Dynamics of Self-Sustained Turbulence in Astrophysics:

MRI-Driven Turbulence in Accretion Disks

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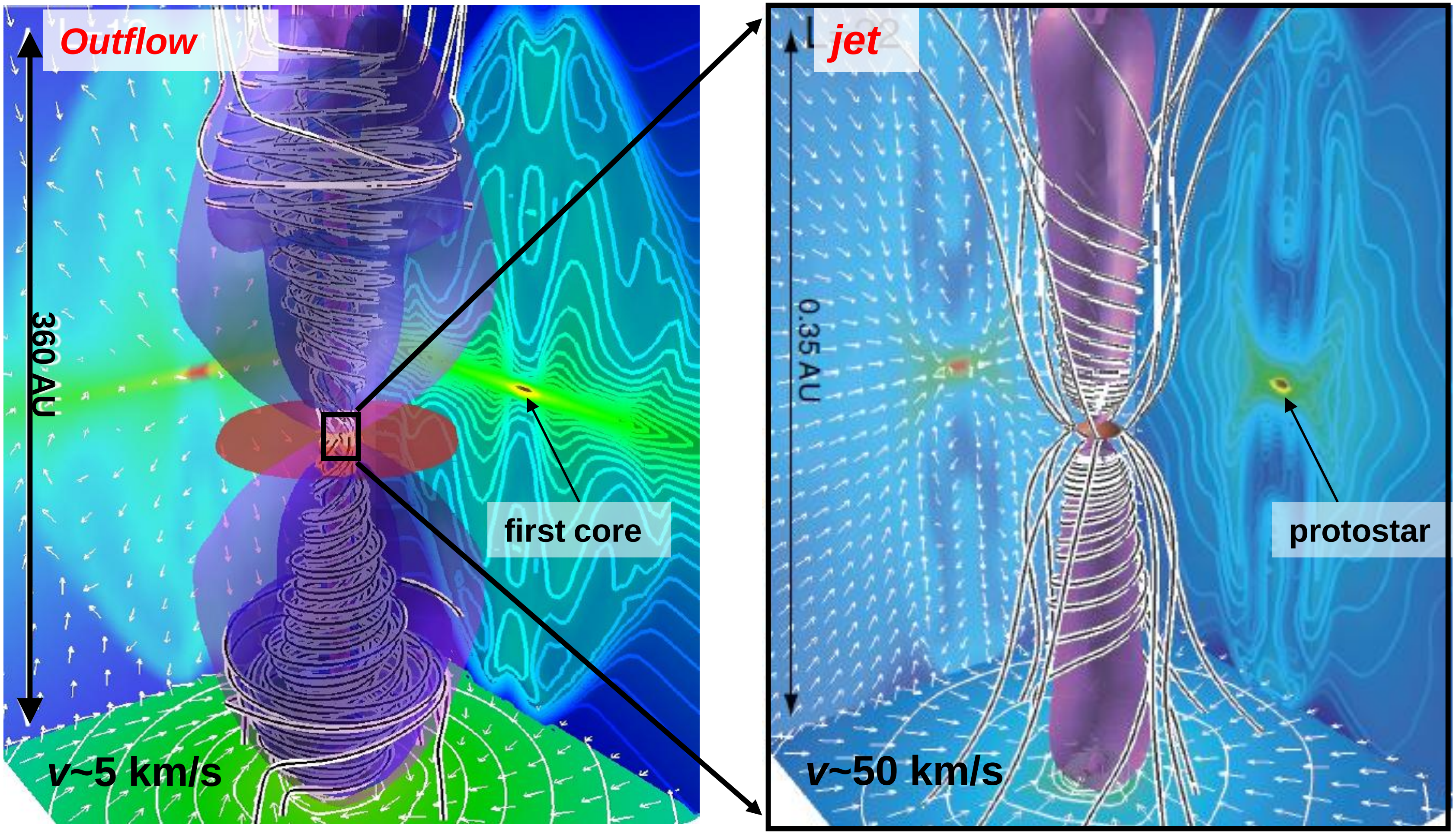
special focus on Net B_z Case

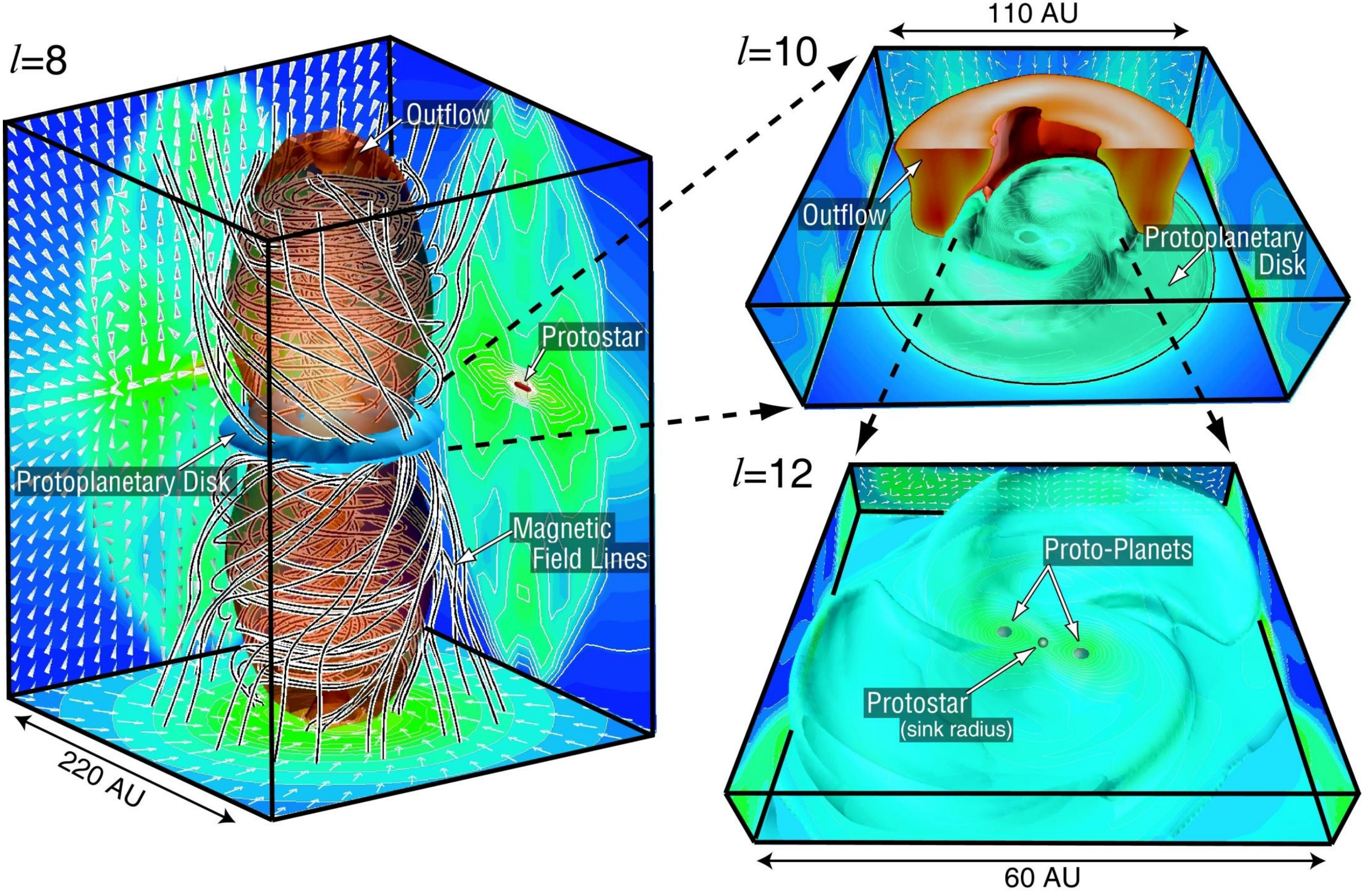
Early Phase of Protostar Formation



Machida, SI, & Matsumoto (2006-2010)

Outflows & Jets are Natural By-Products!

Early Phase of Disk Formation



Formation & Evolution of Discs

Further Evolution of Protostars

= Accretion of Gas from the envelope &

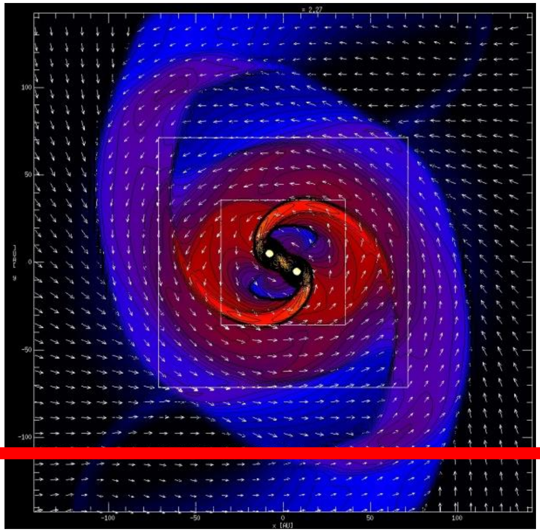
Gas Accretion through the Discs

Early Phase

Rapid Gas Accretion due to

Gravitational Torque of

"m=2" Spiral Mode

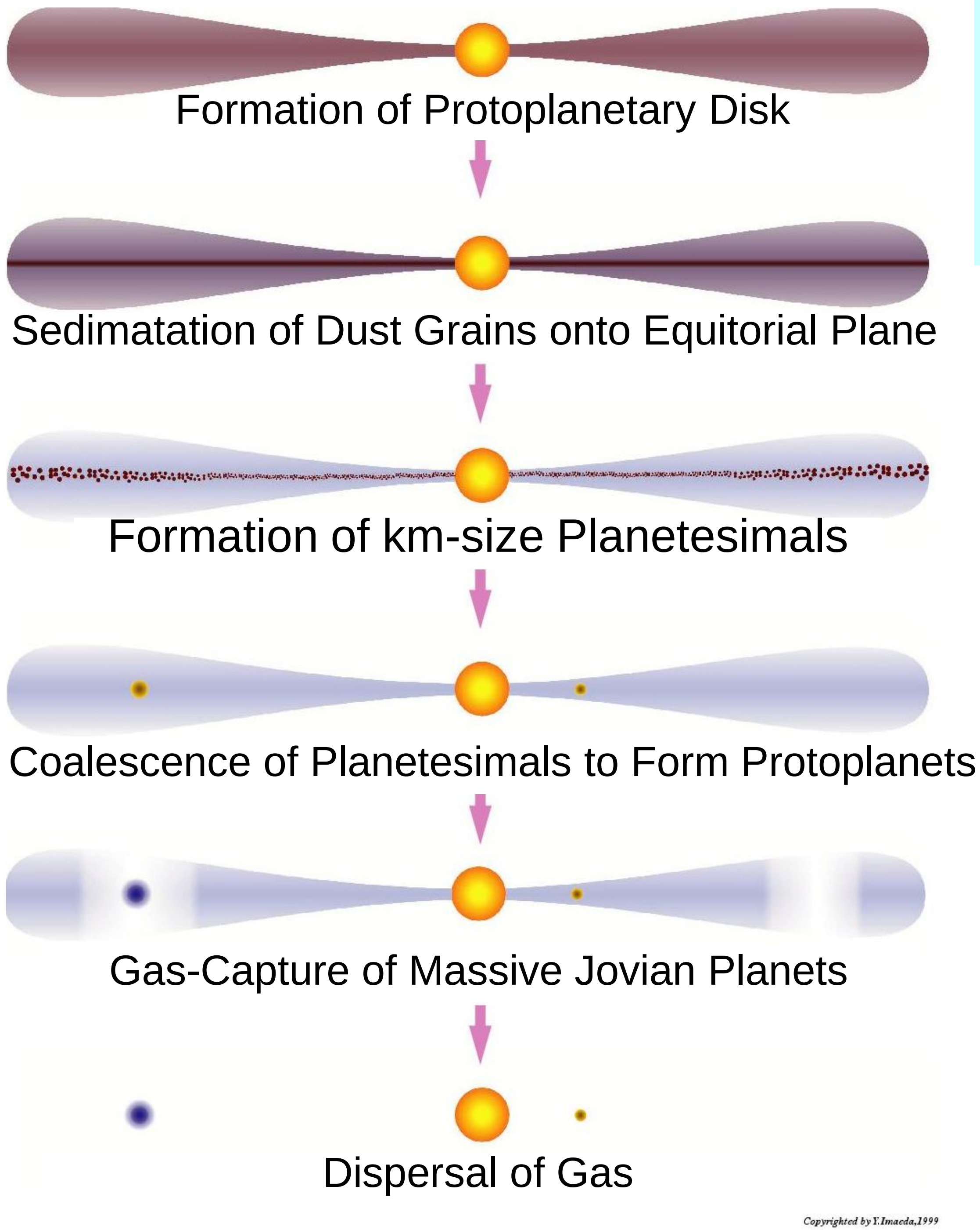


Later Phase

Slow Accretion due to Magnetorotational Instability

Velikhov 1959, Chandrasekhar 1961, Balbus & Hawley 1991

Standard Model of Planet Formation



Basic Energetics 1

specific angular momentum:

