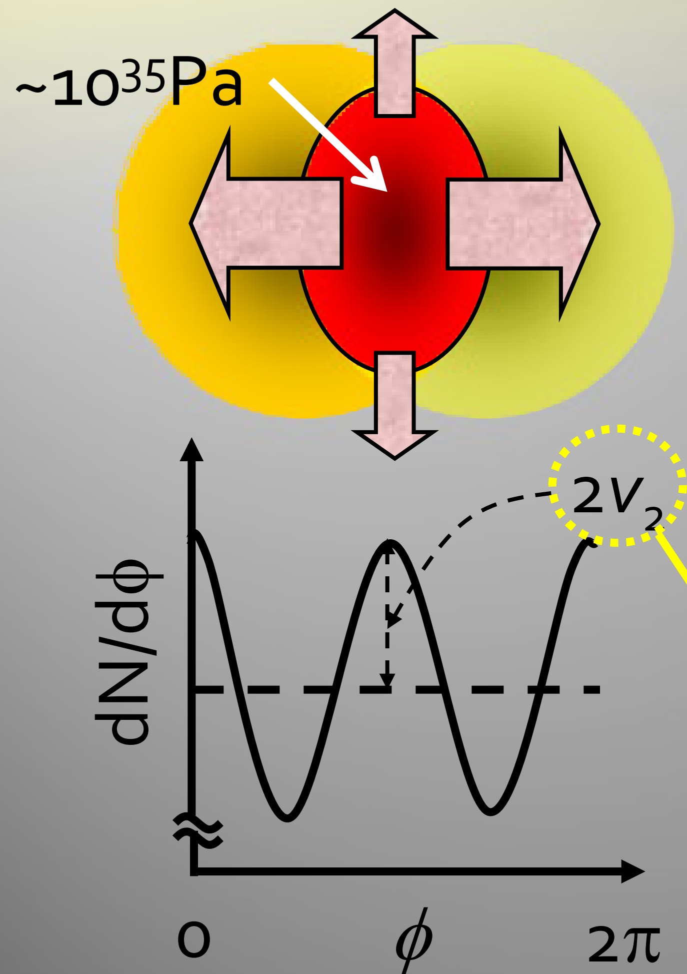
Hydrodynamic Simulation of a Au+Au Collision

http://youtu.be/p8_2TczsxjM

Time scale ~ 10^{-22} sec

YouTube | quark gluon plasma

Elliptic Flow

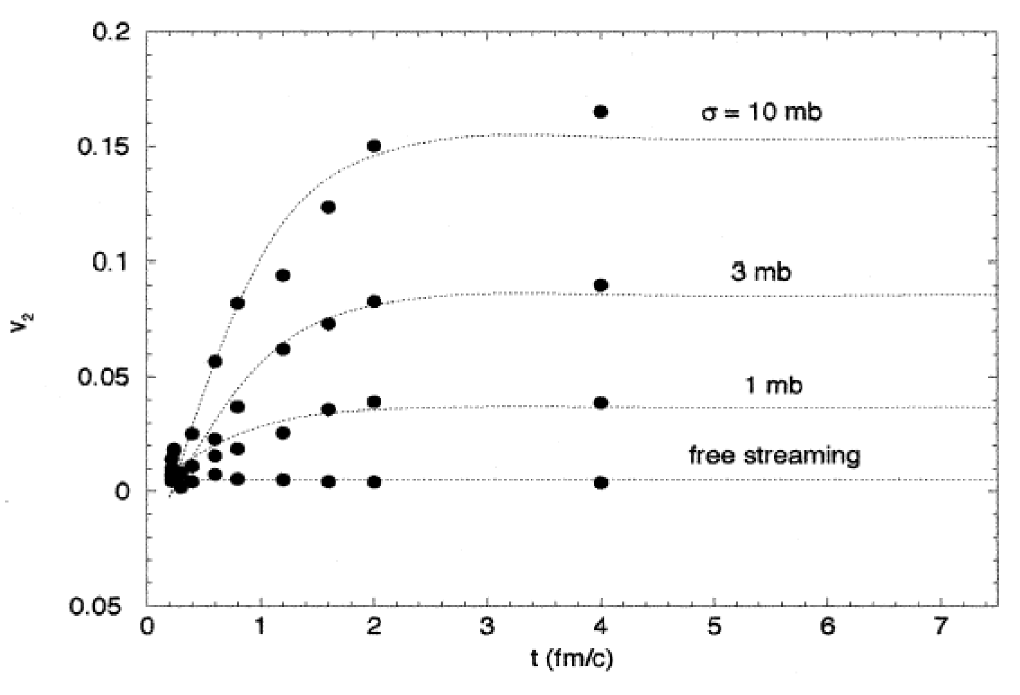


Elliptic flow (Ollitrault, '92)

- Momentum anisotropy as a response to spatial anisotropy
- Known to be sensitive to properties of the system
  - (Shear) viscosity
  - Equation of state

