Magnetic Field Amplification in SNR by Richtmyer-Meshkov Instability

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parallel shock

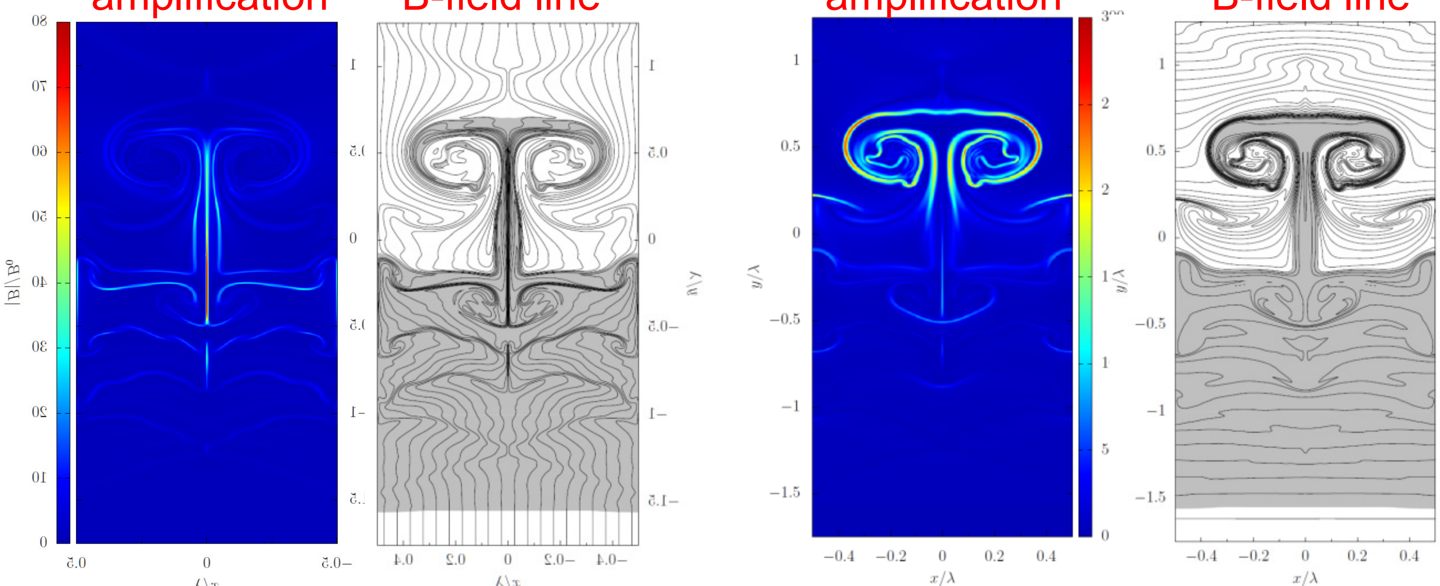
amplification

B-field line

amplification

B-field line

perpendicular shock



NDAMS, Kyoto, 2011/10/31-11/3

Outline of my talk

1. Introduction and background of the research
   - Recent observations indicate strong magnetic field amplification ($\geq$ 100 times) in SNR (Supernova Remnant).
   - Richtmyer-Meshkov instability: nonuniform velocity shear left by rippled shocks (Wouchuk & Nishihara PoP (97), Nishihara et al Phi. Trans. R. Soc. A (10))

2. 2D MHD simulation results of B-field amplification ($\geq$ 100 times)
   - Three cases: a shock perpendicular, parallel and oblique to B-field

3. Physical mechanism of the magnetic field amplification
   - Stretching of the interface and spike due to RMI along the B-field

Rapid variation of synchrotron X-ray intensity (~ 1 yr.) indicates strong magnetic field amplification (~100) in young SNR

X-ray map of SNR: RXJ1713, Uchiyama (07)

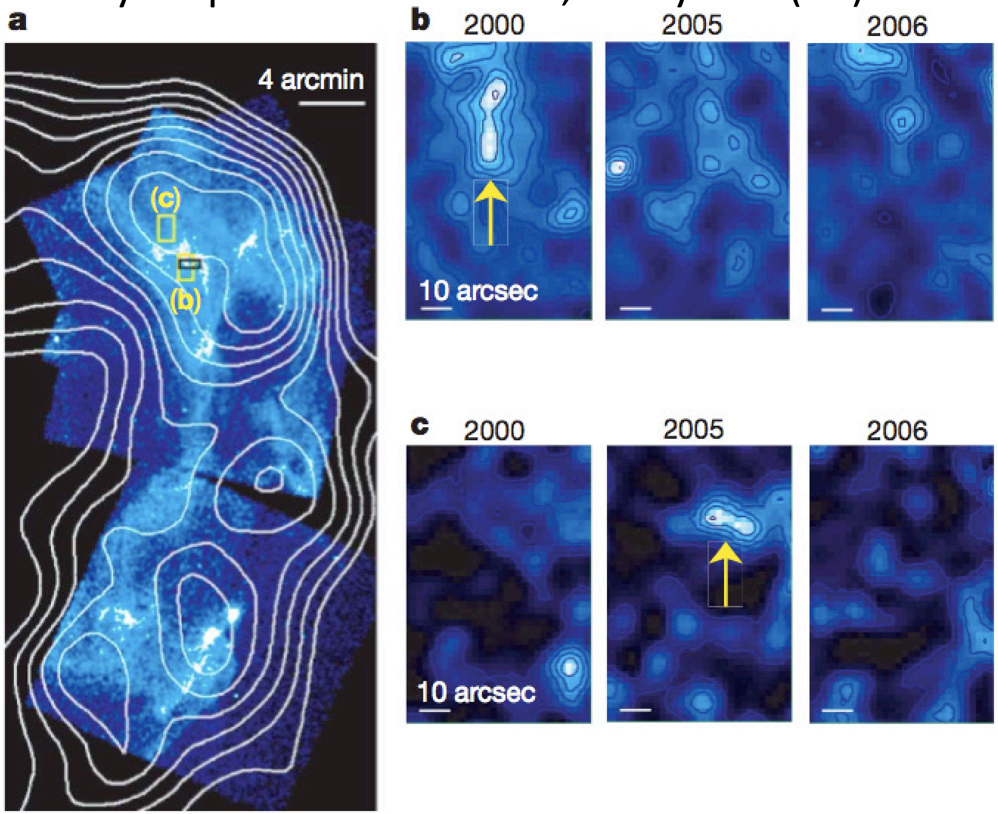


Figure 1 | Chandra X-ray images of the western shell of SNR

Synchrotron X-ray variability: ~ 1 yr.
(Uchiyama (07))

Synchrotron cooling rate:

$$t_{synch} \approx 1.5\left(\frac{B}{mG}\right)^{-1.5}\left(\frac{\varepsilon}{keV}\right)^{-0.5} \text{yr}$$

→ $B \approx 0.1 - 1mG$

$(B_{ISM} \sim 5\mu G)$

Nonuniform Inter Stellar Matter
X-ray image (color scale)
<-> Synchrotron emission
CO (j=1-0) line emission (iso-contour)
<-> molecular cloud ($n \sim 100$ cm^{-3})

B-field amplification: ~100

ISM (Inter Stellar Matter) consists of CNM (Cold Neutral Medium) and WNM (Warm Neutral Medium)

ISM: an open system radiation heating / cooling

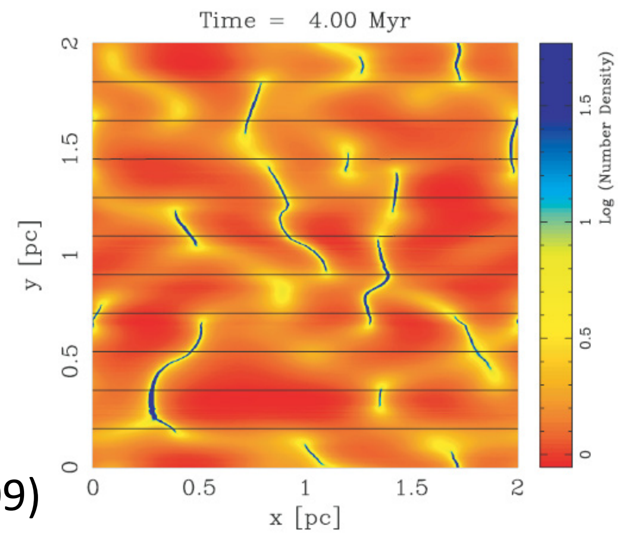
Heating:
radiation and cosmic rays from stars

⇨ Heating rate $\propto n$

Cooling:
line emission from excited atoms and molecular

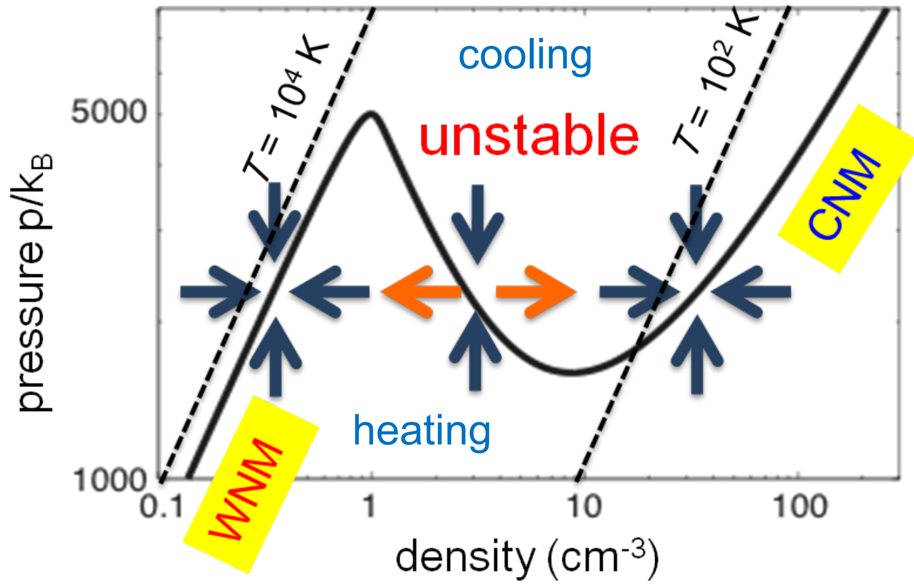
T<10³K: atomic fine structure (ε~0.01eV)
CO rotational transition
T>10³K: electron transition (ε~1eV)
(Ly-α, C, O, Fe etc)

⇨ Cooling rate $\propto n^2 e^{-\varepsilon/kT}$



Inoue, Yamazaki, Inutsuka (09)

equilibrium states (Field(69); Wolfir(95))
(balance between heating and cooling)