$h = r\,v_\phi$

specific energy:

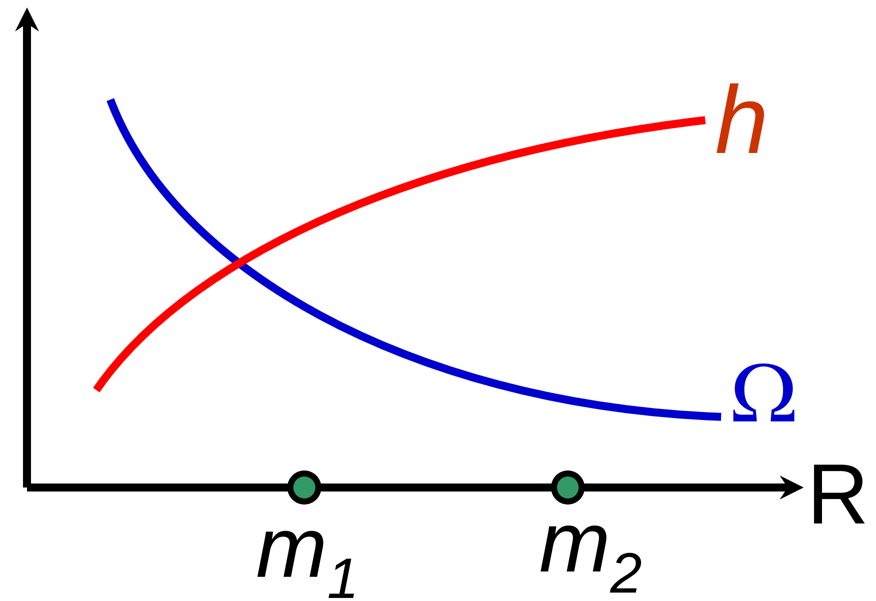
$e = v_\phi^2/2 + \psi = (h/r)^2/2 + \psi(r)$

total energy:

$E = m_1 e_1 + m_2 e_2$

total angular momemtum:

$H = m_1 h_1 + m_2 h_2$



Transfer angular momentum (dh) between 1 & 2 :

$dH = m_1\, dh_1 + m_2\, dh_2 = 0$

$dE = m_1 (\partial e/\partial h)\, dh_1 + m_2 (\partial e/\partial h)\, dh_2$

$= m_1\, dh_1 (\Omega_1 - \Omega_2) \quad < 0 \quad \text{for } dh_1 < 0$

i.e., Outward transfer of Angular Momentum may be unstable.

Lynden-Bell & Pringle 1974

Basic Energetics 2

Next, transfer mass & angular mom.

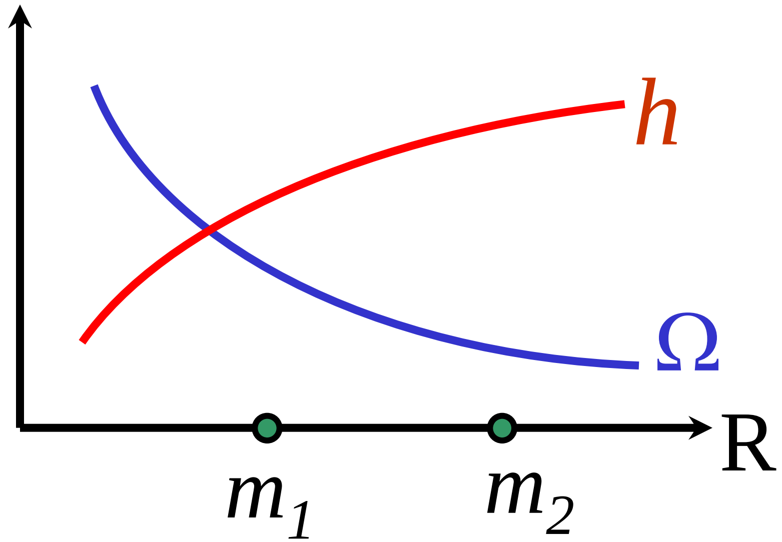
$$dM = dm_1 + dm_2 = 0$$

$$dH = d(m_1 h_1) + d(m_2 h_2) = 0$$

$$dE = d(m_1 e_1) + d(m_2 e_2)$$

$$= dm_1 \{ (e_1 - h_1 \Omega_1) - (e_2 - h_2 \Omega_2) \}$$

$$+ d(m_1 h_1)(\Omega_1 - \Omega_2)$$



where

$$d(e - h\Omega)/dr = d(-v_\phi^2/2 + \psi)/dr = -r\, v_\phi\, d\Omega/dr > 0$$

Thus,

$$(e_1 - h_1 \Omega_1) - (e_2 - h_2 \Omega_2) < 0$$

$dE < 0$ for $dm_1 > 0$ and $d(m_1 h_1) < 0$

How to transfer of Angular Momentum and Mass?

Basic Eq.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{v}\right) = 0$$

$$\frac{d\vec{v}}{dt} + \frac{1}{\rho}\nabla\left(P + \frac{B^2}{8\pi}\right) - \frac{1}{4\pi\rho}\left(\vec{B}\cdot\nabla\right)\vec{B} + \nabla\Phi = 0$$

$$\frac{\partial \vec{B}}{\partial t} - \nabla \times \left(\vec{v} \times \vec{B}\right) = \eta \nabla^2 \vec{B}$$

$$\rho T \frac{ds}{dt} = \frac{\eta}{4\pi}\left(\nabla \times \vec{B}\right)^2$$

No Cooling!

where,

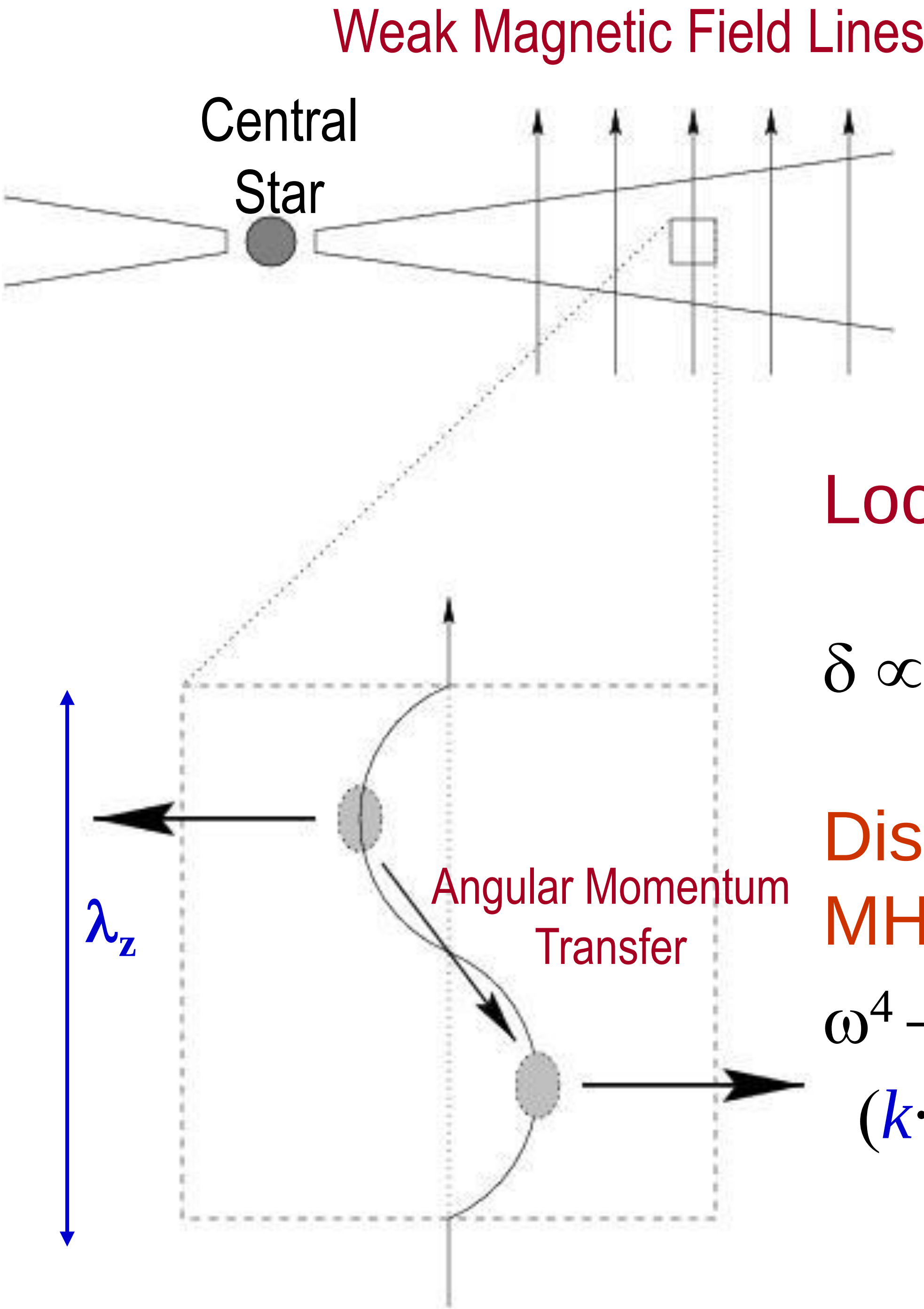
d/dt : Lagrangian Derivative

Φ : Gravitational Potential

η : Magnetic Diffusivity

s : Enthoropy per Unit Mass

Magneto-Rotational Instability (MRI)



Local Linear Analysis
with Bousinesq approx.

$\delta \propto e^{i(\boldsymbol{k} z + \omega t)}$, $\boldsymbol{k} = 2\pi/\lambda_z$

Dispersion Relation in Ideal MHD (η=0) Case

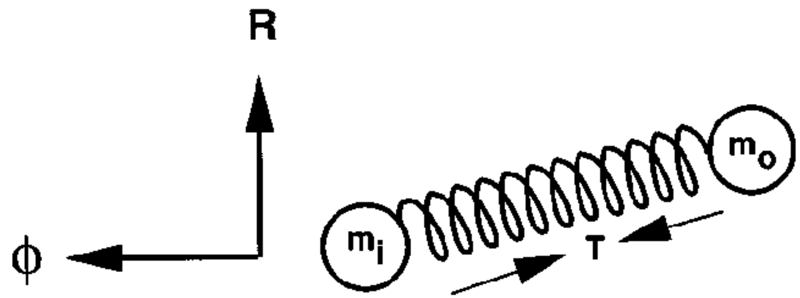
$$\omega^4 - \omega^2[\, \kappa^2 + 2(k \cdot v_A)^2 \,] + (k \cdot v_A)^2 \,[\, (k \cdot v_A)^2 + R\, d\Omega^2/dR \,] = 0$$

Simple Explanation for Instability

Equivalent Model with a Spring!