2nd harmonics (elliptic flow)
→ Indicator of hydrodynamic behavior

Relativistic Boltzmann Simulations



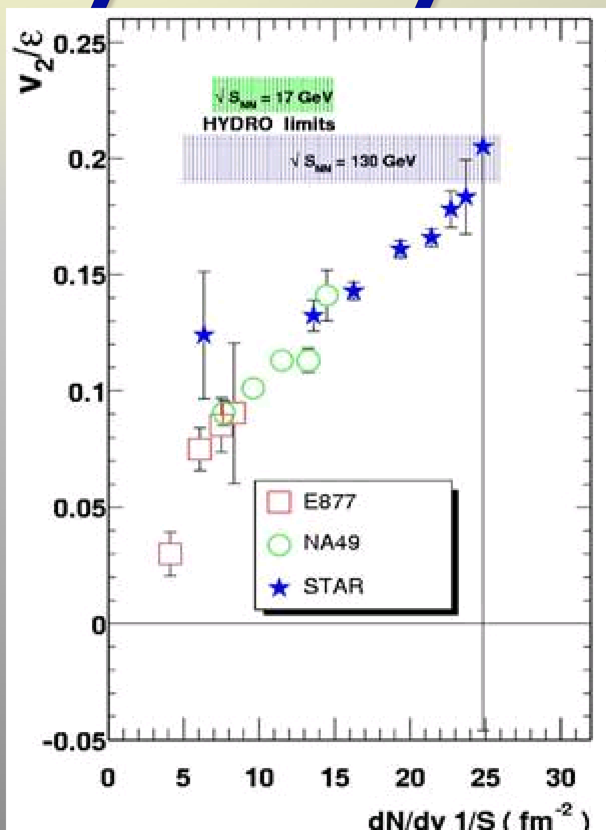
$$\lambda = \frac{1}{\sigma\rho} \propto \eta$$

λ: mean free path
η: shear viscosity

v_2 is
- generated through secondary collisions
- saturated in the early stage
- sensitive to cross section (~1/m.f.p.~1/viscosity)

Zhang-Gyulassy-Ko('99)

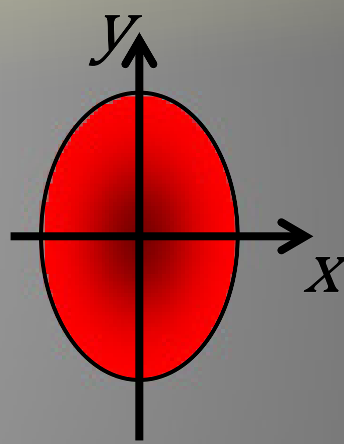
"Hydrodynamic Limit"



$\frac{v_2}{\varepsilon}$ vs. transverse particle density

Eccentricity

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

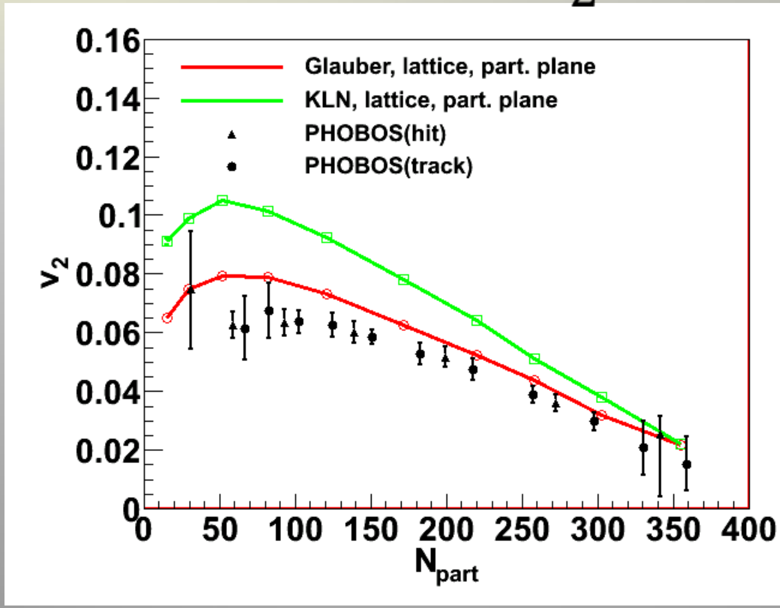


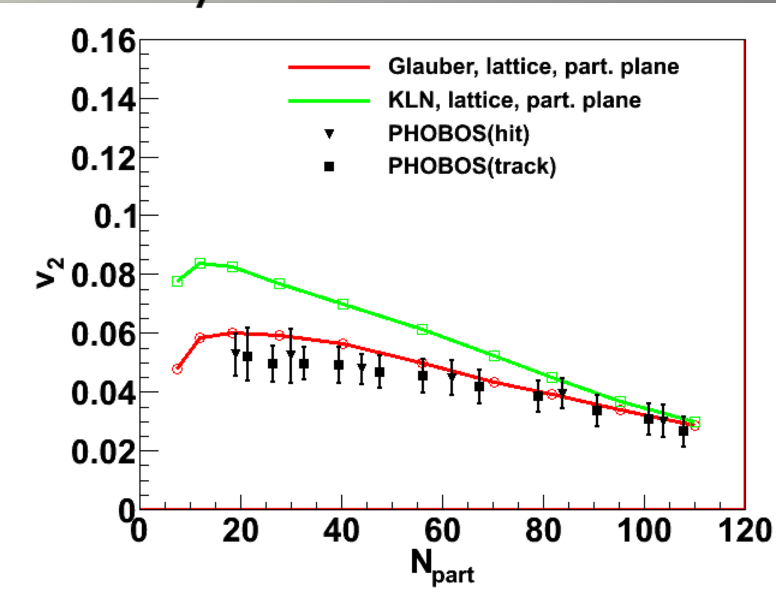
$$\frac{v_2}{\varepsilon} = \frac{\text{Momentum Anisotropy}}{\text{Spatial Anisotropy}}$$

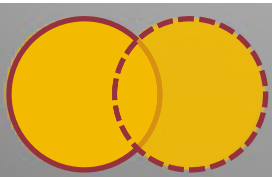
⇒ Response of the system reaches "hydrodynamic limit"