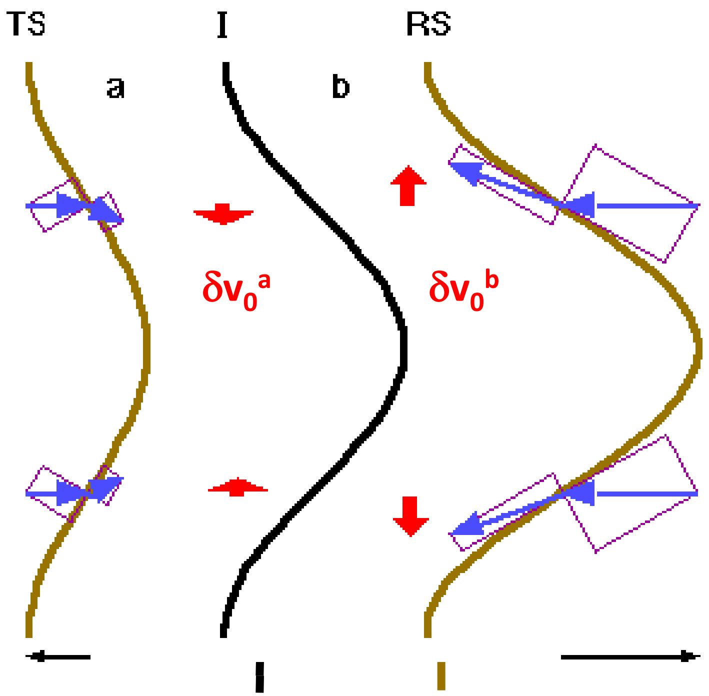
WNM (low density n~1cm⁻³): stable
CNM (high density n>10cm⁻³): stable

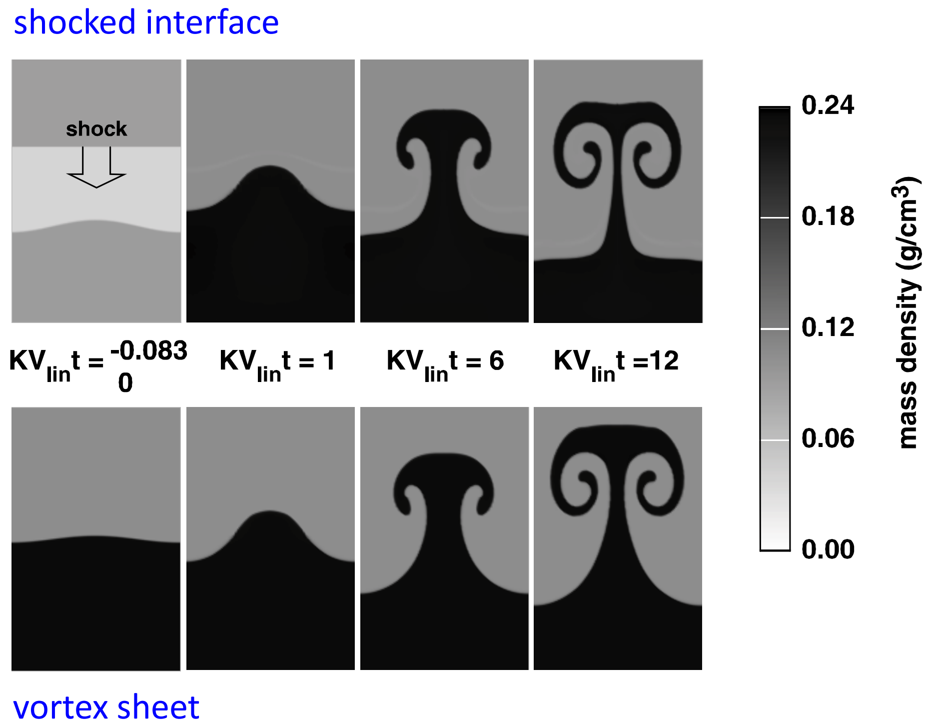
unstable domain
for iso-pressure perturbations

$$\left[\frac{\partial}{\partial T}\left(\frac{\mathcal{L}}{T}\right)\right]_p < 0$$

$\mathcal{L}(\rho, T)$: cooling rate per unit mass

After an incident shock hits a corrugated interface, ripples on reflected and transmitted shocks are induced and RM instability is driven by velocity shear left by the rippled shocks.





from linearized relation of the shock Rankin-Hugoniot

$$\delta v_o^a = k\xi_o\left(1 - \frac{u_{st}}{u_{si}}\right)v_i, \quad \delta v_o^b = k\xi_o\left(1 - \frac{u_{sr}}{u_{si}}\right)(v_i - v_1)$$

Matsuoka, Nishihara Fukuda (PRE(03))
A=0.376, ξ_0/λ=0.02

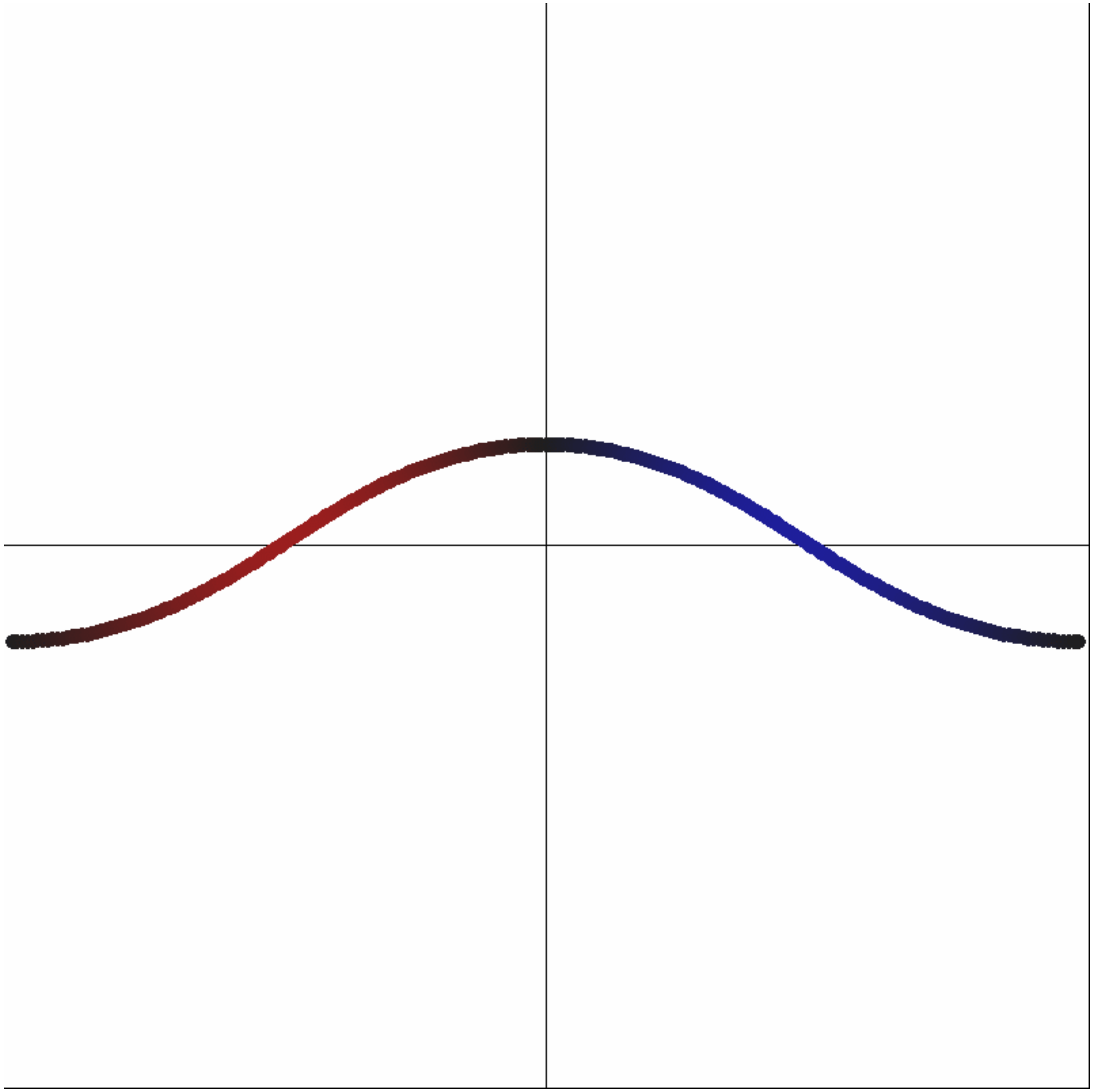
where ξ_0, k; amplitude of the initial interface corrugation and its wave number,

u_{si}, u_{st}, u_{sr}; incident, transmitted and reflected shock speeds, and

v_i, v_1; interface speed after the interaction and fluid velocity behind the incident shock.

Fully nonlinear evolution with vortex sheet model: Double spiral structure is observed as Jacobs & Sheeley experiment.