Connect two bodies

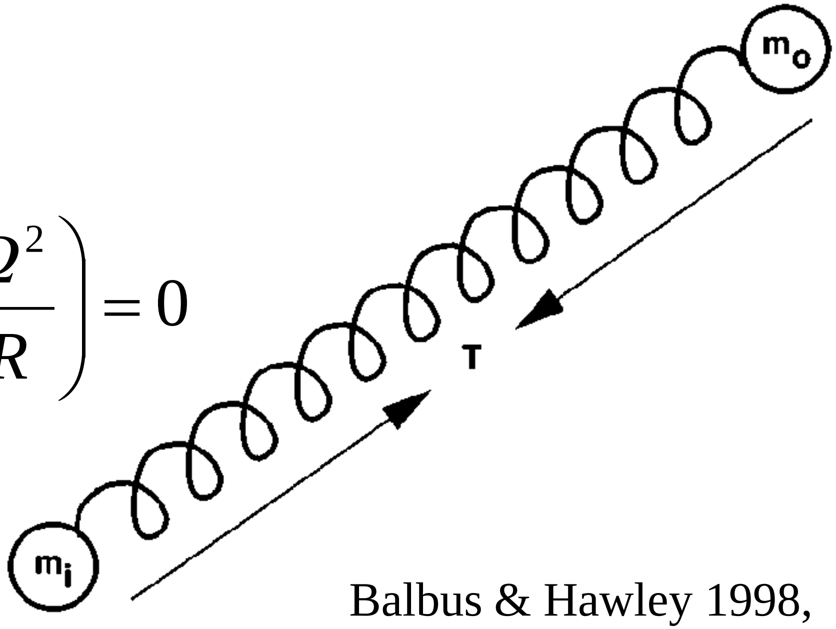
with a spring $K_s=(k\,v_A)^2$.



$$\ddot{x} - 2\Omega\,\dot{y} = -xR\frac{d\Omega^2}{dR} - K_s\,x$$

$$\ddot{y} + 2\Omega\,\dot{x} = -K_s\,y$$

$$\rightarrow \omega^4 - \omega^2\left(\kappa^2 + 2K_s\right) + K_s\left(K_s + R\frac{d\Omega^2}{dR}\right) = 0$$



If $K_s=(k\,v_A)^2$, this is equiv. to

$$\omega^4 - \omega^2[\,\kappa^2 + 2(k\cdot v_A)^2\,] + (k\cdot v_A)^2\,[\,(k\cdot v_A)^2 + R\,d\Omega^2/dR\,] = 0$$

Balbus & Hawley 1998,
Rev. Mod. Phys. **70**, 1

Basics of MRI

Ideal MHD

Linear Growth Rate:

$\omega_{max} \approx (3/4)\,\Omega_{kepler}$

Exponential Growth from Small Field

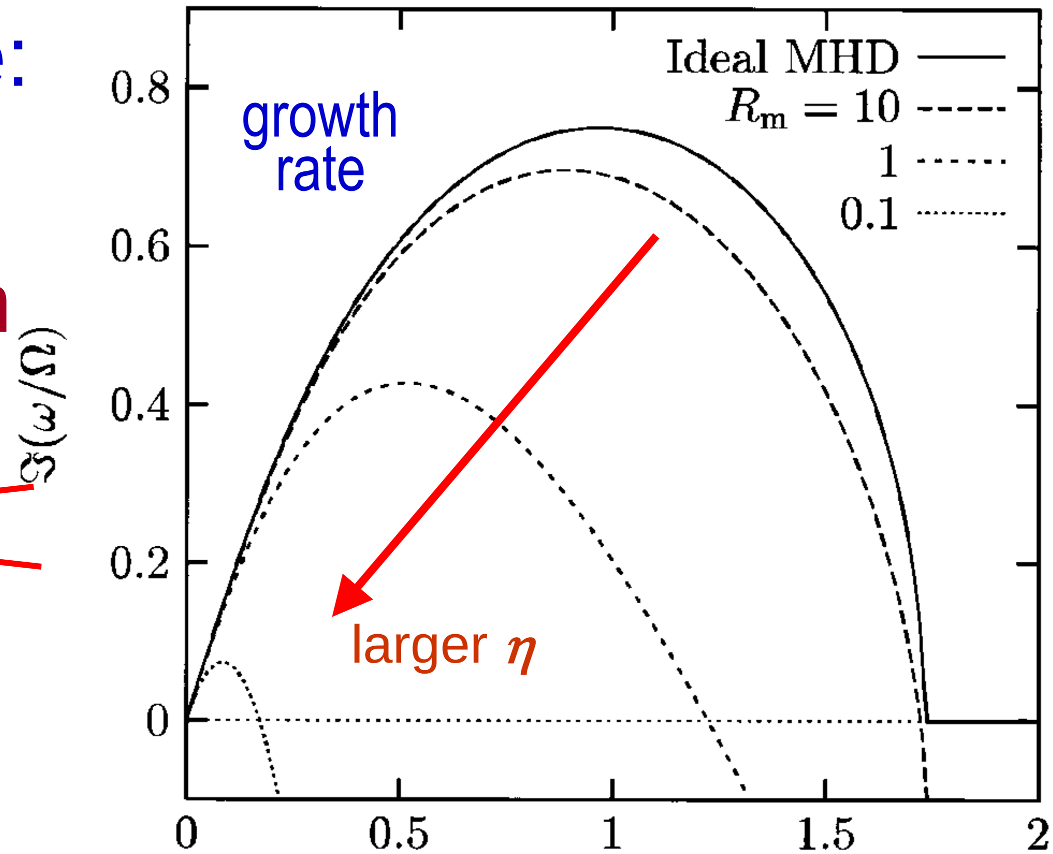
➔ ~~Kinetic Dynamo~~

$\lambda_{max} \approx 2\pi\, v_a/\Omega_{kepler}$

⇒ *"Inverse Cascade"*

k_x=0 axisymmetric (m=0) mode

$R_m \equiv v_A (v_A/\Omega) / \eta$



wavenumber $k_z v_{Az}/\Omega$

Sano & Miyama 1999, ApJ **515**, 776

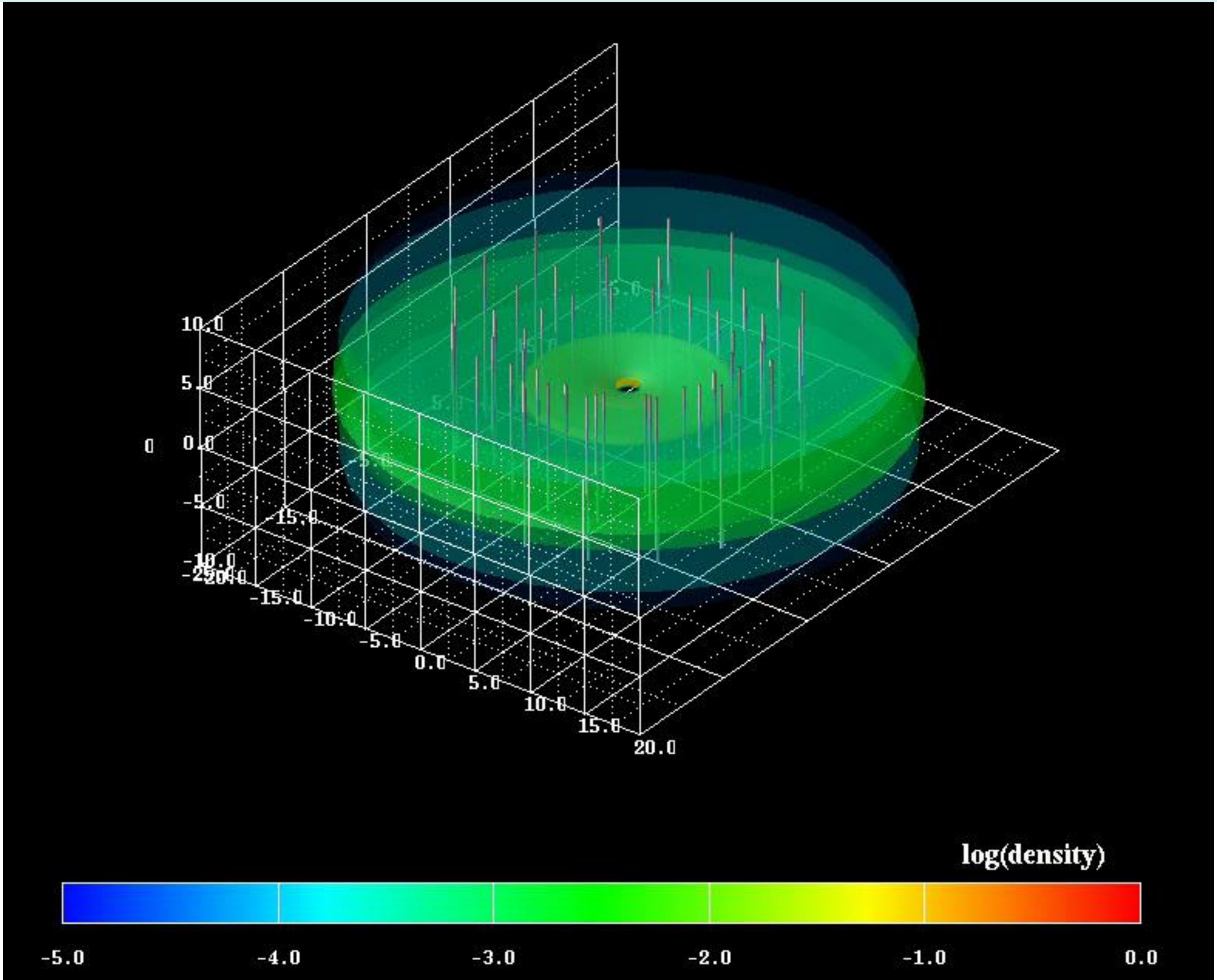
Non-Linear Stage of MRI

- Hawley & Balbus (1991)
- Hawley, Gammie & Balbus (1995, 1996)
- Matsumoto & Tajima (1995)
- Brandenburg et al. (1995)

  ….

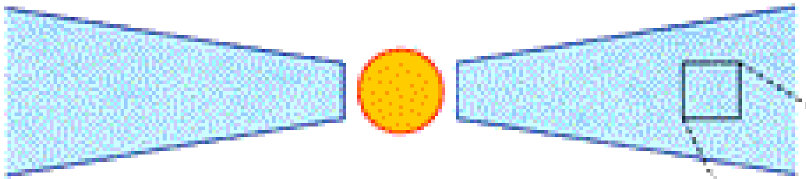
Balbus & Hawley (1998) Rev. Mod. Phys. **70**, 1

Global Disk Simulation



MHD Simulations including Ohmic Dissipation

A Keplerian Disk + Uniform Vertical Fields B_0



On the Flame
Rotating with Local
Angular Velocity Ω

Local Approximation:

Box $<$ Disk Thickness H

Density ρ_0, Pressure P_0,

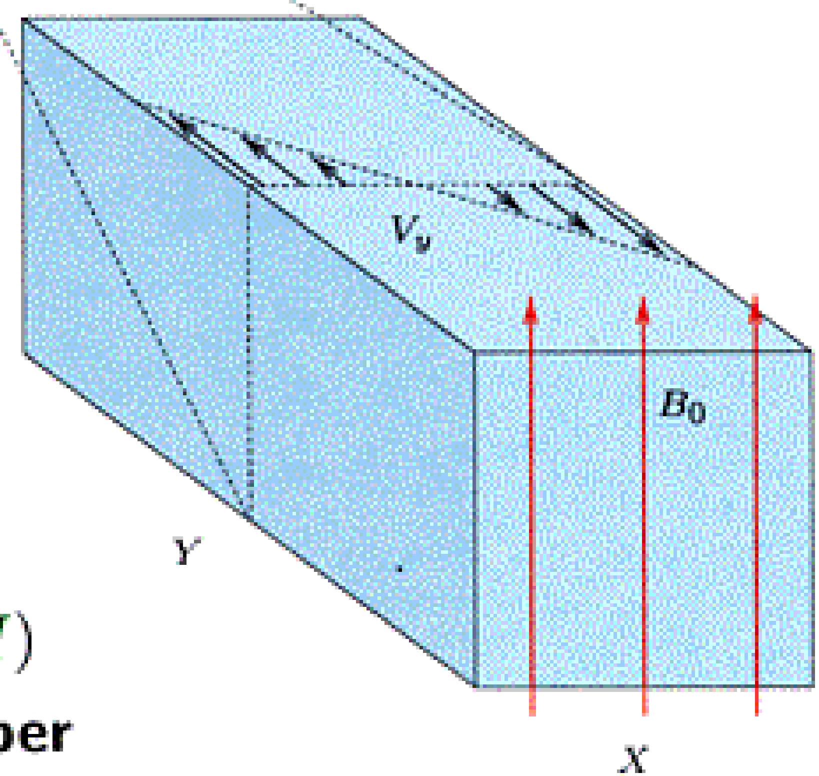
Magnetic Diffusivity η are Uniform

Boundary Conditions: Periodic

Size: $(x, y, z) =$

$$(0.5H, 2H, 0.5H) \sim (2H, 8H, 2H)$$

$= (64, 256, 64)$: **Grid Number**



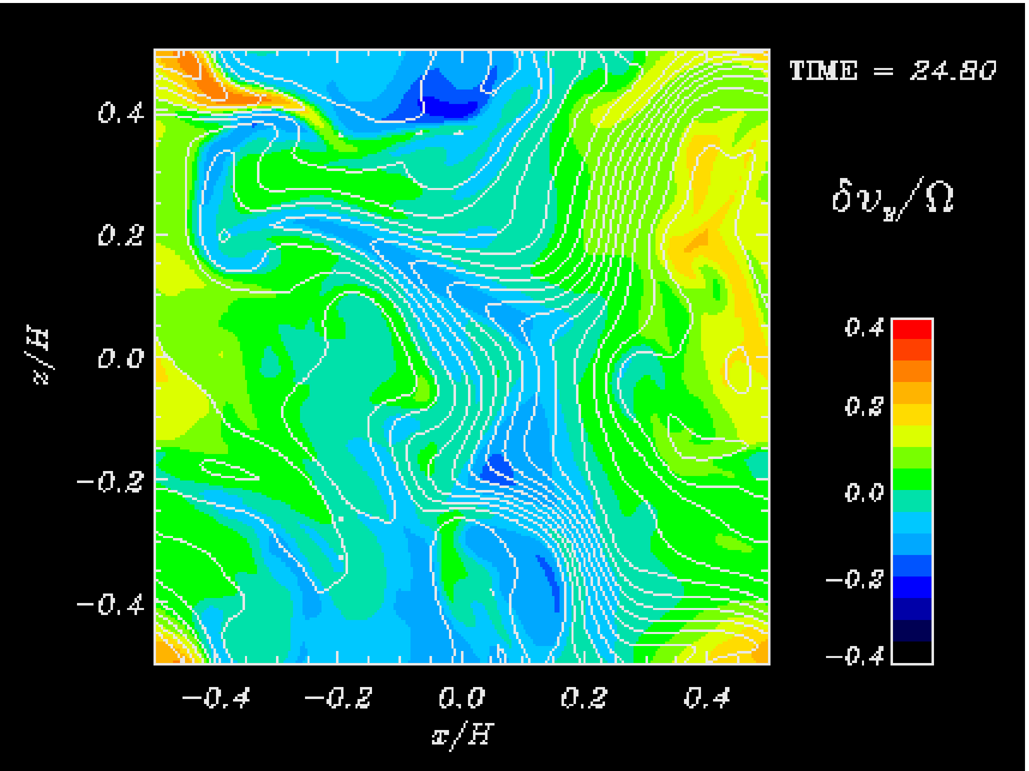
2nd-order Godunov Method + MoC CT

2D Axisymmetric Calculation

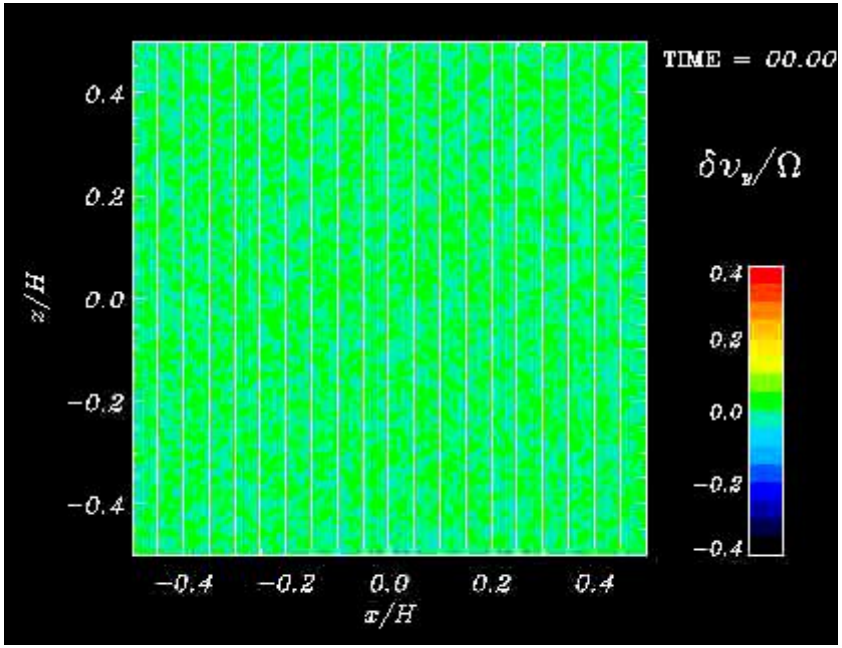
Magnetic Raynolds Number: $R_M < 1$

"Uniformly Random" Turbulent State

$\Rightarrow$ η-Dependent Saturation Level



$\beta_0 = 3200$, $R_m = 0.5$



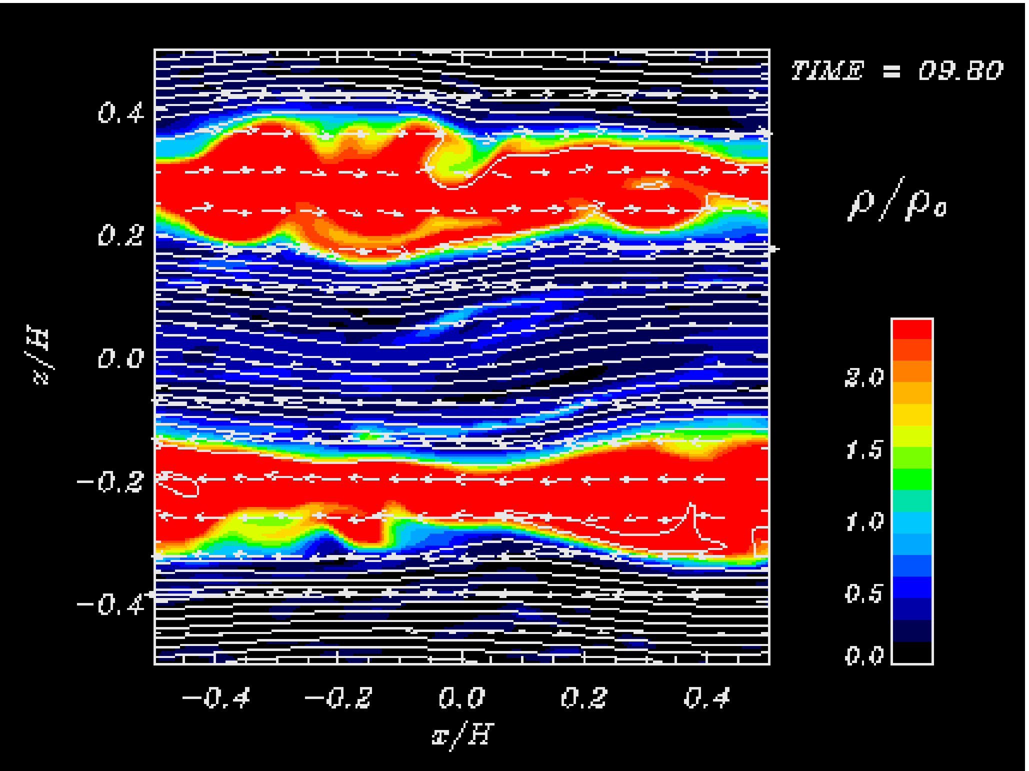
2D Axisymmetric Calculation

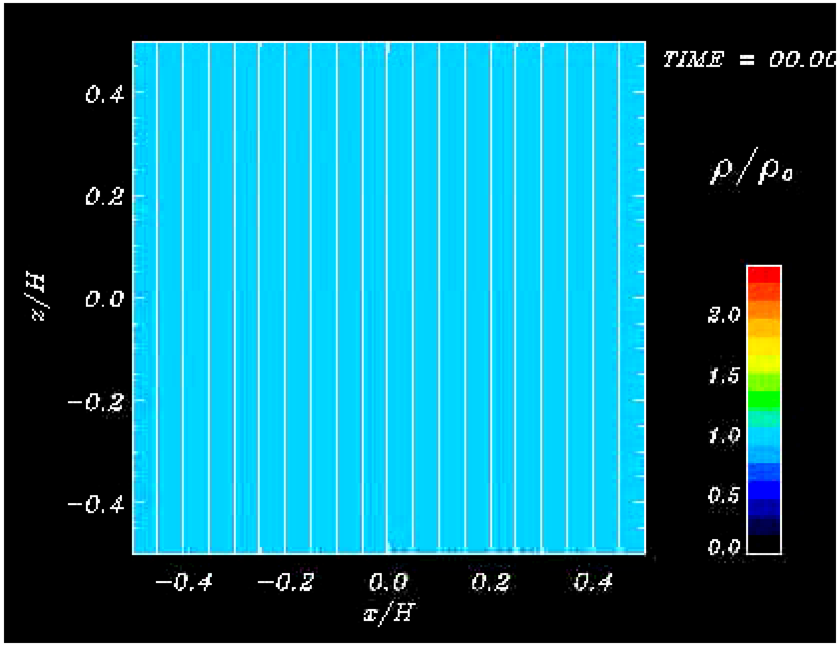
Magnetic Raynolds Number: $R_M > 1$

simple growth of the most unstable mode

⇒ Channel Flow… indefinite growth of B

$\beta_0 = 3200, R_m = 1.5$





2D Axisymmetric Calculation

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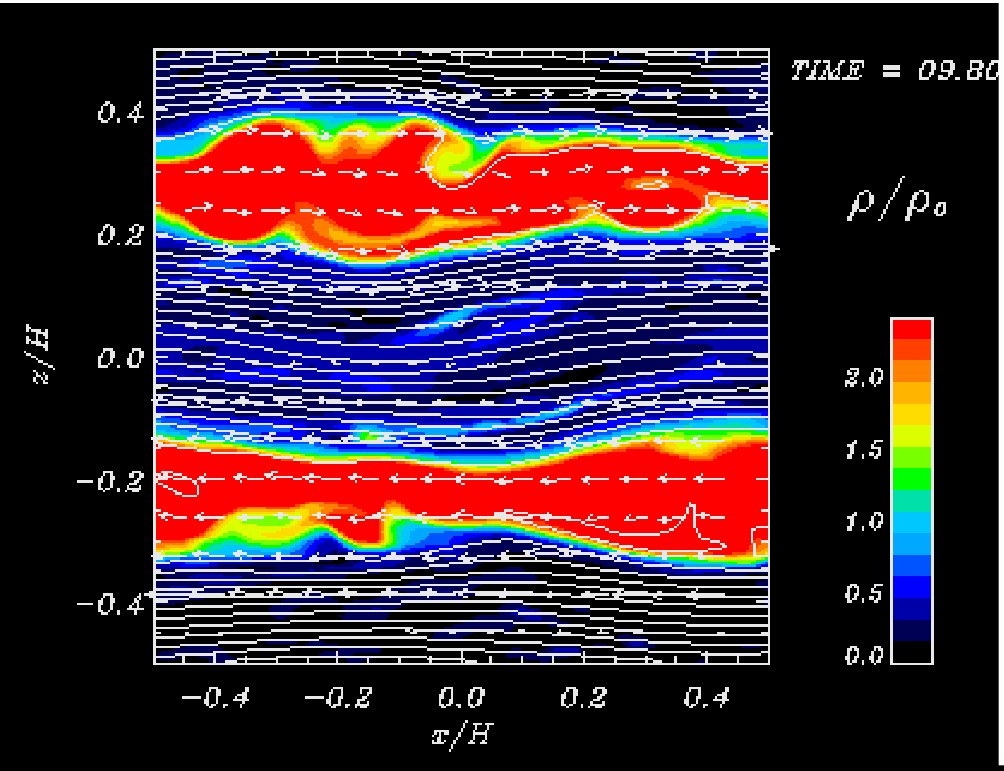
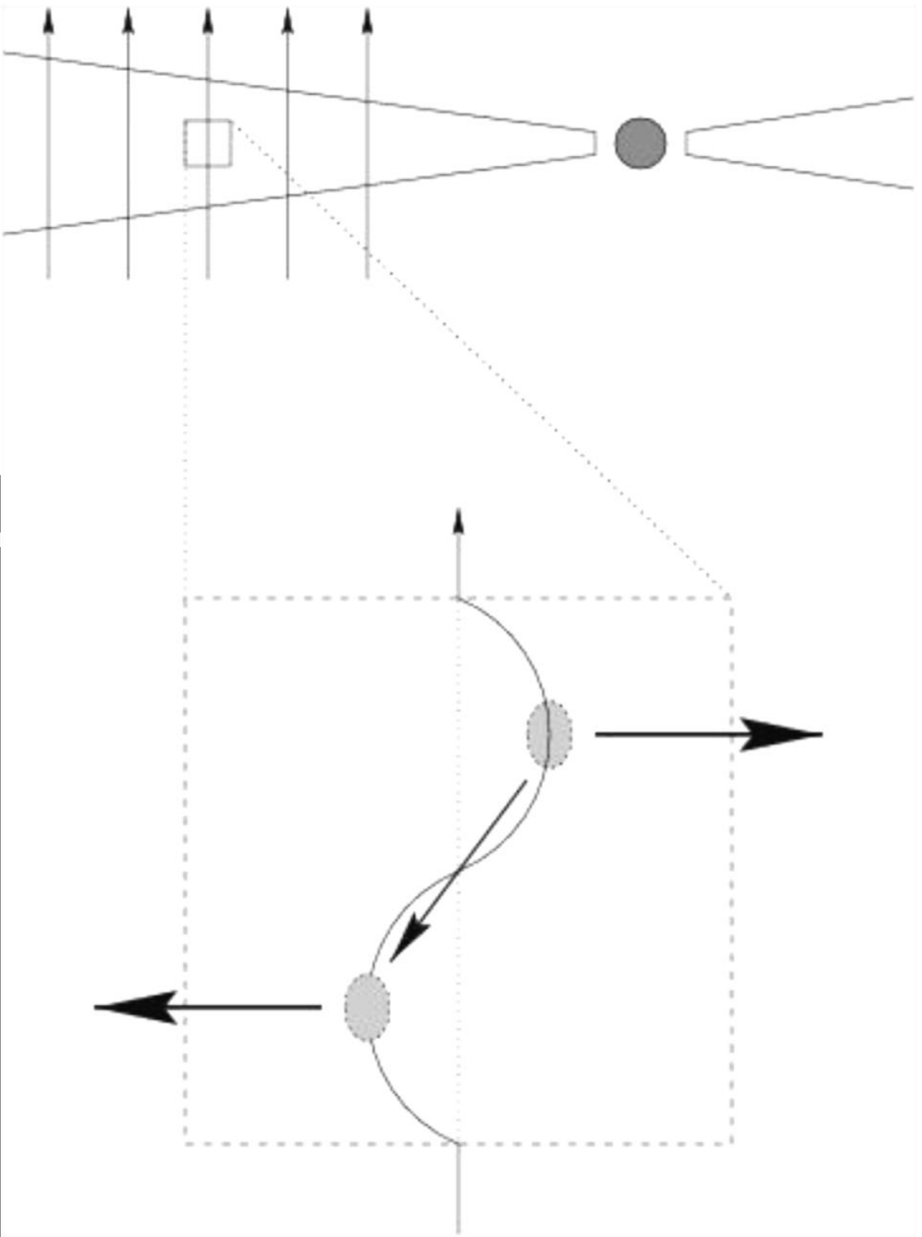