Ideal Hydrodynamics at Work

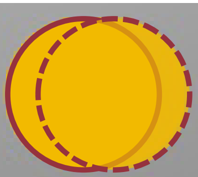
v_2 vs. centrality

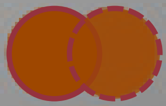






Au+Au



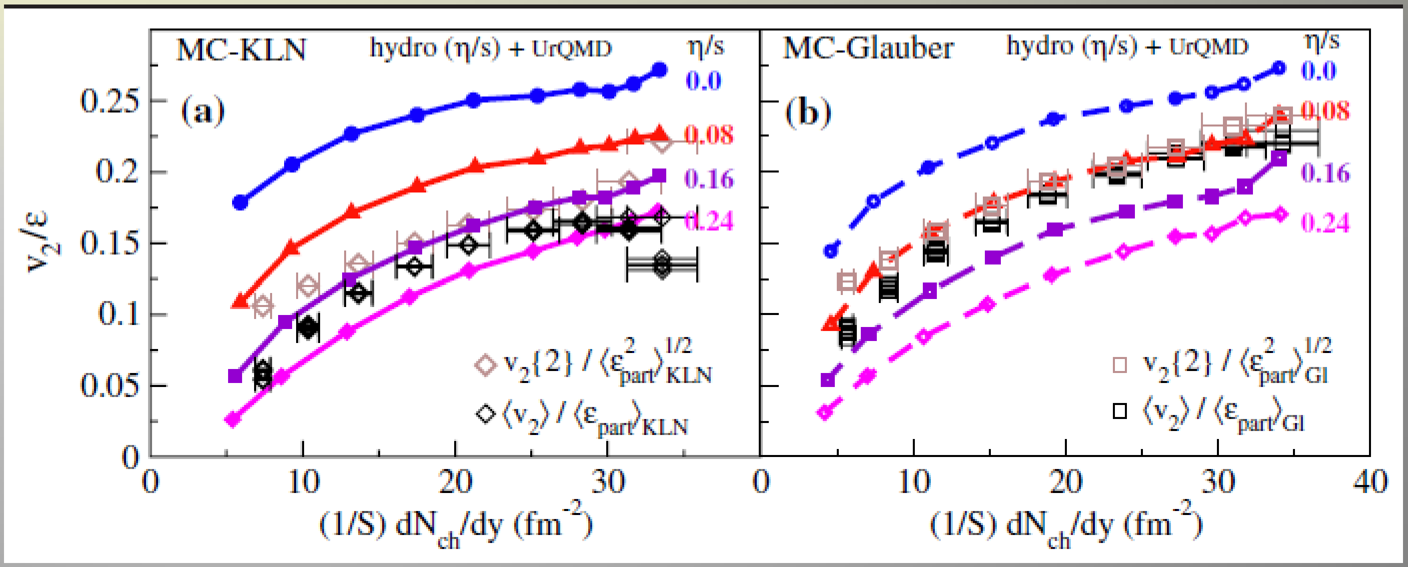


Cu+Cu

T.Hirano et al. (in preparation)

H.Song et al., PRL106, 192301 (2011)

Viscous Fluid Simulations



$\frac{v_2}{\varepsilon}$ vs. transverse particle density

Ratio of shear viscosity to entropy density

$$\left(\frac{\eta}{s}\right)_{\mathrm{QGP}} = (0.08{\sim}0.16)\,\frac{\hbar}{k_B} \approx \frac{1}{380}\left(\frac{\eta}{s}\right)_{\mathrm{Water}} \approx \frac{1}{9}\left(\frac{\eta}{s}\right)_{\mathrm{Liquid\ He}}$$

Strong Coupling Nature of the QGP

Large expansion rate of the QGP in relativistic heavy ion collisions

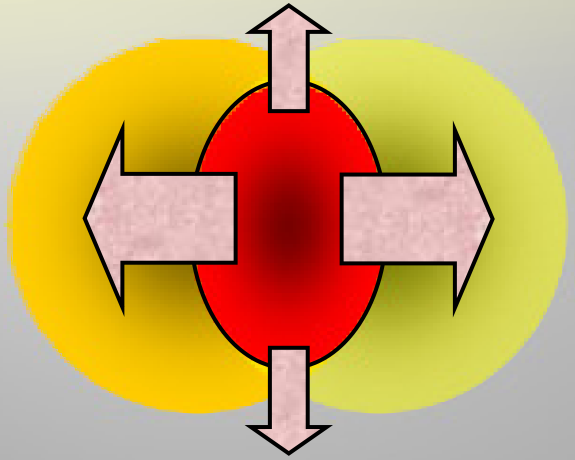
$$\frac{\dot{V}}{V} = \mathrm{div}\,\vec{v} \approx 10^{22}\ /\mathrm{sec}$$

→ Tiny viscosity when hydrodynamic description of the QGP works in any ways

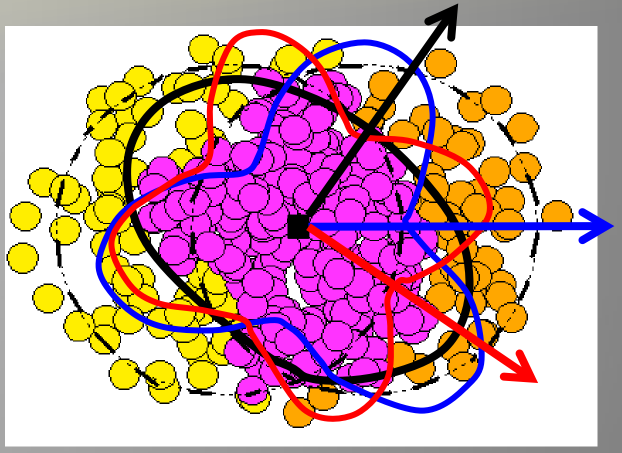
→ Manifestation of the strong coupling nature of the QGP