Color shows the vorticity (Matsuoka (06))



asymptotic linear growth rate v_{lin} (weak shock limit)

$$v_{lin} = \frac{\rho_{bf}\delta v_{yb}^{0} - \rho_{af}\delta v_{ya}^{0}}{\rho_{bf} + \rho_{af}}$$

Wouchuk (97)

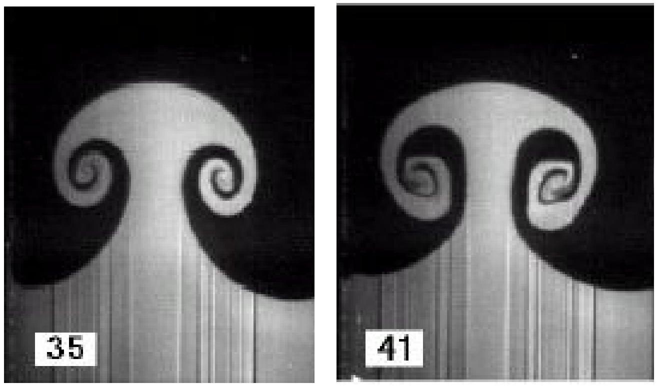
Parameters

$A = 0.155$

$k\xi_0 = 0.2$

$kv_{lin}t = 0, 1, 2,,,,12$

Jacobs



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   - Richtmyer-Meshkov instability: nonuniform velocity shear left by rippled shocks (Wouchuk (97), Nishihara (10))

2. 2D MHD simulation results of B-field amplification
   - Three cases: a shock perpendicular, parallel, and oblique to B-field
   - A shock wave propagates through a sinusoidal corrugated interface.
   - Amplification factor of magnetic field ($\geq$ 100 times)
   - parameter dependence of amplification factor

3. Physical mechanism of the magnetic field amplification
   - Stretching of the interface and spike due to RMI along the B-field

Initial Condition of 2-d MHD Simulations

- Density Jump: $\delta = \frac{\rho_2}{\rho_0} = 10$
- Mach Number of the Shock: $M = \frac{V_s}{c_{s0}} = 10$
- Initial Corrugation Amplitude: $\xi = \frac{\alpha}{\lambda} = 0.1$
- Field Strength: $\beta_0 = \frac{8\pi P_0}{B^2} = 10^8$

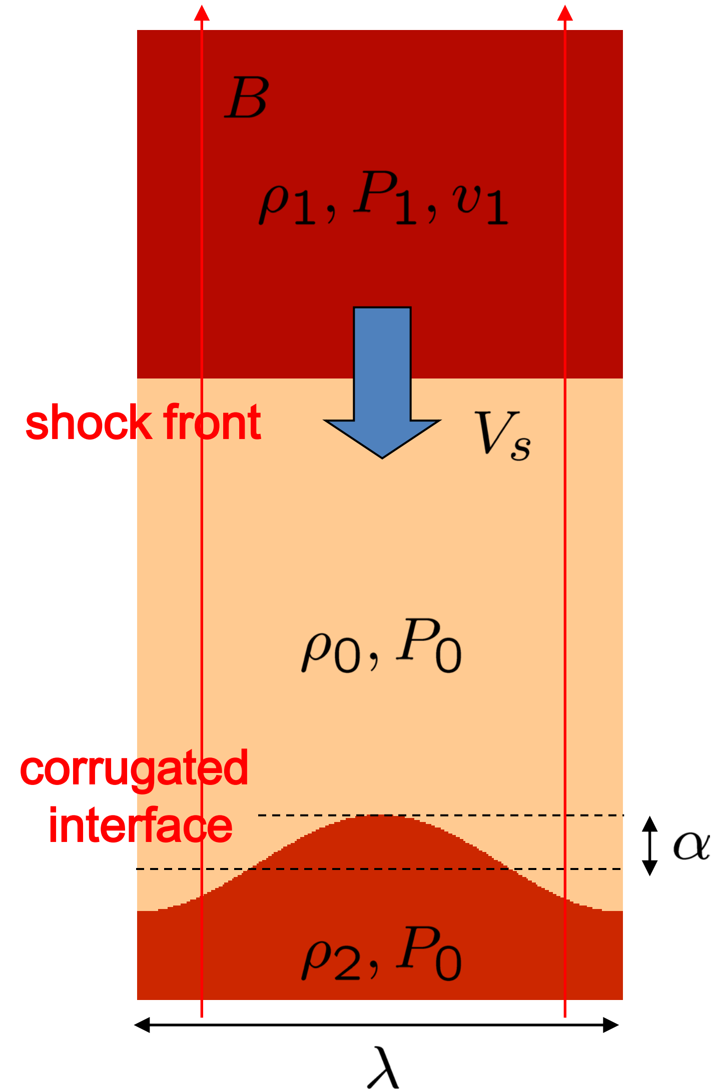
three cases

perpendicular shock (to B-field)
(B-field lines // interface)

parallel shock (to B-field)
(B-field lines ⊥ interface)

0blique shock (to B-field)
(B-field lines ∠ interface)

parallel shock

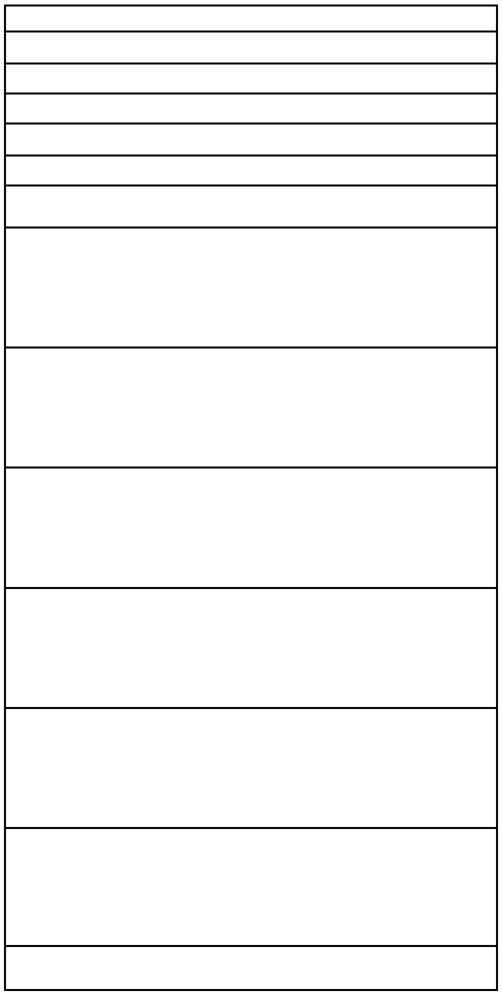


(B-field lines ⊥ interface)