3D Simulations

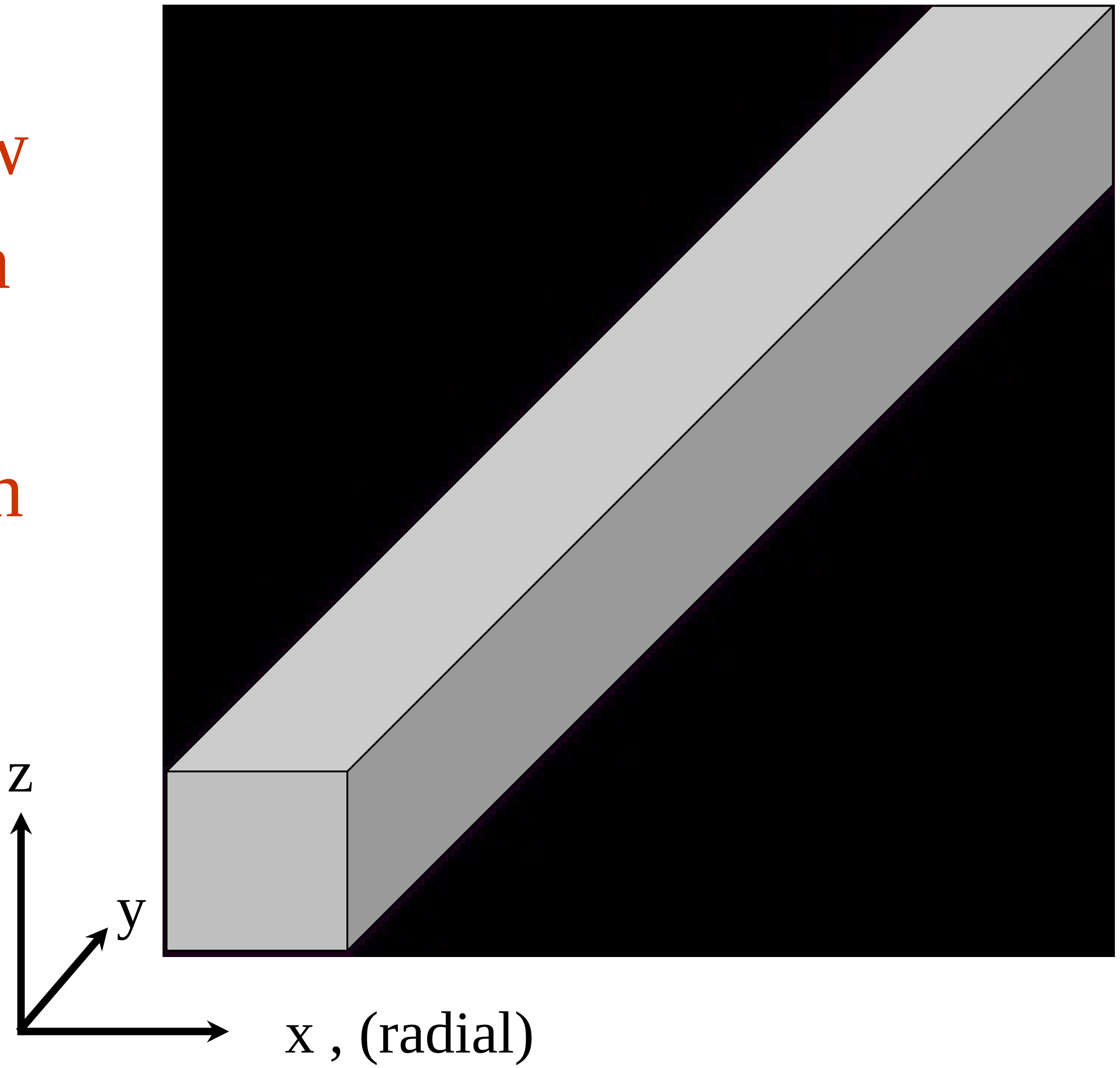
$R_m > 1$

Channel Flow

Break-Down

by

Reconnection

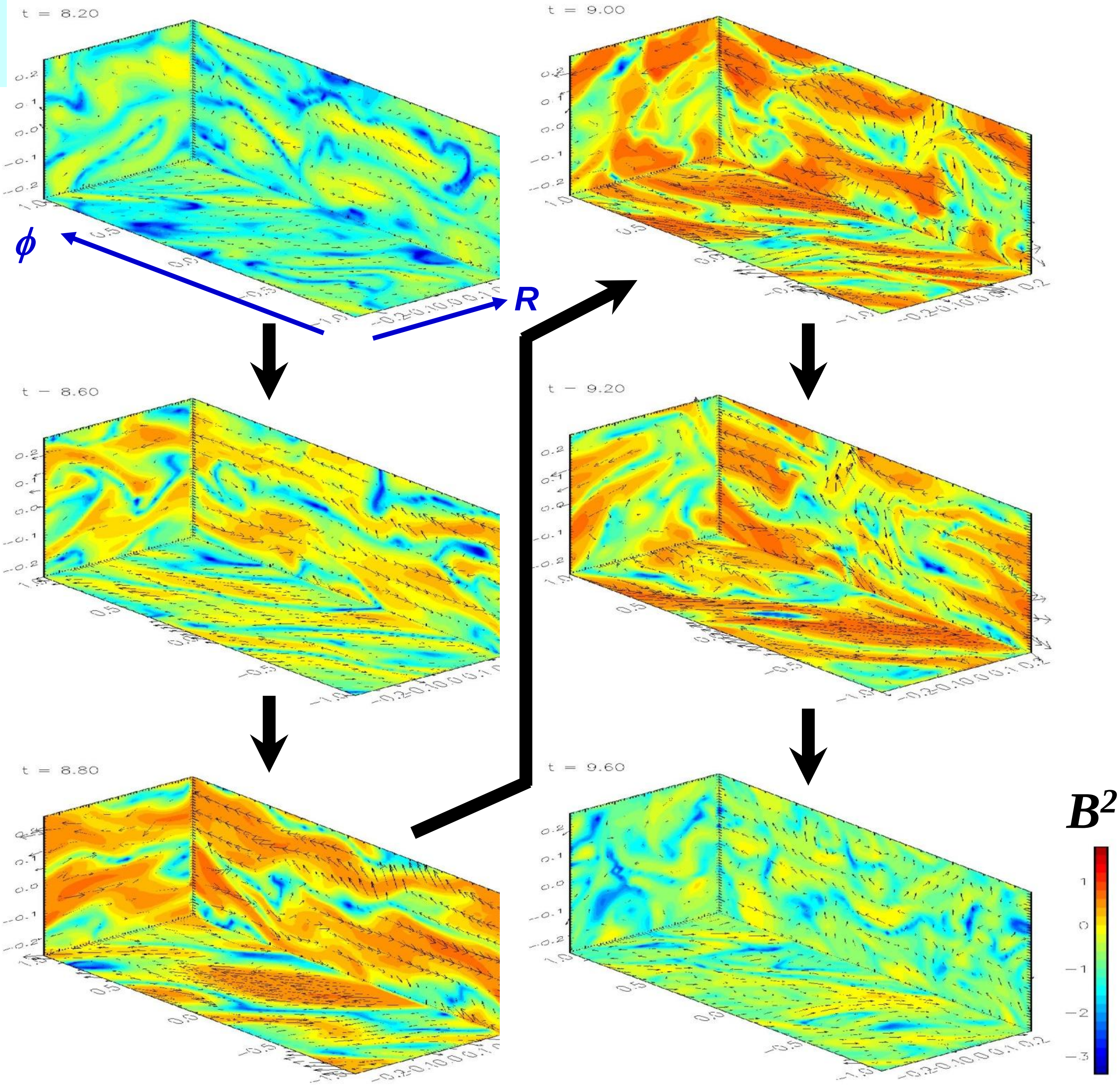


3D Calculations
$Re_{\mathrm{M}} > 1$

Exponential Growth of Most Unstable Mode

⇒ channel flow

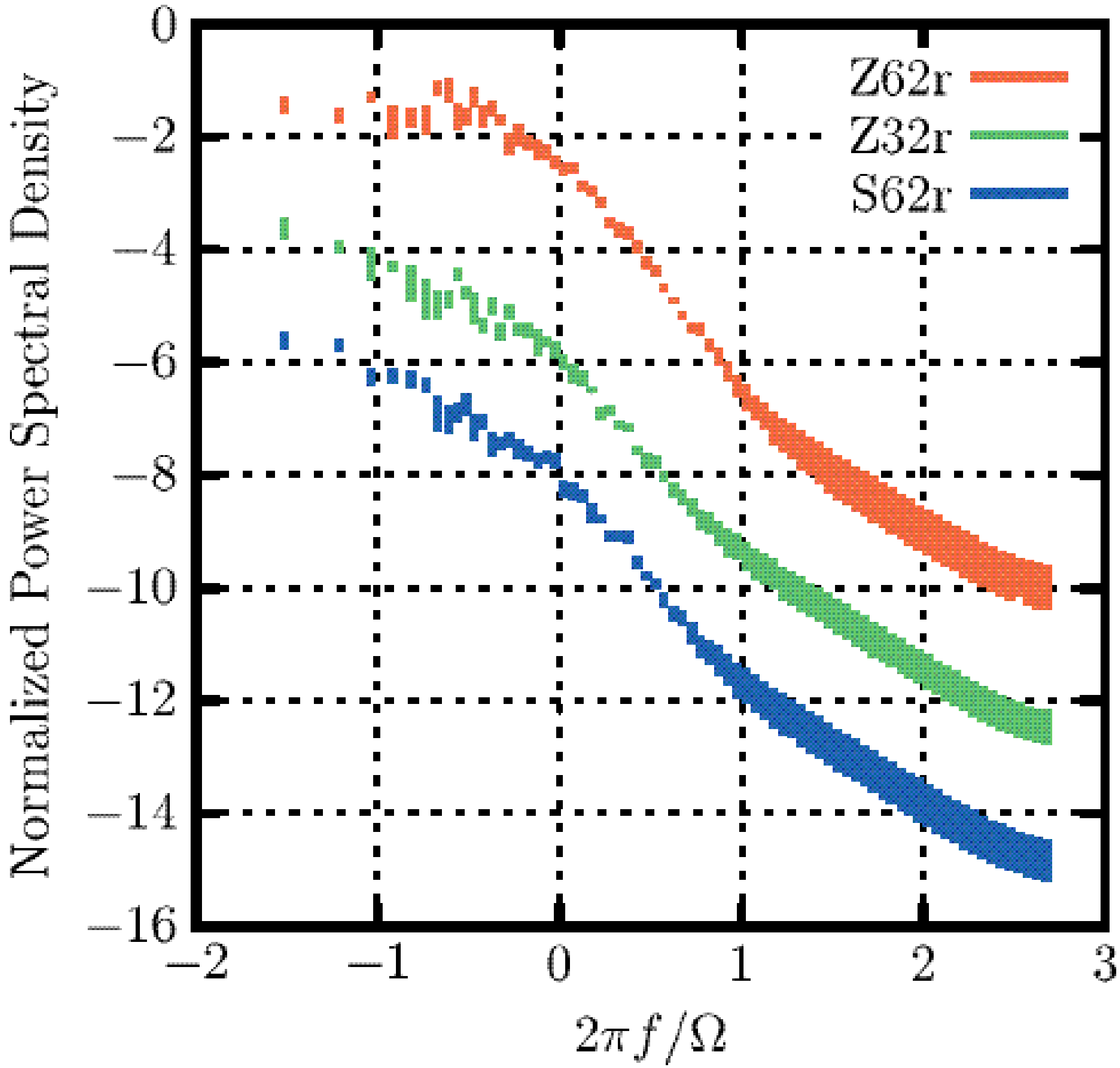
⇒ dissipation due to reconnection



Sano, SI, Turner, & Stone 2004, ApJ **605**, 321

Turbulence Spectrum

Power Spectrum of Gas Velocity, $v_{gas}(t)$



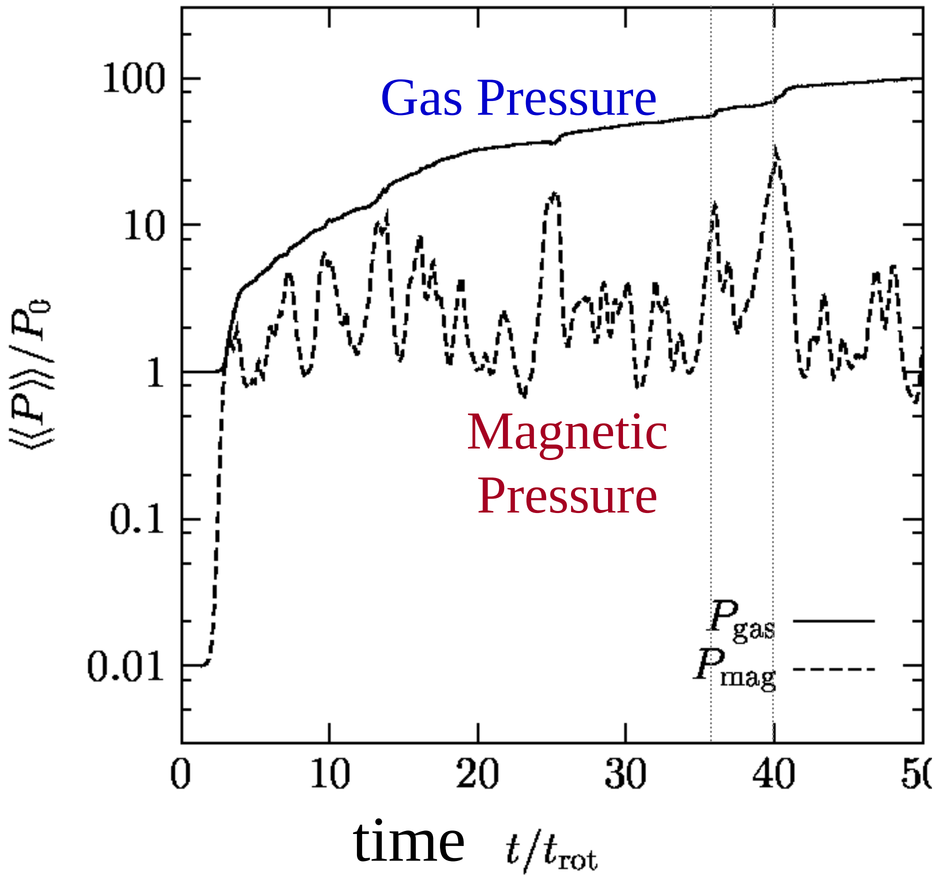