Note: Underlying theory → Quantum ChromoDynamics (theory of strong interaction)

Event-by-event Fluctuation



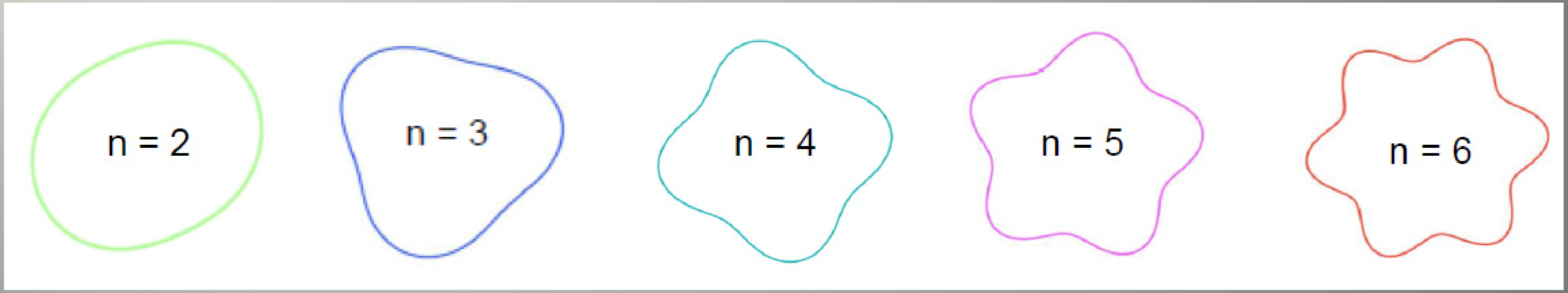
Ideal, but unrealistic?
OK on average(?)



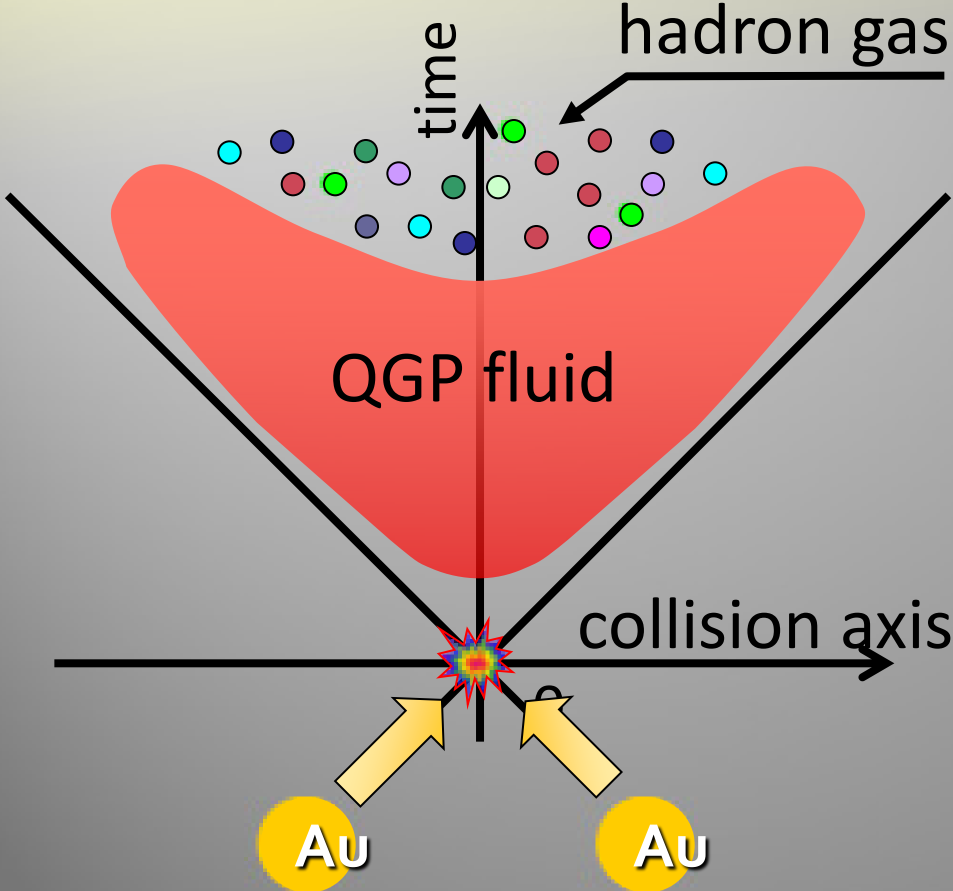
Actual collision?
→ Higher order deformation