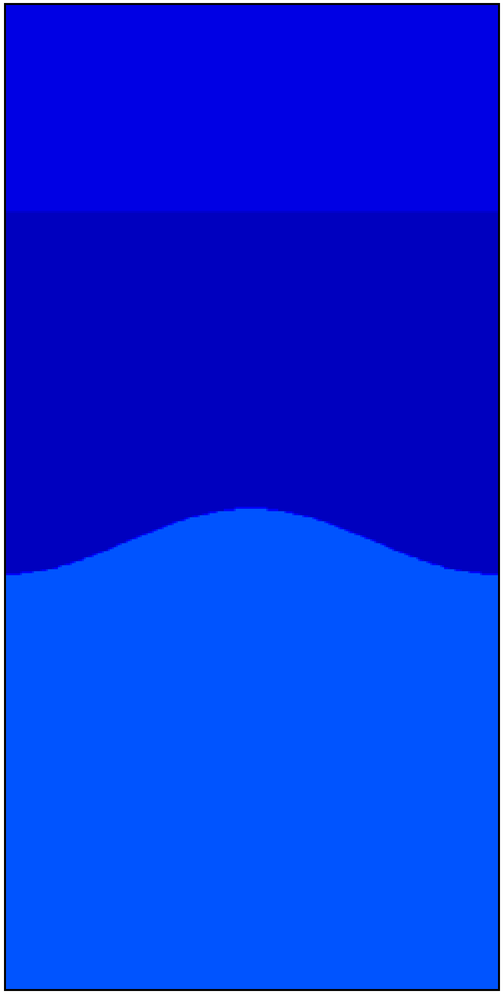
2d MHD 2
B // interface

2-d MHD simulation of a shock perpendicular to B-field

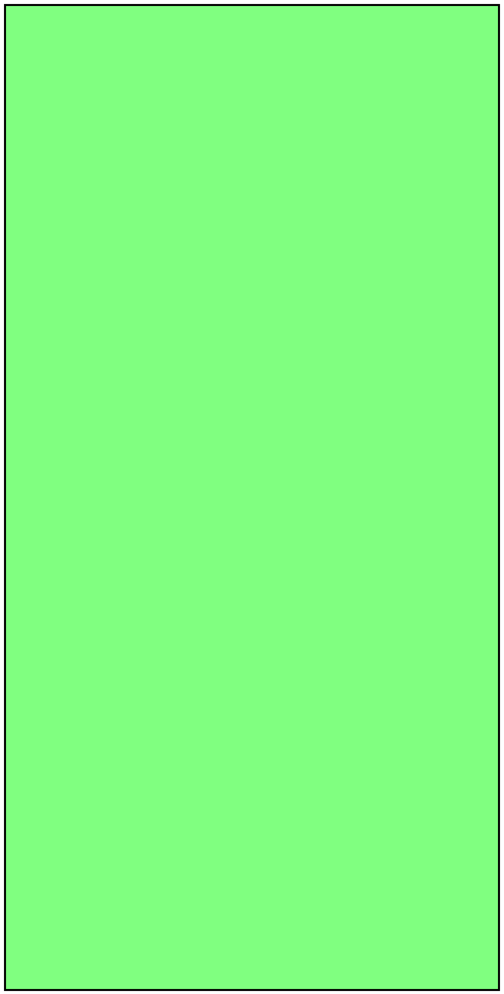
B-field lines



mass density



vorticity

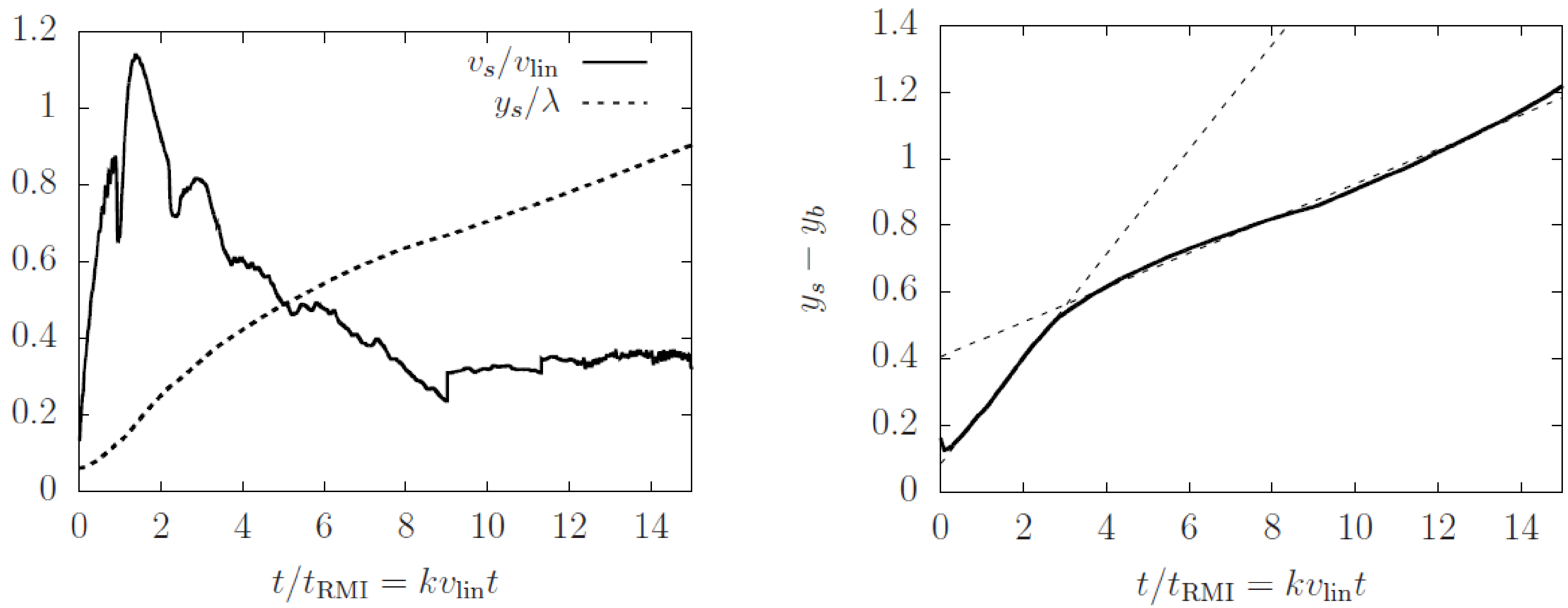


2d MHD 3
B // interface

RMI growth in 2-d MHD simulation of a shock perpendicular to B-field

time evolution of
RMI growth rate and spike height

normalized length
between spike top and bubble top



growth rate normalized by V_{lin}
length normalized by wavelength

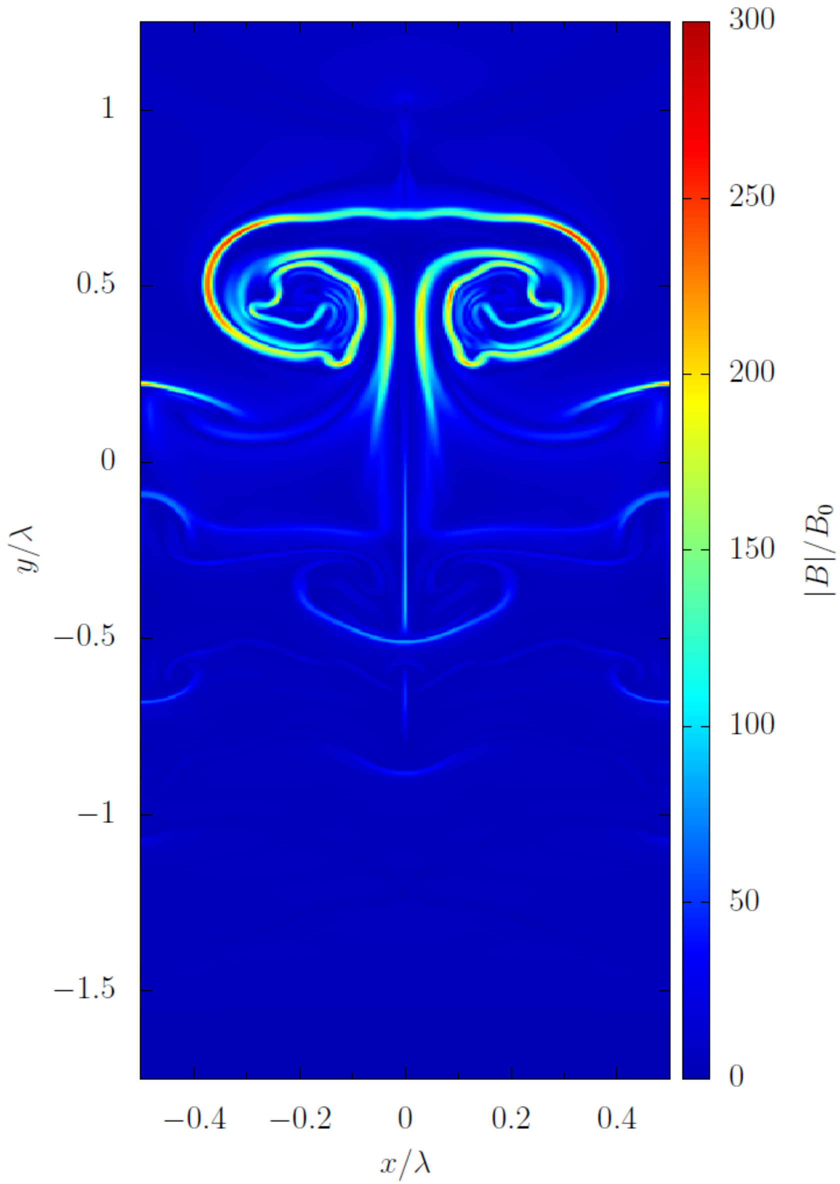
Growth rate reaches its maximum around $kv_{lin}t = 1.5$ and decreases with time

2d MHD 5
B // interface

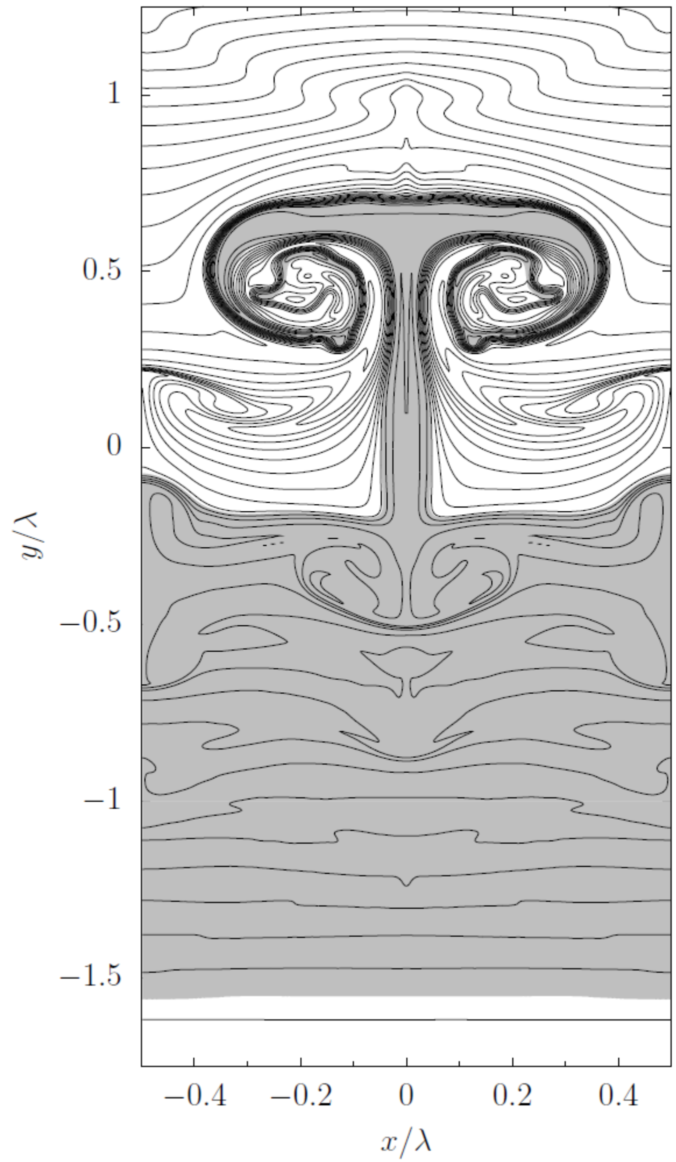
B-field amplification for a shock perpendicular to B-field, Strong amplification (~300) appears at mushroom umbrella

$kv_{lin}t = 10$

B-field ($|B| / B_0$)