Sano, SI, Turner, & Stone
2004, ApJ **605**, 321

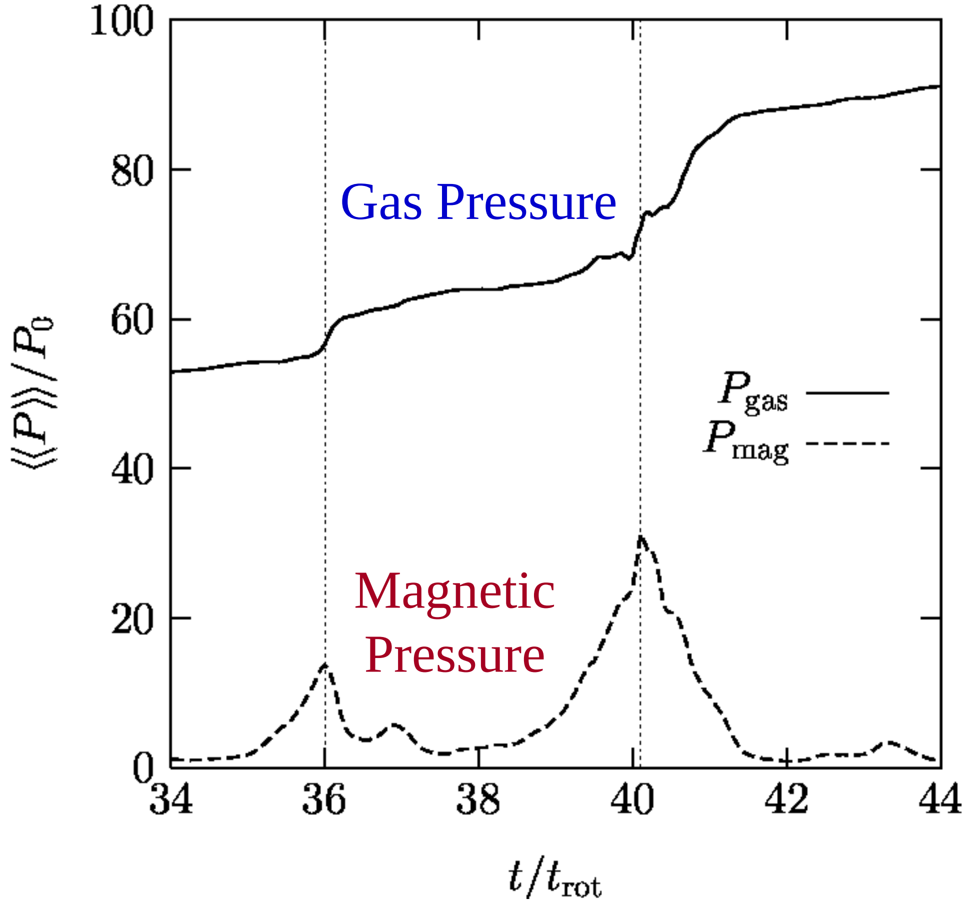
Nonlinear Time Evolution

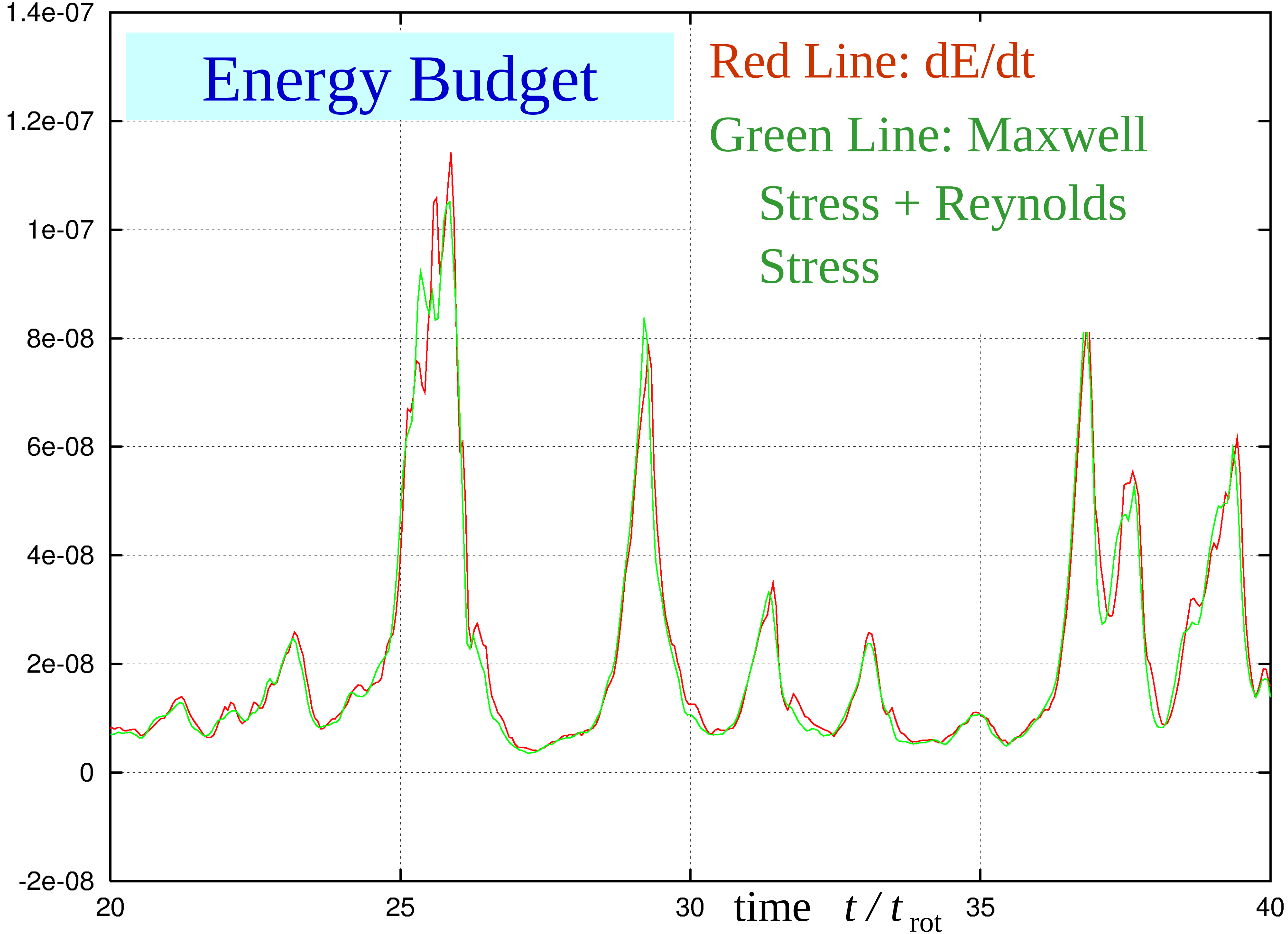
When $Re_M > 1$,

Spicky Feature in Time Evolution of Energy

= Recurrence of Exponential Growth and Magnetic Reconnection







Energy Budget
Red Line: dE/dt
Green Line: Maxwell Stress + Reynolds Stress
1.4e-07
1.2e-07
1e-07
8e-08
6e-08
4e-08
2e-08
0
-2e-08
20
25
30
35
40
time t / t_{rot}