Figure adapted from talk by J.Jia (ATLAS) at QM2011

Deformation at Higher Order



Dynamical Modeling of Relativistic Heavy Ion Collisions*



Relativistic Boltzmann

↑

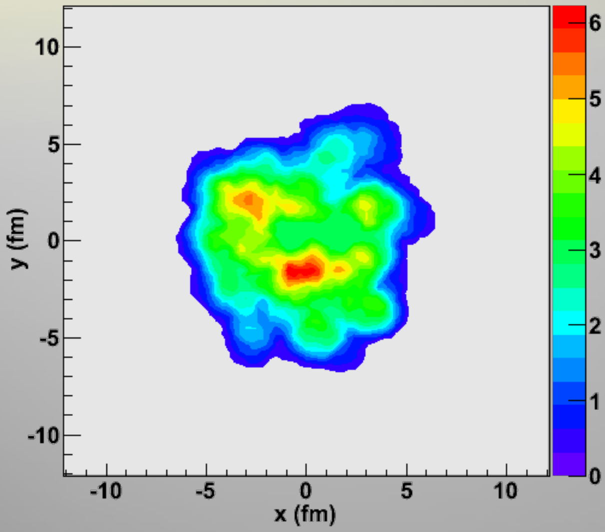
Relativistic Ideal Hydro

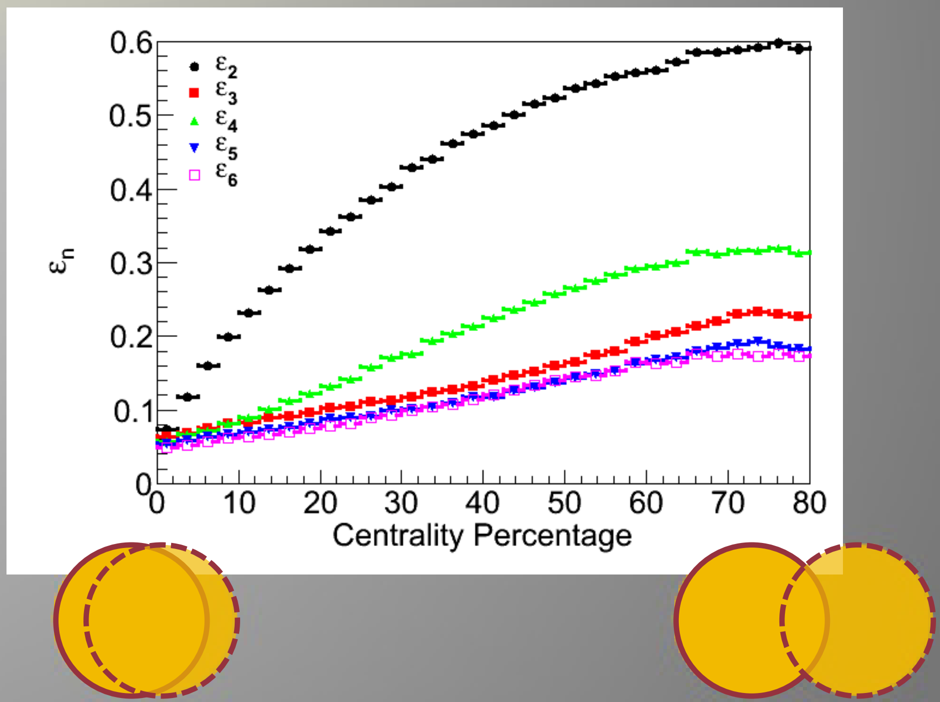
↑

Monte Carlo I.C.

*K.Murase *et al.* (in preparation)

Deformation in Model Calculations



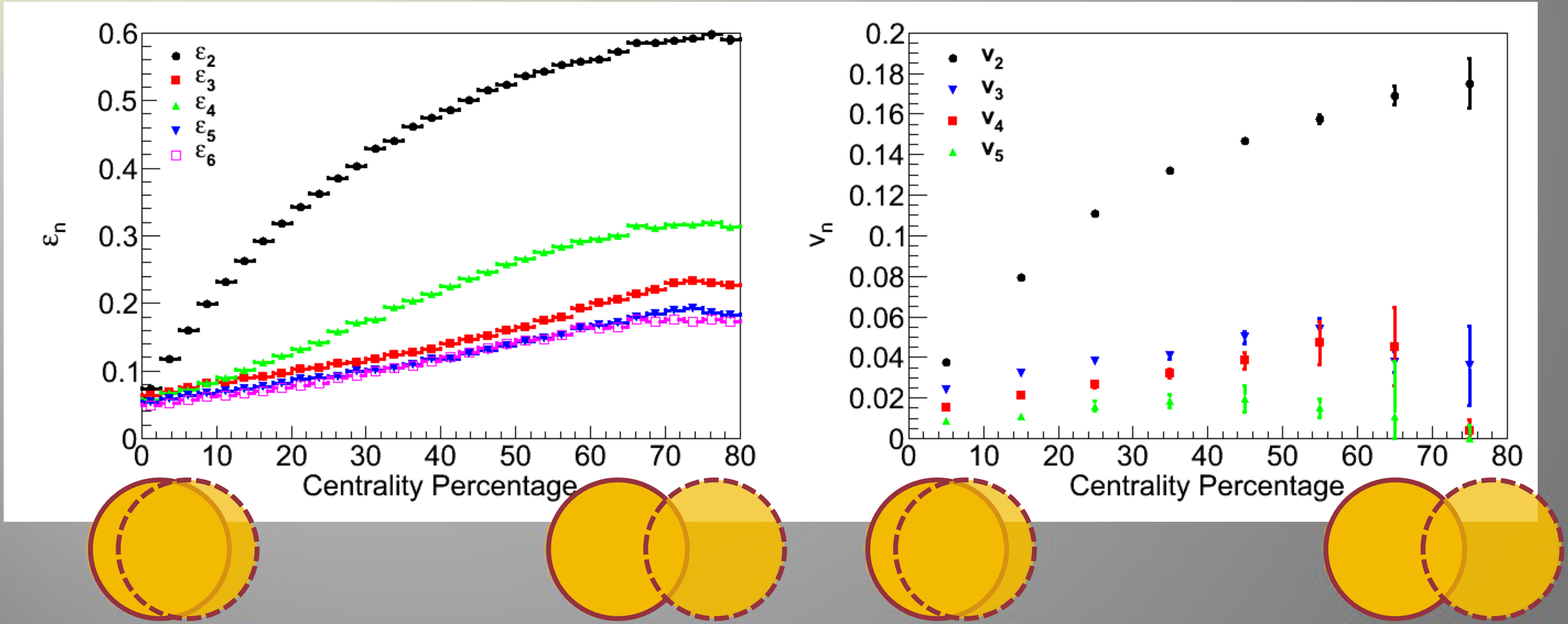


Sample of entropy density profile in a plane perpendicular to collision axis

$$\varepsilon_n = \frac{\left|\langle r^2 e^{in\phi} \rangle\right|}{\langle r^2 \rangle}$$