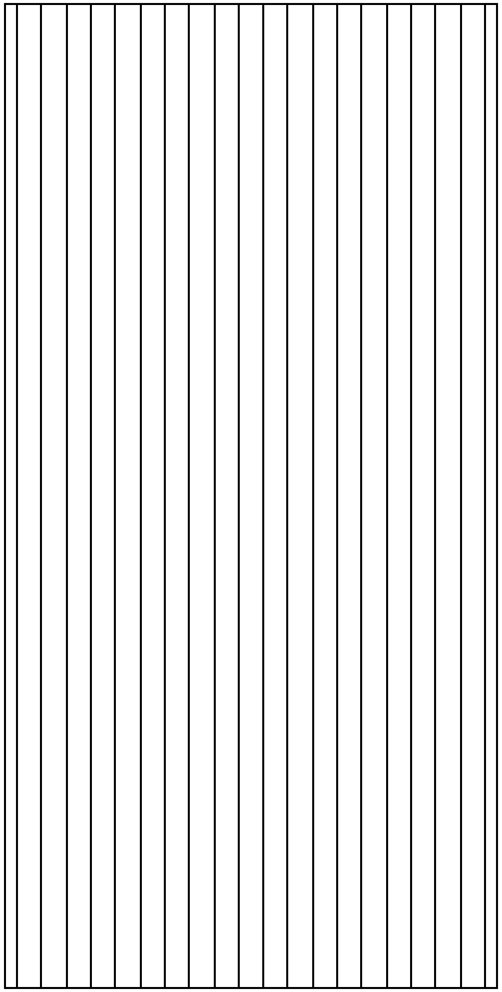
B-field lines



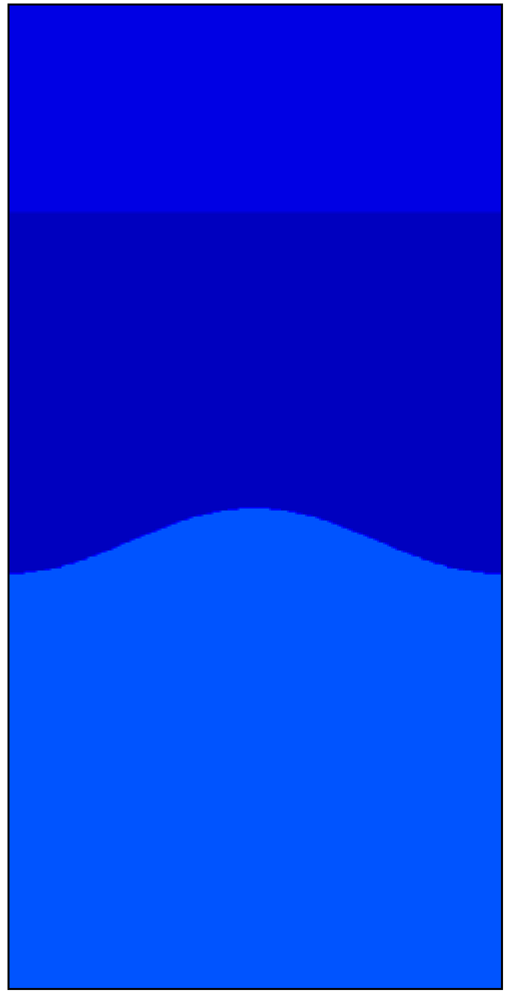
2d MHD 6
B ⊥ interface

2-d MHD Simulation for a shock parallel to B-field

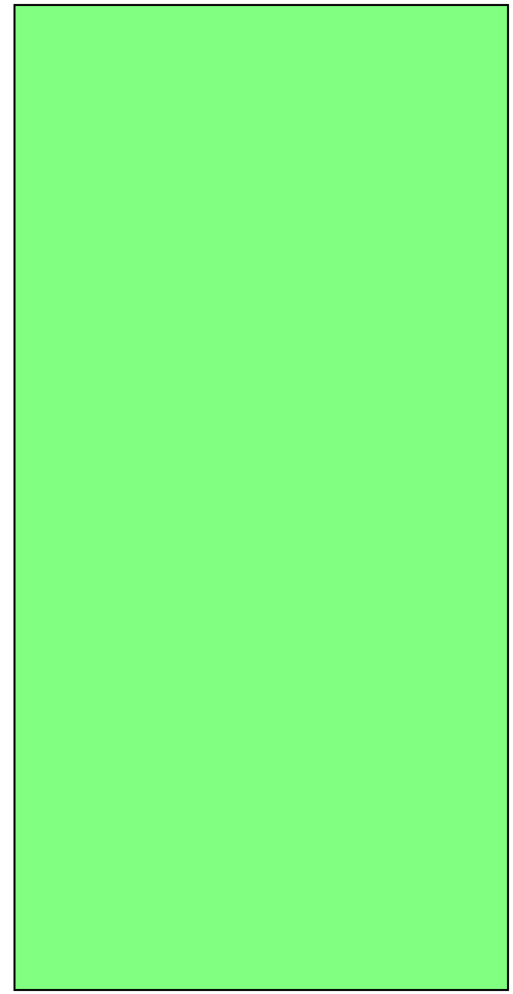
B-field lines



mass density



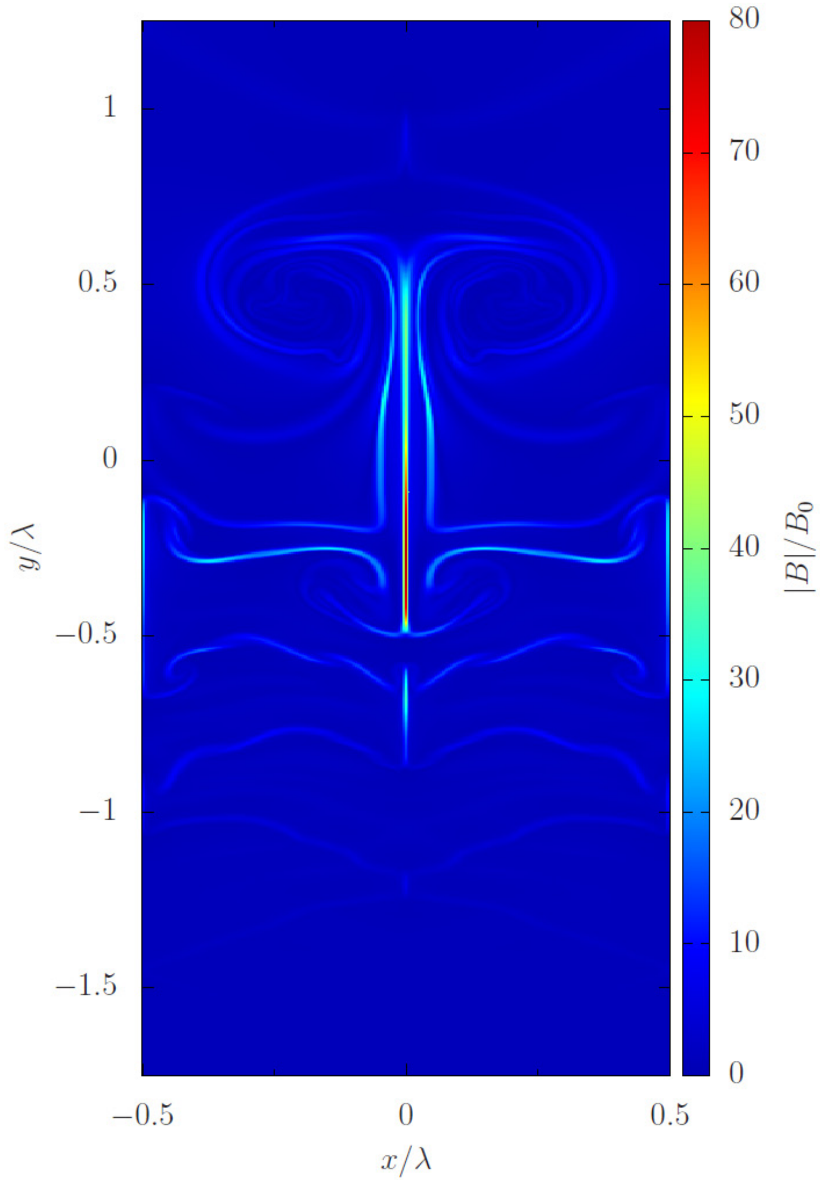
vorticity



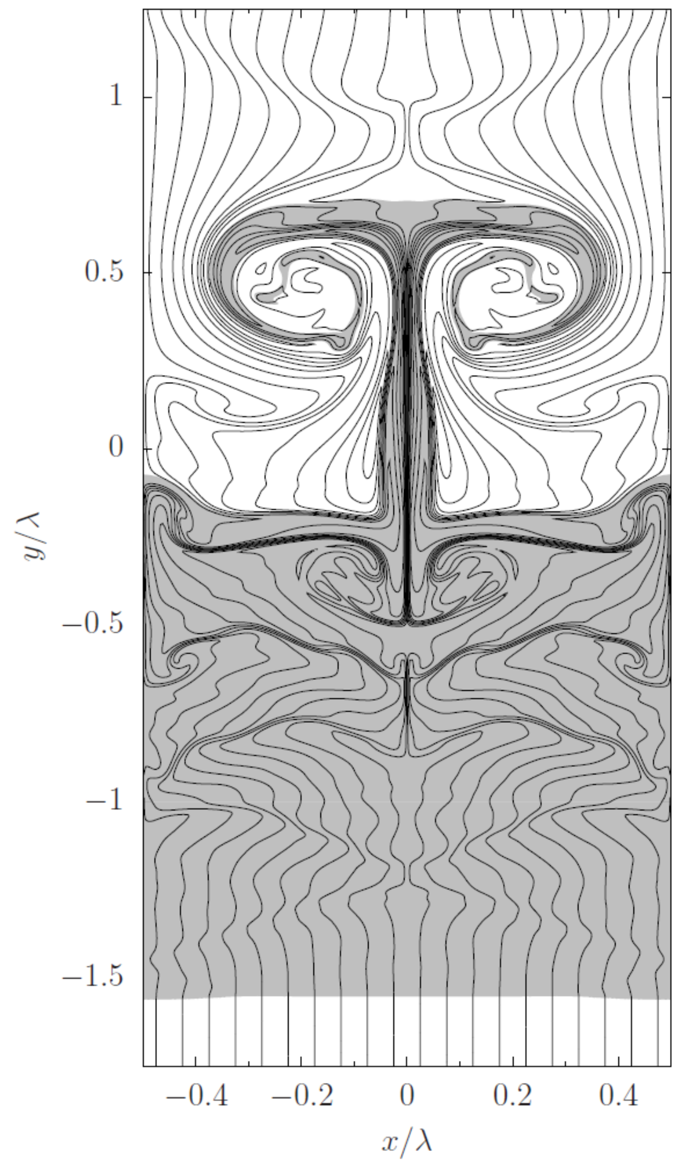
B-field amplification for for a shock parallel to B-field, Large amplification (~80) appears at the mushroom stem

$kv_{lin}t = 10$

B-field ($|B| / B_0$)