Fluctuation vs Dissipation

$$\Gamma \equiv \iiint \left[\rho\left(\frac{1}{2}v^2 + u + \psi\right) + \frac{B^2}{8\pi} \right] dV$$

Thermal Energy

Hawley et al. 1995

$$\frac{d\Gamma}{dt} \equiv \iint \left[\rho \ \vec{v} \ \left(\frac{1}{2}v^2 + u + \frac{P}{\rho} + \psi\right) + \vec{S} \right] \cdot \overrightarrow{dA} = \frac{3}{2}\Omega \ L_x \iint_{yz面} \left(\rho \mathrm{v}_x \delta \mathrm{v}_y - \frac{B_x B_y}{4\pi} \right) dA$$

Poynting Flux

Stress Tensor, *Wxy*

$$\dot{M} \propto W_{R\phi} \equiv \rho \mathrm{v}_R \delta \mathrm{v}_\phi - \frac{B_R B_\phi}{4\pi} \propto \frac{d\Gamma}{dt} \ .$$

If saturated, $\left\langle\left\langle \frac{\partial \mathrm{v}^2}{\partial t} \right\rangle\right\rangle = \left\langle\left\langle \frac{\partial B^2}{\partial t} \right\rangle\right\rangle = 0$, then, $\left\langle \frac{d\Gamma}{dt} \right\rangle = \left\langle\left\langle \frac{\partial \rho u}{\partial t} \right\rangle\right\rangle = \frac{3\Omega}{2} \left\langle\left\langle \rho \mathrm{v}_R \delta \mathrm{v}_\phi - \frac{B_R B_\phi}{4\pi} \right\rangle\right\rangle$,

where $\langle \ \rangle$ denotes time-average, and $\langle\langle \ \rangle\rangle$ denotes time- and spatial- average.

Note that $\langle \mathrm{v}_R \rangle = \langle \delta \mathrm{v}_\phi \rangle = \langle B_R \rangle = \langle B_\phi \rangle = 0$.

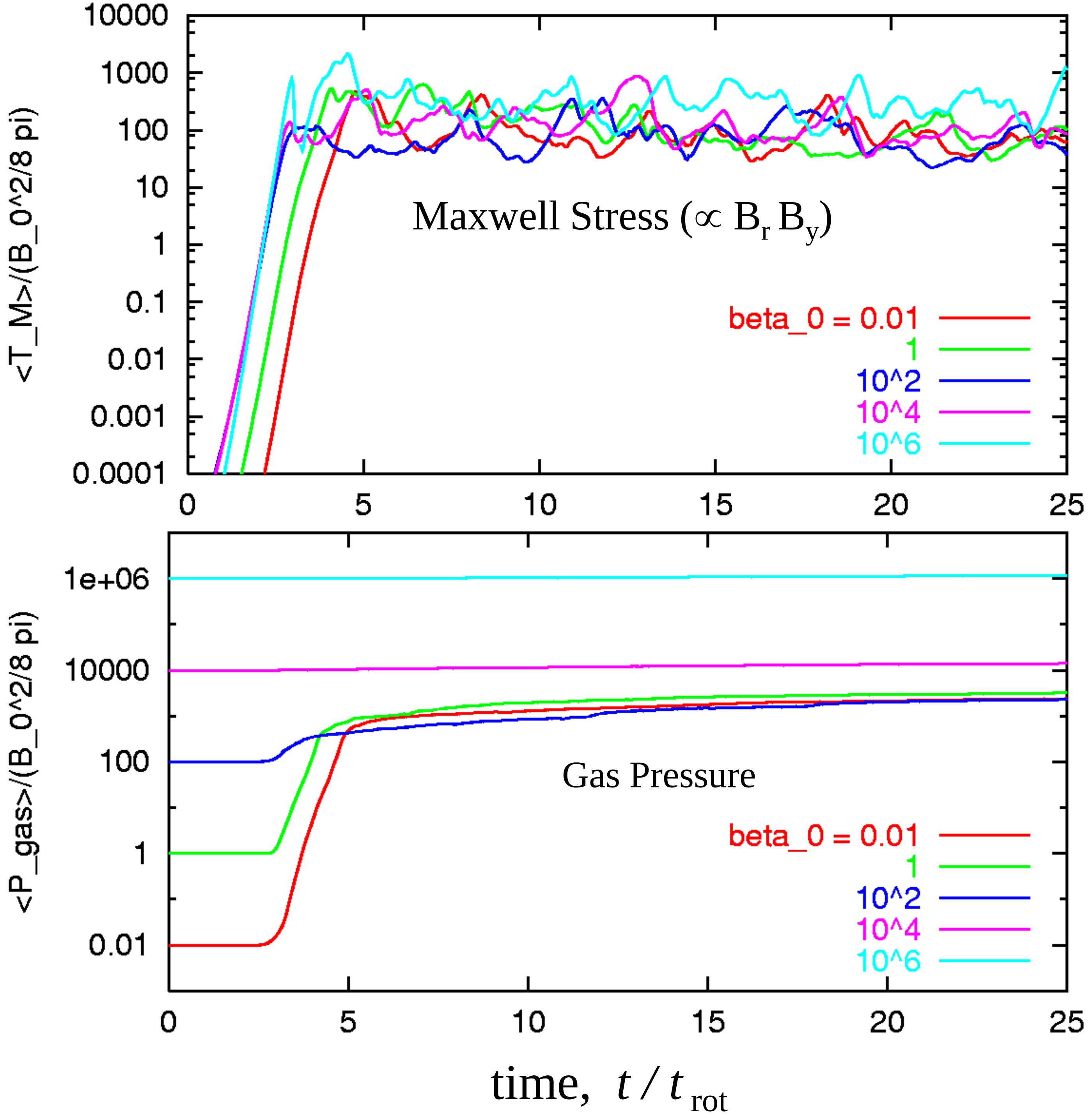
Sano & SI (2001) ApJ **561**, L179

Saturation Value of $\langle\langle B^2 \rangle\rangle \Rightarrow$ Dissipation Rate $\approx 0.03\Omega \langle\langle B^2 \rangle\rangle$

SI & Sano (2005) ApJL **628**, L155

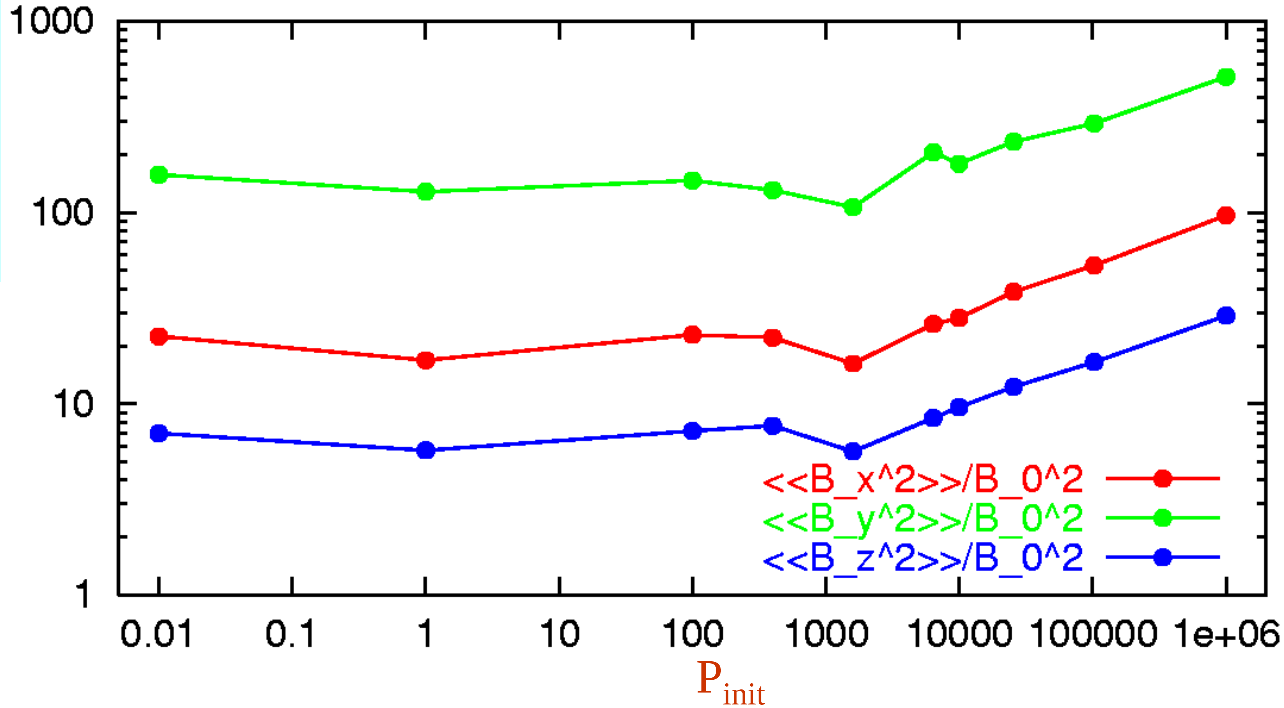
Evolution of Pressure

Monotonic Increase of Pressure because of no cooling

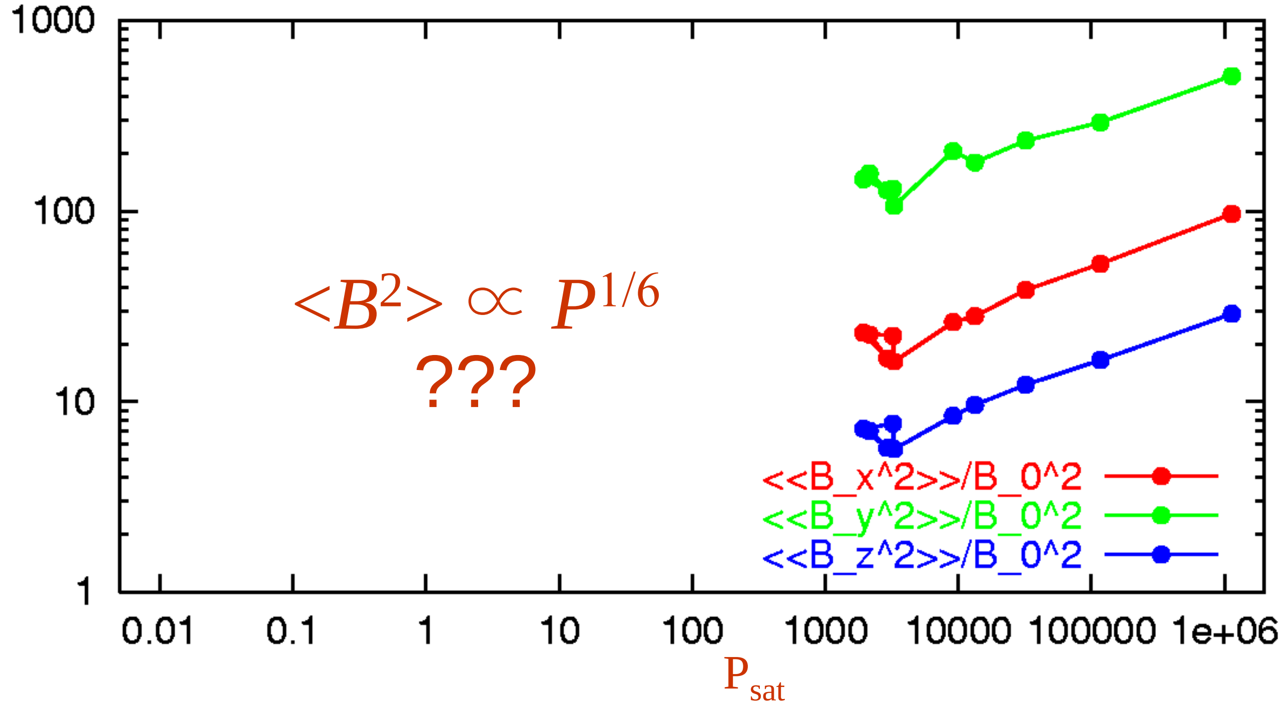


Saturation Level

dependence on the <u>initial</u> pressure



dependence on the <u>resultant</u> pressure



Discussion 1: Saturation Level?