Higher Harmonics from Ideal QGP Fluids

ε_n vs. centrality (input) v_n vs. centrality (output)

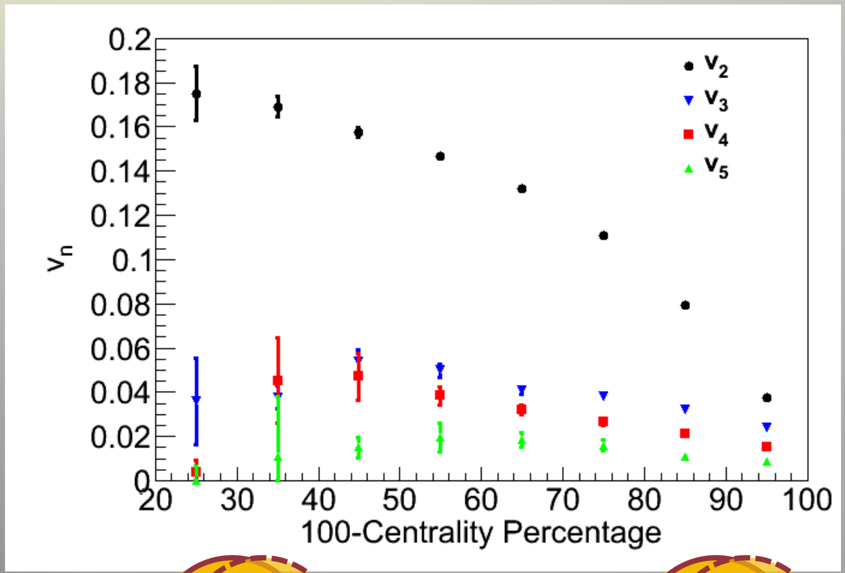


Response of the QGP to initial deformation

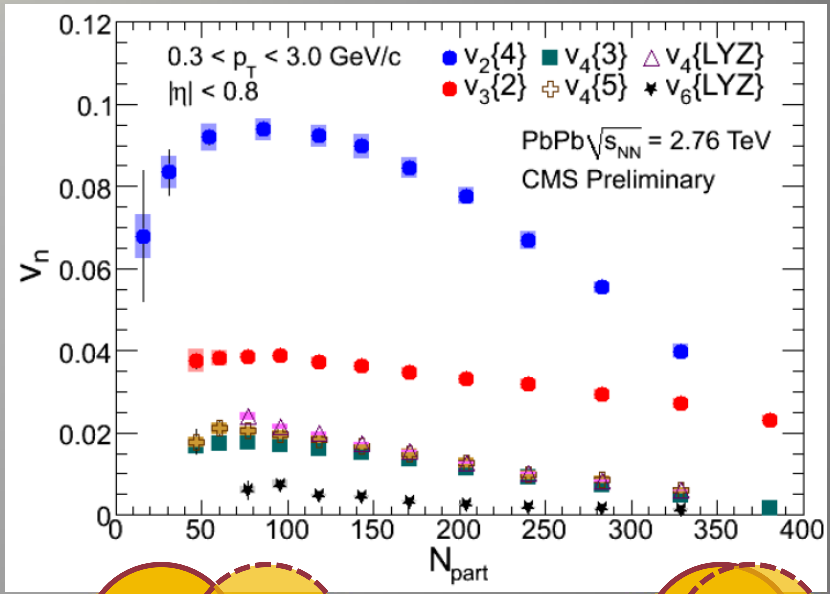
v_n roughly scales with ε_n

Comparison of v_n with Data

Theory



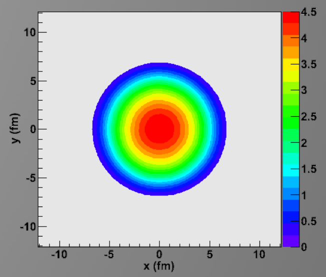
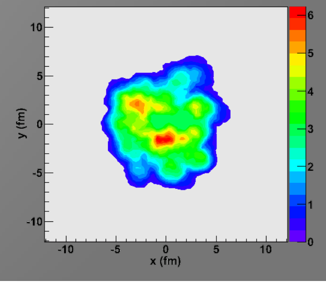
Experiment