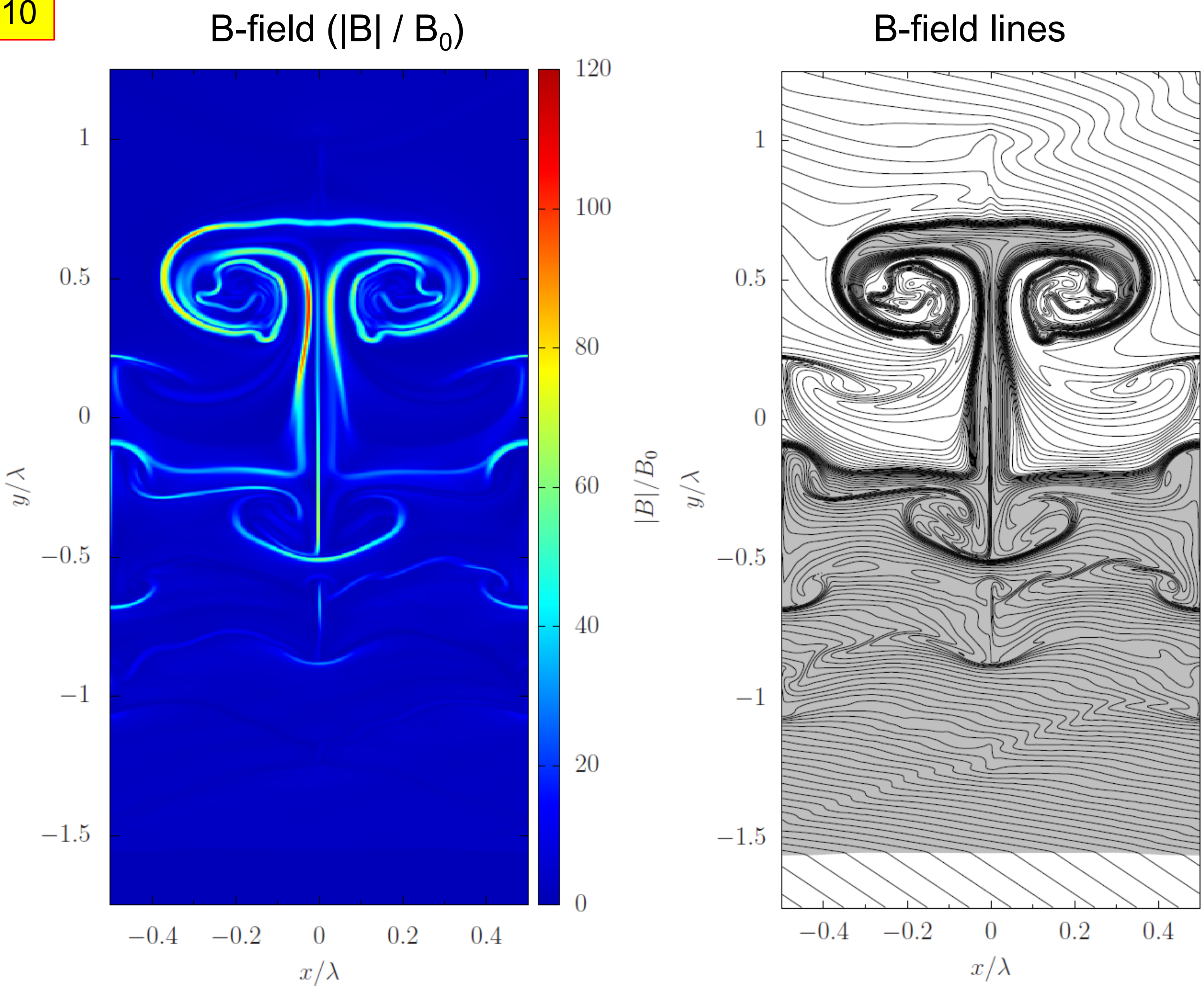
B-field lines



2d MHD 8
B∠ interface

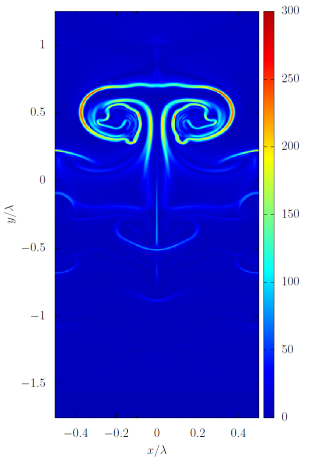
B-field amplification for the case of a shock oblique to B-field
Strong amplification (~120) appears at interface aligned to B-field

$kv_{lin}t = 10$

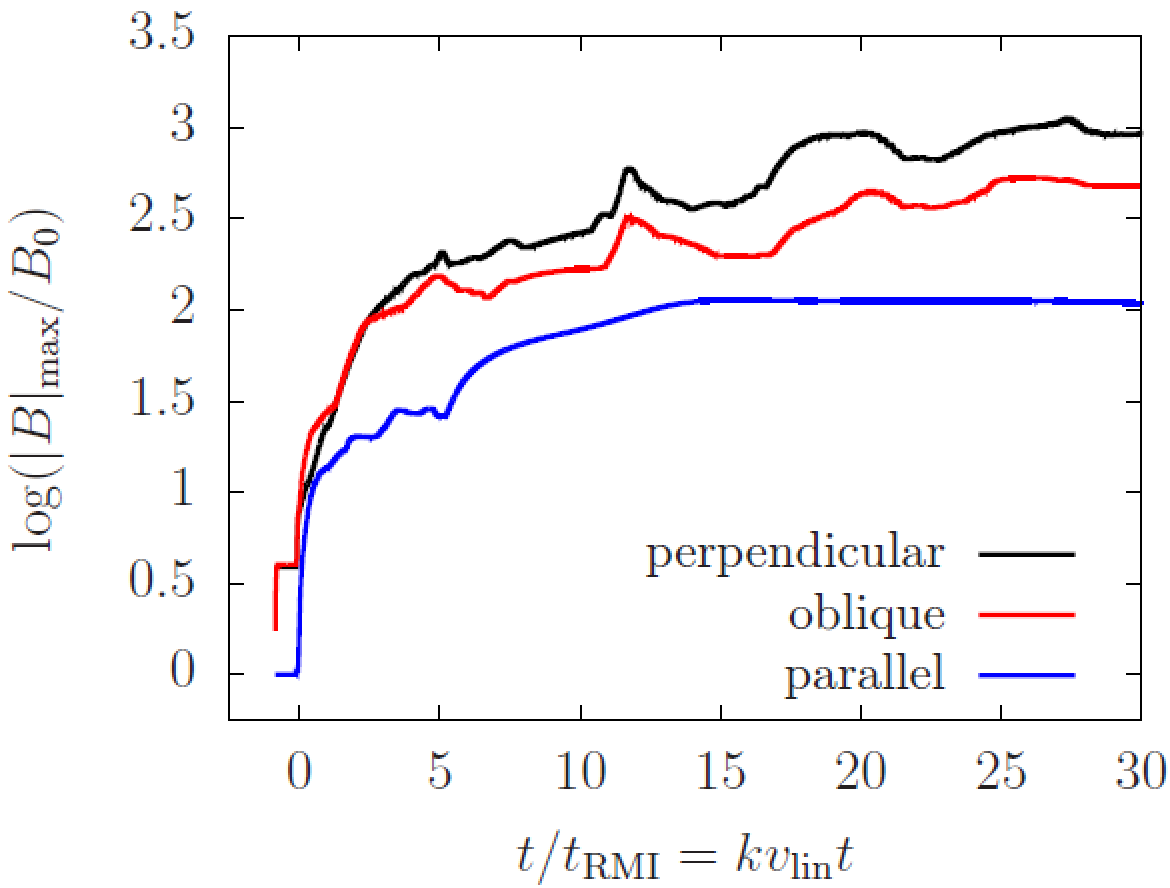


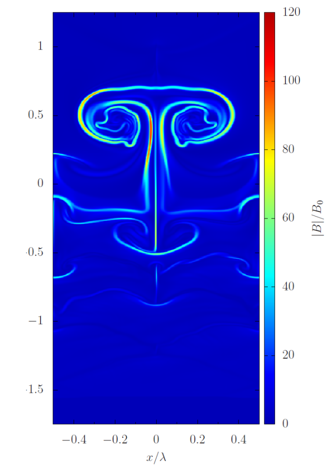
B-field amplification factors for three different cases become about 100 to 1000

B // interface

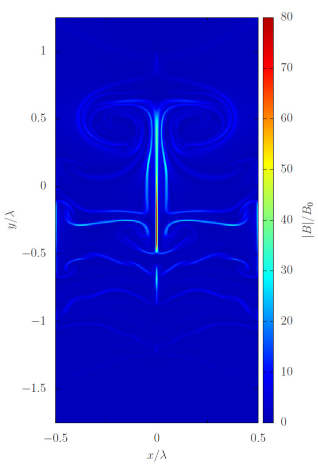


B∠ interface





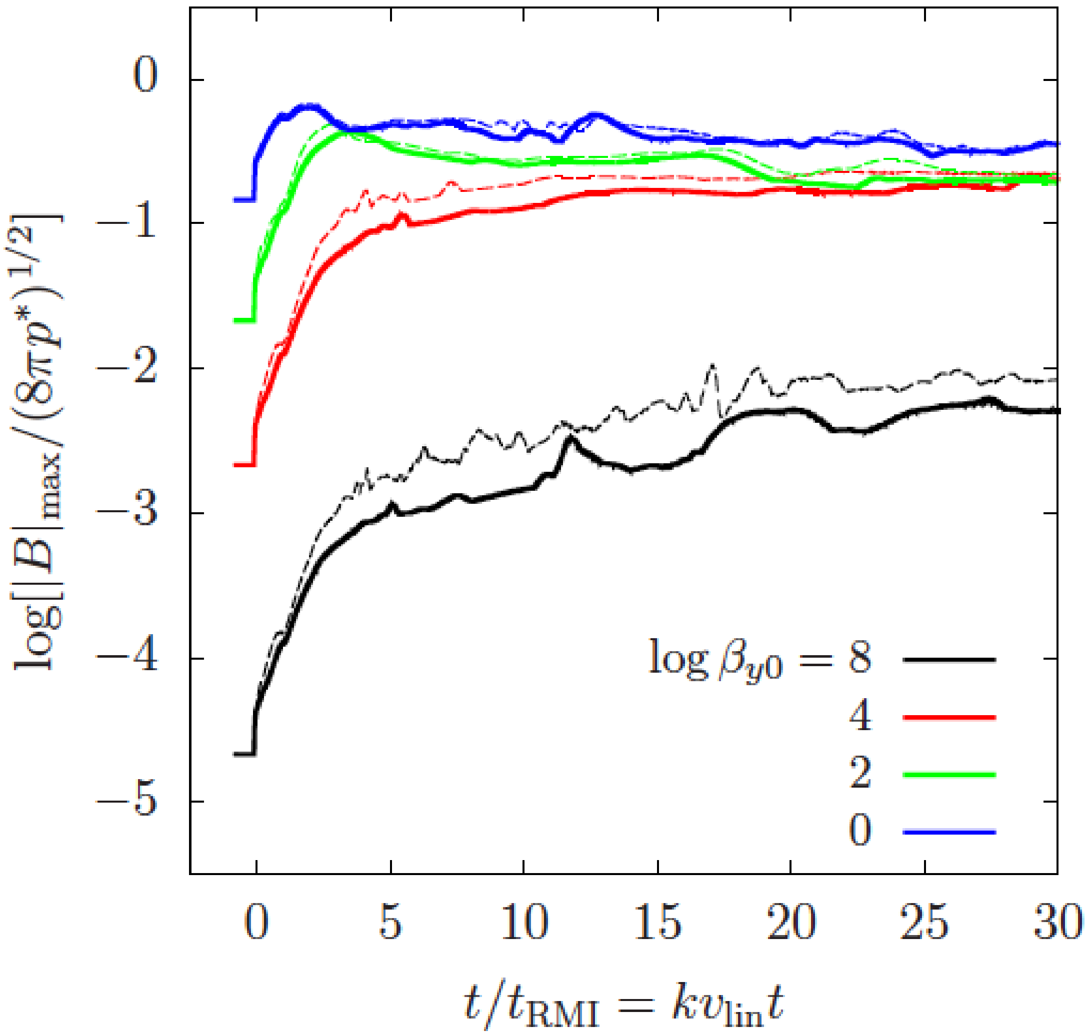
B⊥interface



Largest amplification factor is obtained for B-field // to the interface (perpendicular shock).