$$\langle\langle \rho v_x \delta v_y - B_x B_y / 4\pi \rangle\rangle \equiv \langle\langle \boldsymbol{B}^2 \rangle\rangle_{\mathbf{sat}} (\eta, B_{z,init}, P, L_z, \ldots) \propto \langle\langle B_z^{\ 2} \rangle\rangle$$

In the case with Net B_z

- $\mathrm{Re_m} < 1$…Strong Dependence on Resistivity ≈ 2D evolution
  Sano, SI, & Miyama, ApJ **506**, L57, 1998

- $\mathrm{Re_m} > 1$… recurrence of Channel Flow & Reconnection

  Sano, SI, Turner & Stone (2004)

  $$\langle\langle \boldsymbol{B}^2 \rangle\rangle_{\mathbf{sat}} \approx v_{Az,init}\, \rho\, L_z\, \Omega\, (P_{gas}/P_c)^{1/6} \quad \ldots\text{Why?}$$

Discussion 2: Saturation Level?

Lesur & Longaretti (2007), $\mathrm{Re_m} > 1$

Using Spectral Method for Incompressible Fluid

$$\langle\langle \boldsymbol{B}^2 \rangle\rangle_{\mathbf{sat}} \propto (\mathrm{Pr})^{\delta},\ \delta = 0.25\text{–}0.5$$

where Magnetic Prandtl number is $\mathrm{Pr} \equiv \nu_{\mathrm{viscosity}} / \eta_{\mathrm{resistivity}}$

➔ Importance of Turbulent Reconnection?

cf.) Lazarian & Vishniac (1999)

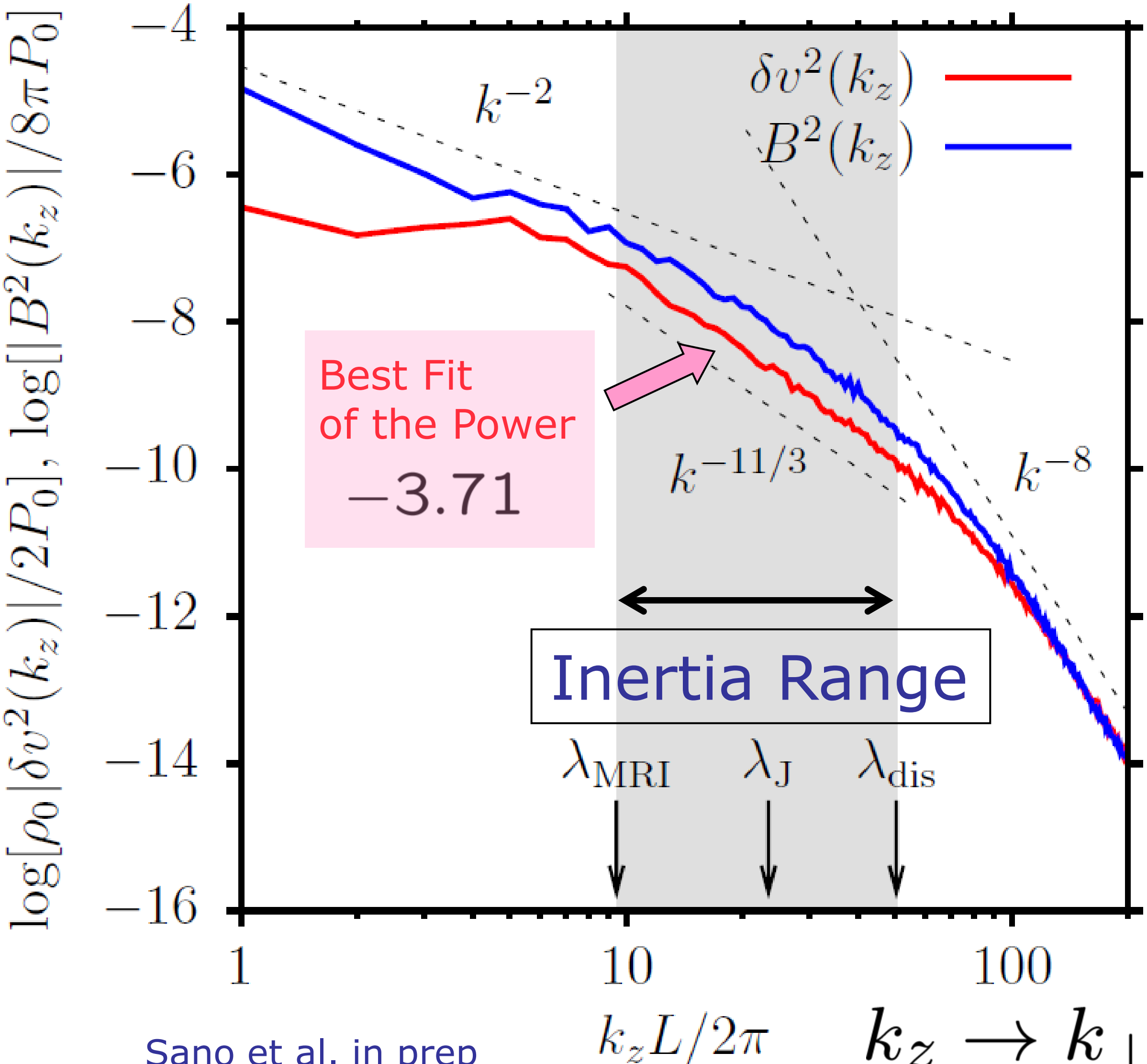
ν, viscosity ↑

➔ Size of Smallest Eddy ↑

➔ Turbulent Reconnection Rate ↓

➔ Saturation Level ↑

Spectrum for Motion ⊥ $\boldsymbol{B}$ field



$k_z \to k_\perp$

MRI Active Range

$$\lambda_{MRI} = 2\pi \frac{\langle v_{Az}^2 \rangle^{1/2}}{\Omega}$$

Dissipation Dominant Range

$$k^2 \eta \sim \Omega$$

$$\lambda_{dis} = 2\pi \sqrt{\frac{\eta}{\Omega}}$$

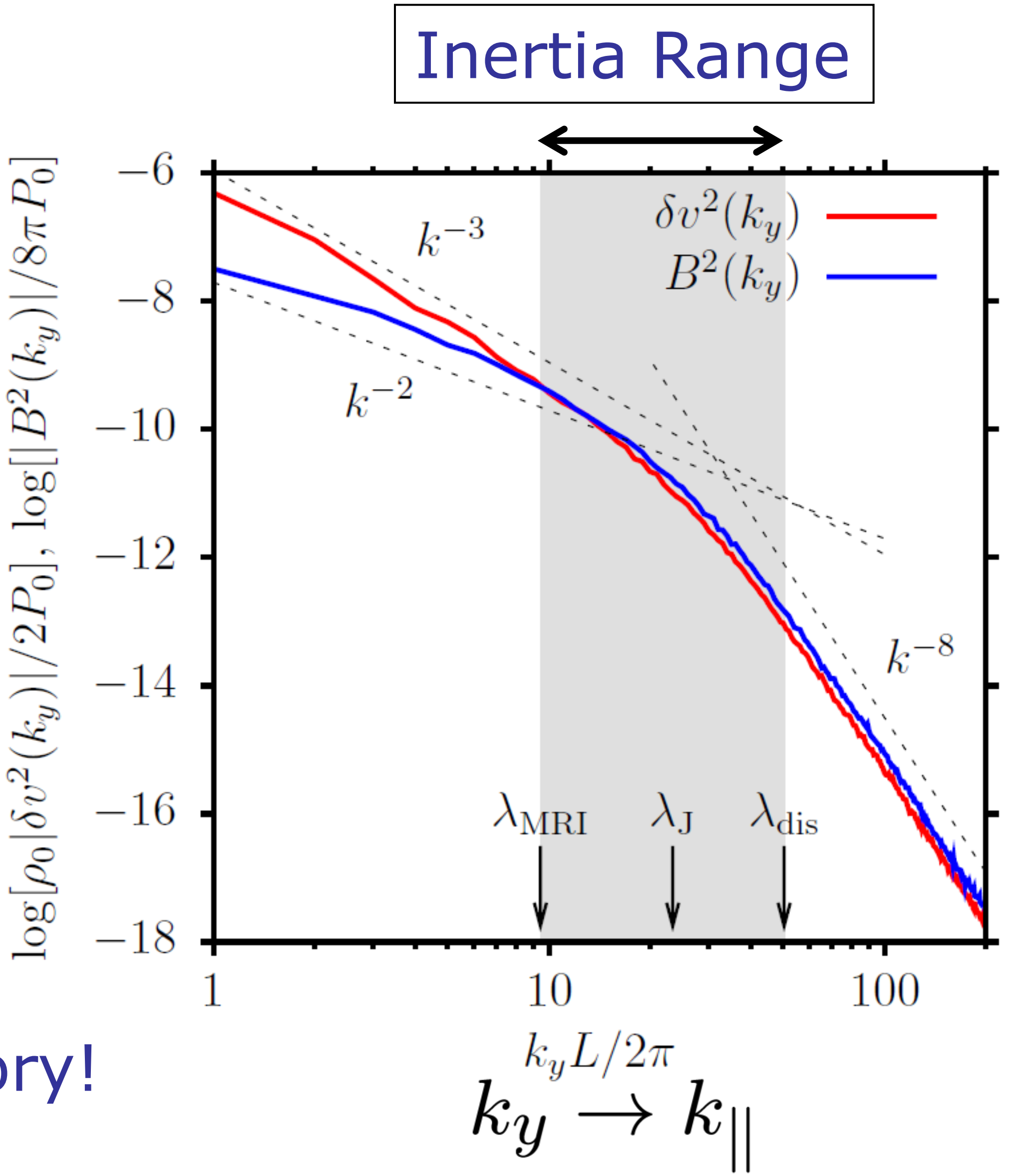
Sano et al. in prep

Spectrum for Motion // *B* field

- **Vertical Direction**
  - Kolmogorov Spectrum
- **Azimuthal Direction**
  - Weaker Power
  - Steeper Decline

- Many Similarities to Goldreich-Sridhar Spectrum

We need turbulence theory!



Summary

Results of 3D Resistive MHD Calculation

When Magnetic Reynolds Number（Re_m）> 1

Exponential Growth from very small *B*

- Growth Rate = $(4/3)\Omega$… independent on *B* Field Strength

cf. Kinematic Dynamo

- $\lambda_{\text{maximum growth}}$ becomes larger as *B* becomes greater.

➔ Inverse Cascade of Energy

Saturated States…≠ Energy Equipartition

Classified by Re_m

- $Re_m < 1$…quasi-steady saturation similar to 2D results
- $Re_m > 1$… recurrence of Channel Flow & Reconnection

Fluctuation-Dissipation Relation

《Energy Dissipation Rate》 ∝ 《$\rho v_x \delta v_y - B_x B_y / 4\pi$》

∝ Mass Accretion Rate