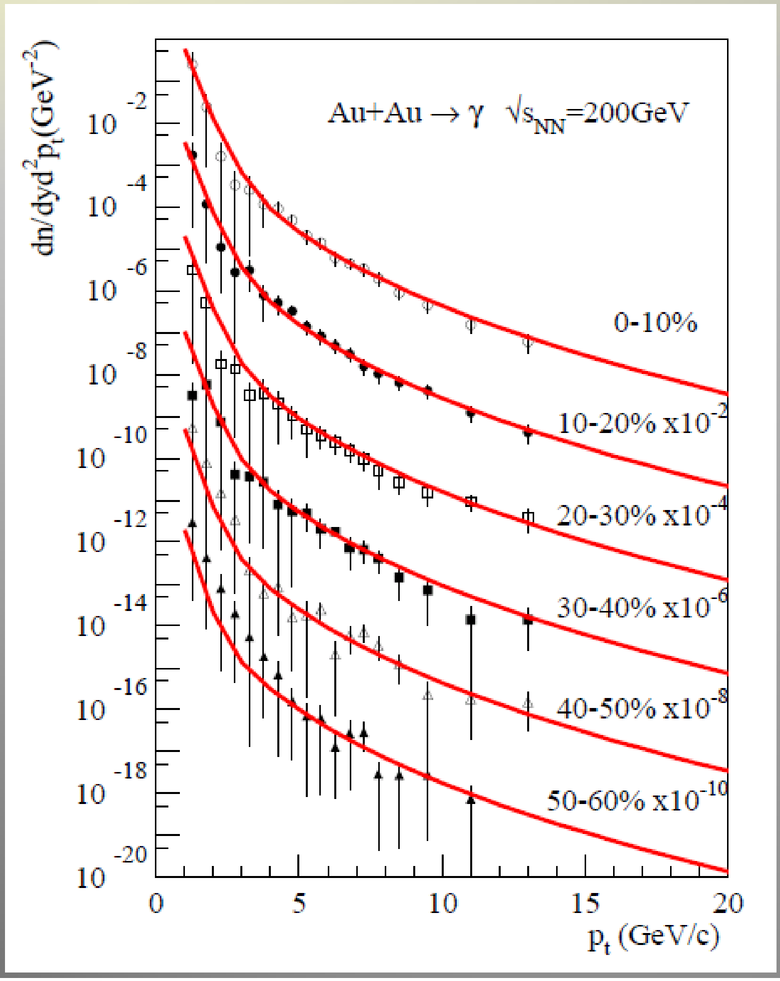
Tendency similar to experimental data

Absolute value ←→ Viscosity

Impact of Finite Higher Harmonics

	coarse graining size	initial profile
• Most of people did not believe hydro description of the QGP (~ 1995)		
• Hydro at work to describe elliptic flow (~ 2001)	$d \lesssim 5\ \mathrm{fm}$	
• Hydro at work (?) to describe higher harmonics (~ 2010)	$d \lesssim 1\ \mathrm{fm}$	

Thermal Radiation from the QGP



Photon spectra in relativistic heavy ion collisions

Blue shifted spectra with $T \sim 200\text{-}300$ MeV $\sim (2\text{-}3) \times 10^{12}$K

RELATIVISTIC HYDRODYNAMICS

Several Non-Trivial Aspects of Relativistic Hydrodynamics

1. Non conservation of particle number nor mass
2. Choice of local rest frame
3. Relaxation beyond Fourier, Fick and Newton laws

Hydrodynamic Equations (Ideal Hydro Case)

$$\partial_\mu T^{\mu\nu} = 0, \partial_\mu N^\mu = 0$$

Energy conservation

$$\frac{\partial}{\partial t} E + \boldsymbol{\nabla} \cdot [(E + P)\boldsymbol{v}] = 0, E = (e + P)\gamma^2 - P$$

Momentum conservation

$$\frac{\partial}{\partial t} M^i + \boldsymbol{\nabla} \cdot \left[M^i \boldsymbol{v}\right] = -\nabla^i P, M^i = (e + P)\gamma^2 v^i$$

Charge conservation (net baryon number in QCD)

$$\frac{\partial}{\partial t} N + \boldsymbol{\nabla} \cdot N\boldsymbol{v} = 0, N = \gamma(n_B - n_{\bar{B}})$$

Several Non-Trivial Aspects of Relativistic Hydrodynamics

1. Non conservation of particle number nor mass
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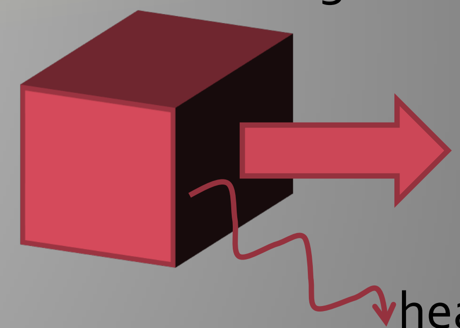
See also talk by Kunihiro

Choice of Local Rest Frame in Dissipative Fluids

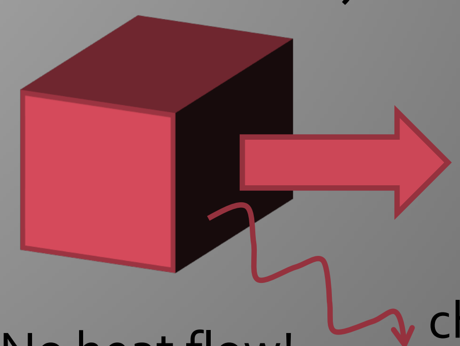
1. Charge flow

$$u^\mu = \frac{1}{N}\sum_i \frac{N_i{}^\mu}{n_i}$$



2. Energy flow
   (Eigenvector of energy-momentum tensor)

$$T^\mu{}_\nu u^\nu = e u^\mu$$



*Energy flow is relevant in heavy ion collisions

Several Non-Trivial Aspects of Relativistic Hydrodynamics

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See also talk by Kunihiro

Relaxation and Causality

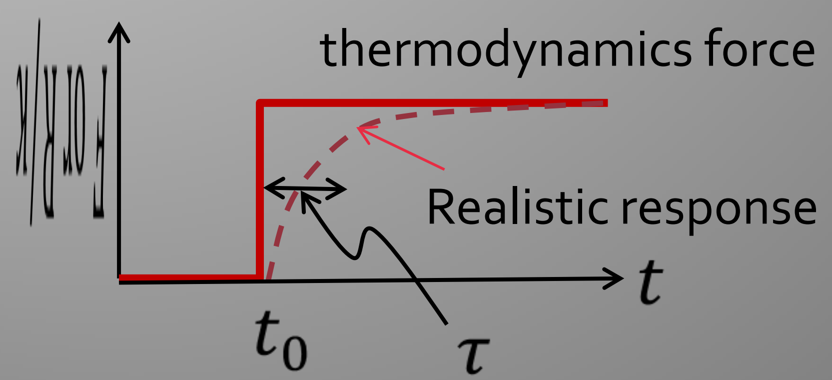
Constitutive equations at Navier-Stokes level

$$\pi^{\mu\nu} = 2\eta\partial^{<\mu}u^{\nu>},$$
$$\Pi = -\varsigma\partial_\mu u^\mu$$