Magnetic pressure does not exceed to plasma pressure even for parallel shock

square root of the ratio of magnetic pressure to plasma pressure
for different its initial values



Amplification factor of B-field becomes > 100 for initial plasma $\beta > 10^3$

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3. Physical mechanism of the magnetic field amplification
   - Stretching of the interface and spike due to RMI mainly results in the amplification along the B-field

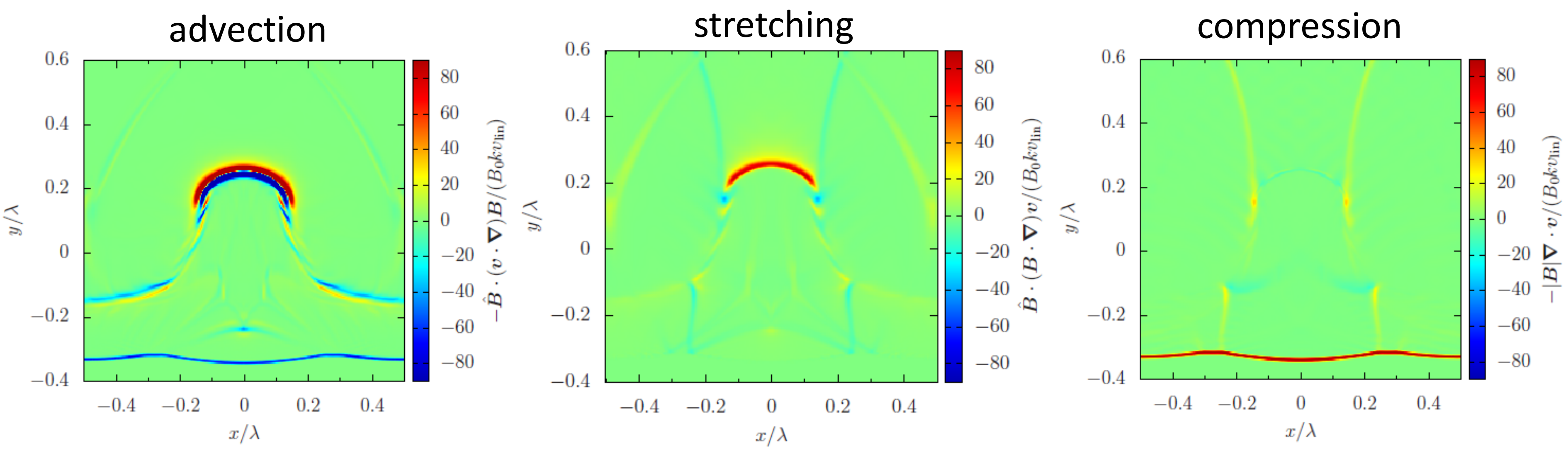
B-amplify 1

B-field amplification in ideal MHD
- advection, stretching and compression along B field line -

Magnetic field amplification

$$\frac{\partial B}{\partial t} = -\nabla \times E$$
$$= \nabla \times (v \times B)$$
$$= \underbrace{-v \cdot \nabla B}_{\text{advection}} + \underbrace{B \cdot \nabla v}_{\text{stretching}} \underbrace{- B\nabla \cdot v}_{\text{compression}}$$

$$\frac{1}{2}\frac{\partial}{\partial t}\left|B^2\right| = -B \cdot (v \cdot \nabla)B \quad \text{advection}$$
$$+ B \cdot (B \cdot \nabla)v \quad \text{stretching}$$
$$- \left|B^2\right| \nabla \cdot v \quad \text{compression}$$

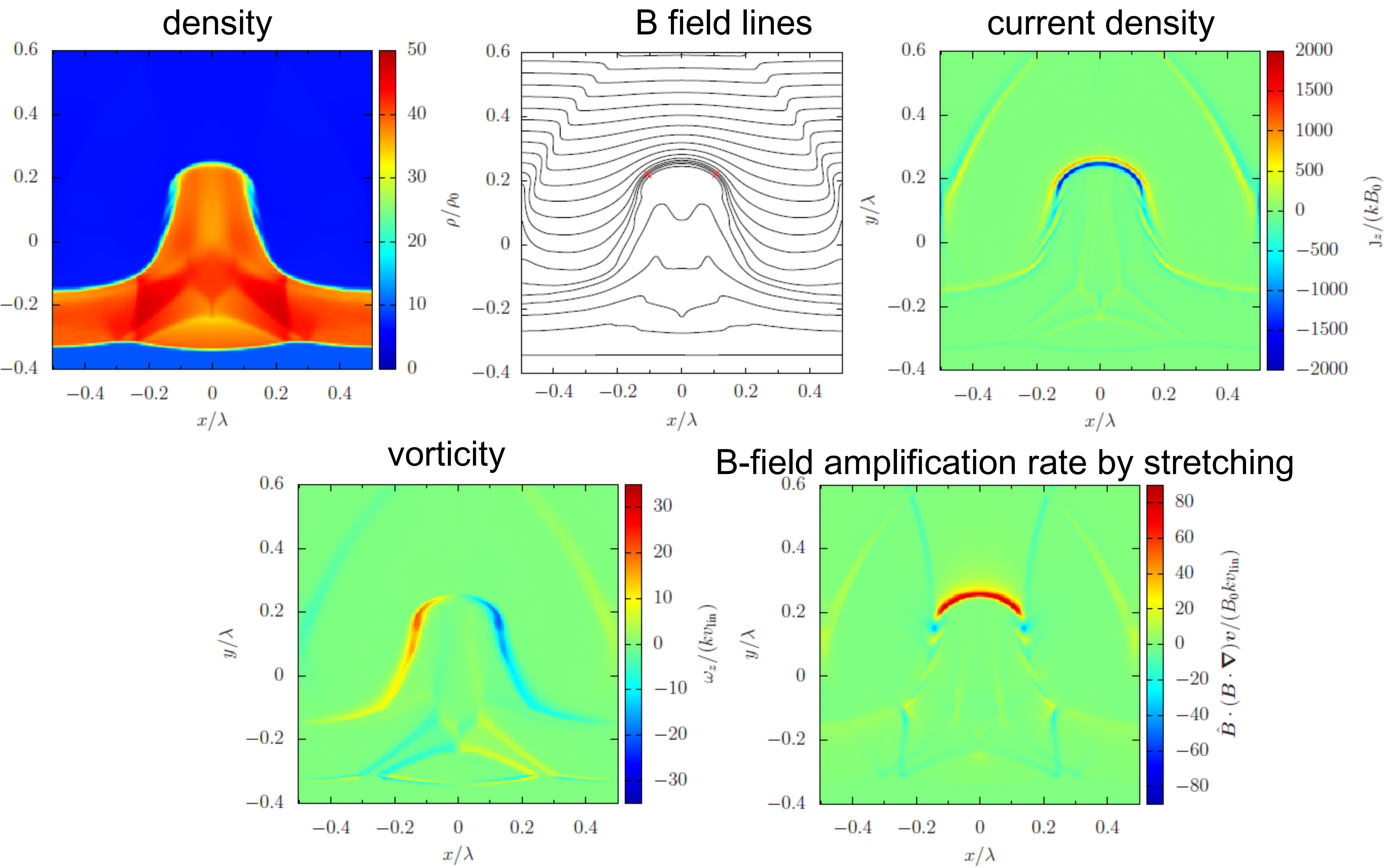


Advection does not increase B-field along the plasma

Stretching of the interface along a magnetic field mainly leads to the magnetic amplification

Stretching of the interface at the top of the spike causes B-field amplification at early stage in the case of B // interface

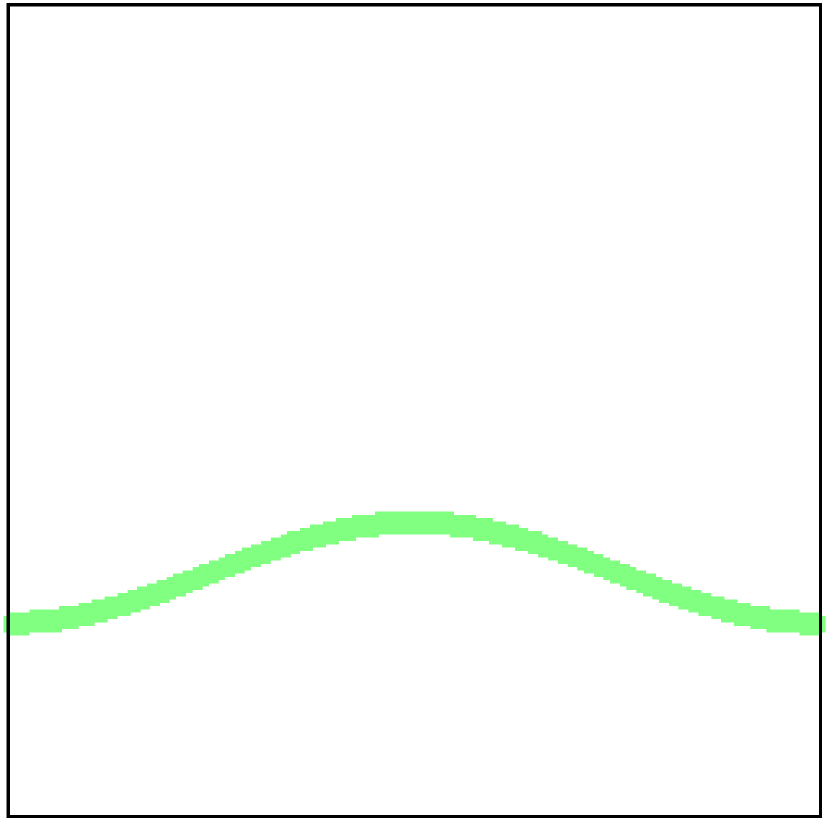
spatial profile (initial B // interface: at $kv_{lin}t$=2.0)



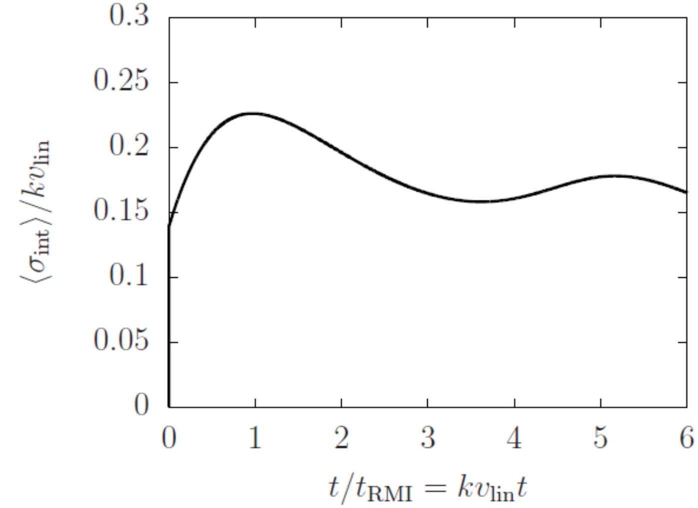
B-amplify 3
B // interface

Interface stretching rate obtained from nonlinear vortex sheet model shows large stretching rate at the top of the spike in early stage.