Instantaneous response violates causality

→ Critical issue in relativistic theory

→ Relaxation plays an essential role



Causal Hydrodynamics

Within linear response

$$R(t) = \int dt' G_R(t,t') F(t')$$

Suppose

$$G_R(t,t') = \frac{\kappa}{\tau} \exp\left(-\frac{t-t'}{\tau}\right) \theta(t-t')$$

one obtains differential form

$$\dot{R}(t) = -\frac{R(t) - \kappa F(t)}{\tau}, \qquad v_{\mathrm{signal}} = \sqrt{\frac{\kappa}{\tau}} < c$$

⇒ Maxwell-Cattaneo Eq.

Relativistic Fluctuating Hydrodynamics (RFH)

Thermal fluctuation in event-by-event simulations

dissipative current

thermodynamic force

$$R(x) = \int d^4x' G(x,x') F(x') + \delta R(x)$$

thermal noise

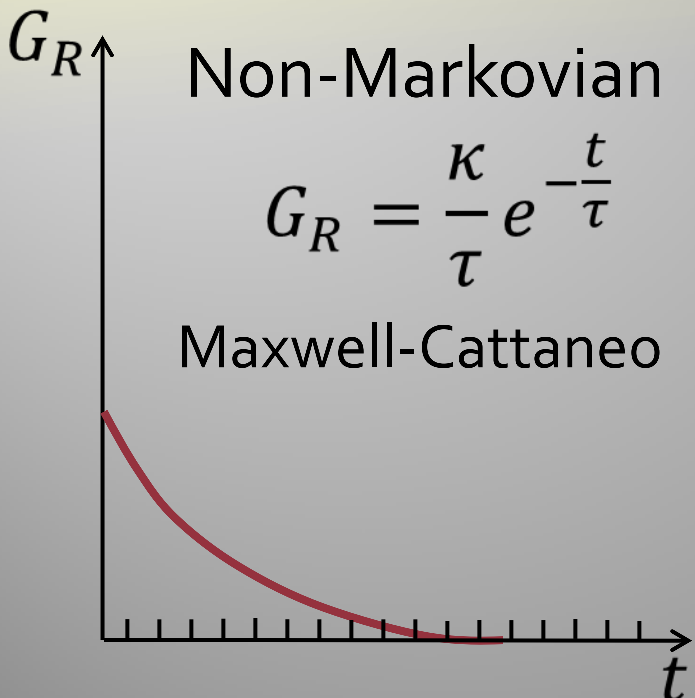
$$\langle \delta R(x) \delta R(x') \rangle \approx G(x,x')$$

Fluctuation ←→ Dissipation

* In non-relativistic cases, see Landau-Lifshtiz, Fluid Mechanics

** Similar to glassy system, polymers, etc?

Coarse-Graining in Time



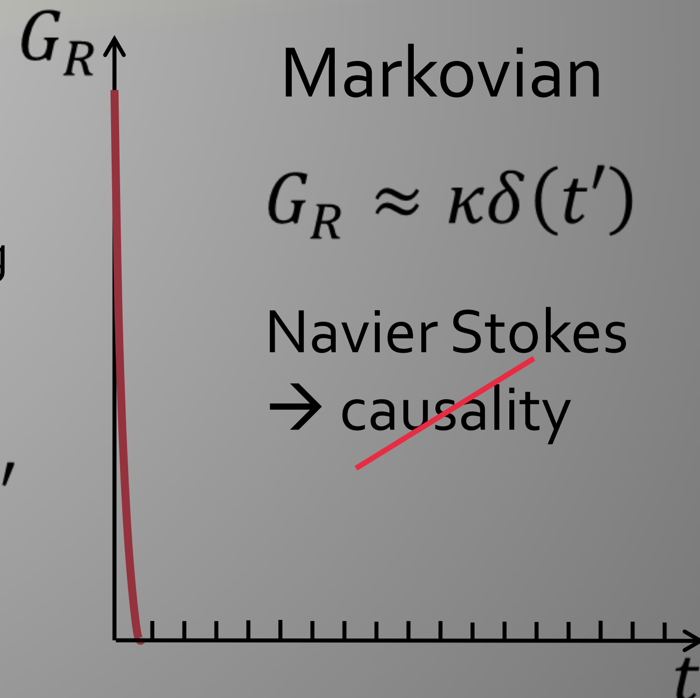
Non-Markovian

$$G_R = \frac{\kappa}{\tau} e^{-\frac{t}{\tau}}$$

Maxwell-Cattaneo

coarse graining ???

$t \rightarrow t'$



Markovian

$$G_R \approx \kappa \delta(t')$$

Navier Stokes

→ ~~causality~~

Existence of upper bound in coarse-graining time (or lower bound of frequency) in relativistic theory???

Finite Size Effect

Fluctuation ←→ Local volume.

$$\xi(x) \propto \frac{1}{\sqrt{\Delta V}}$$

- Information about coarse-grained size?
- Fluctuation term ~ average value?
  → Non-equilibrium small system?
  → Fluctuation would play a crucial role.
- Need to consider (?) finite size effects in equation of state and transport coefficients

Conclusion

Physics of the quark gluon plasma

- Strong coupling nature
- Small viscosity

Physics of relativistic heavy ion collisions

- Playground of relativistic non-equilibrium system
  - Relativistic dissipative hydrodynamics
  - Relativistic kinetic theory
  - Non-equilibrium field theory