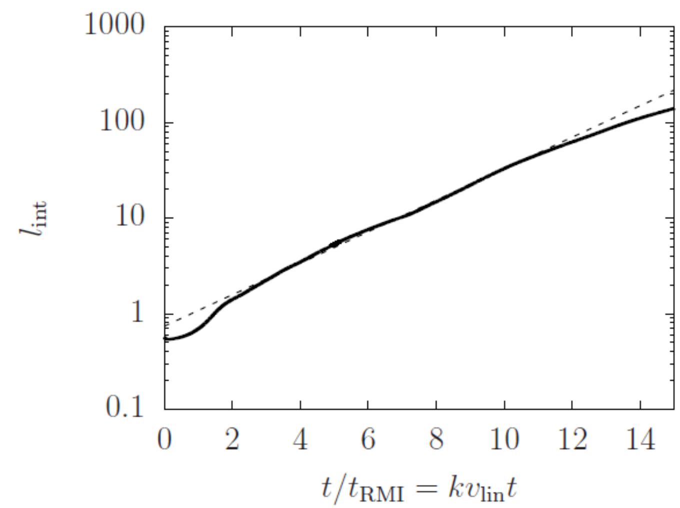
interface stretching rate
from kv_{lin} t=0 to 6



stretching rate of the interface in RMI
(nonlinear vortex sheet model)



interface length vs time
observed in 2d MHD simulation

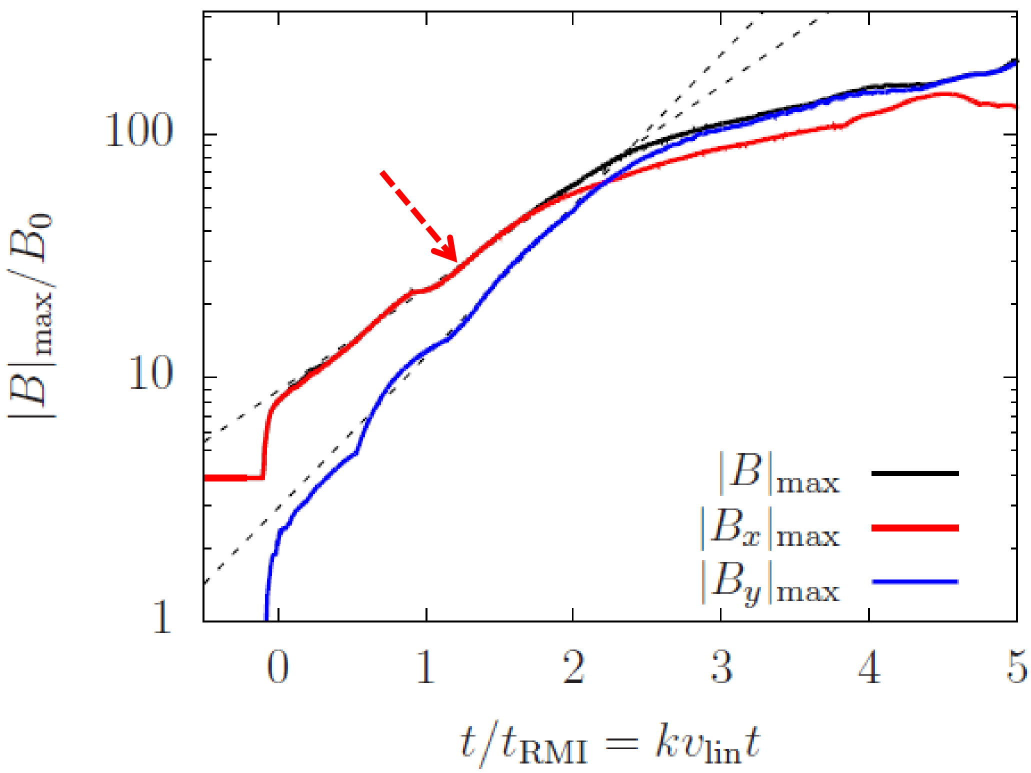


B-amplify 4
B // interface

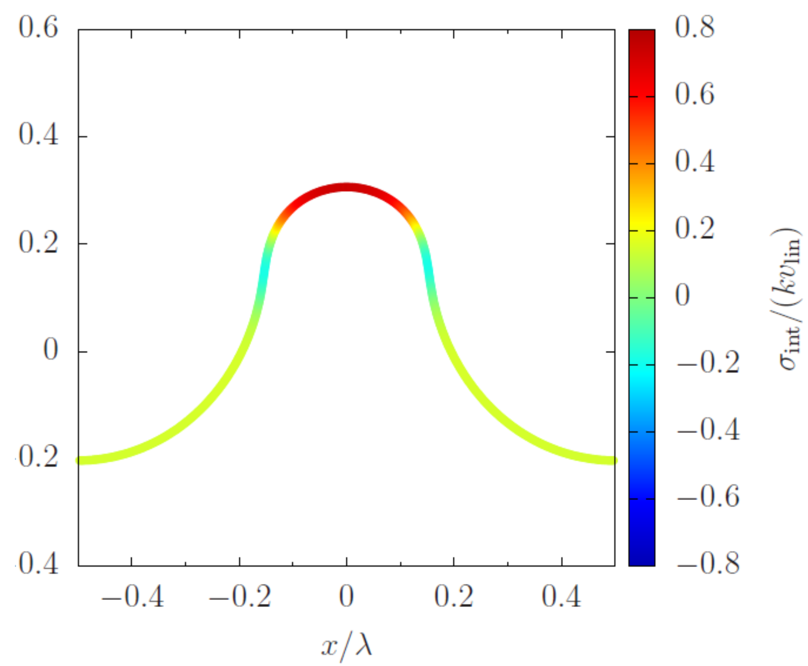
Growth rate of B-field (// interface) observed agrees fairly well with the stretching rate of interface from nonlinear vortex sheet model

Stretching rate at t=2.0,the peak value is about $1/kv_{lin}$, which is comparable to the growth rate of B-field.

B-field growth



local stretching rate of the interface

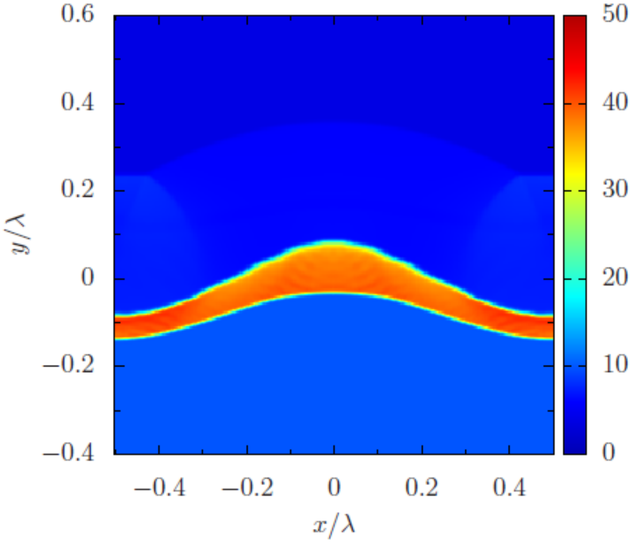


The B-field in the shock propagation direction (B_y) grows later, which corresponds to the nonlinear stretching of the spike.

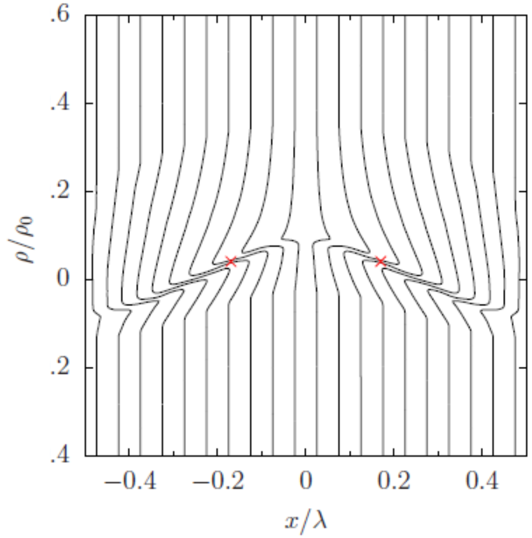
B-amplify 5
B $\perp$ interface

B-field arraigned the interface appears at early stage due to velocity shear at the interface, even for parallel shock.

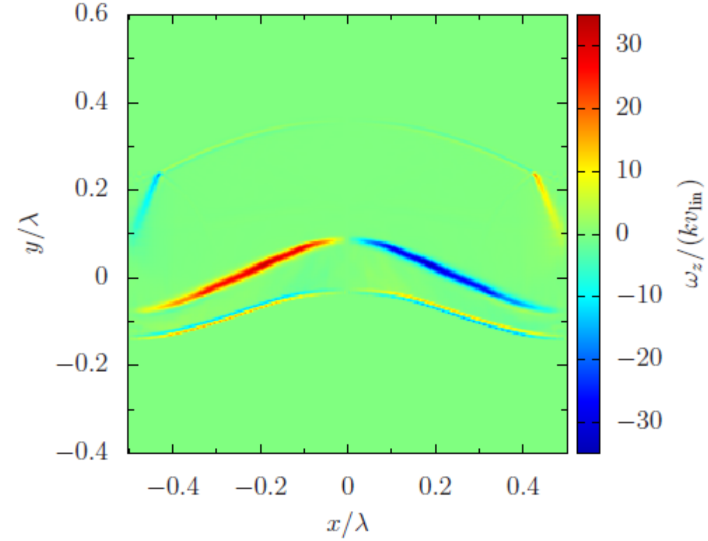
density



B field lines

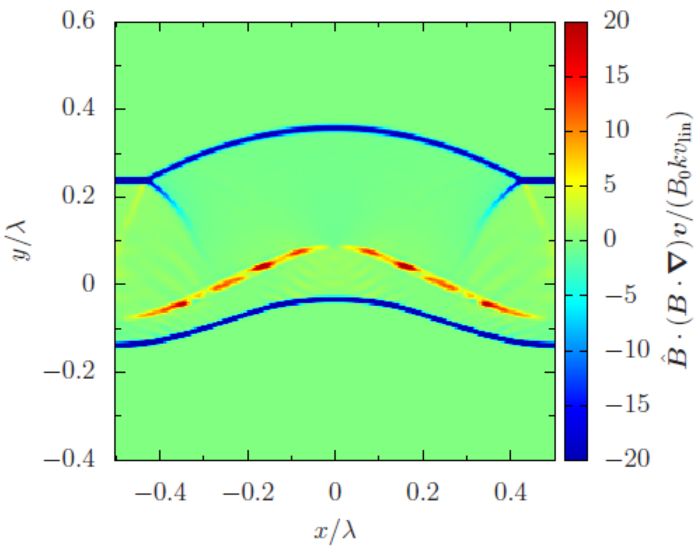


vorticity

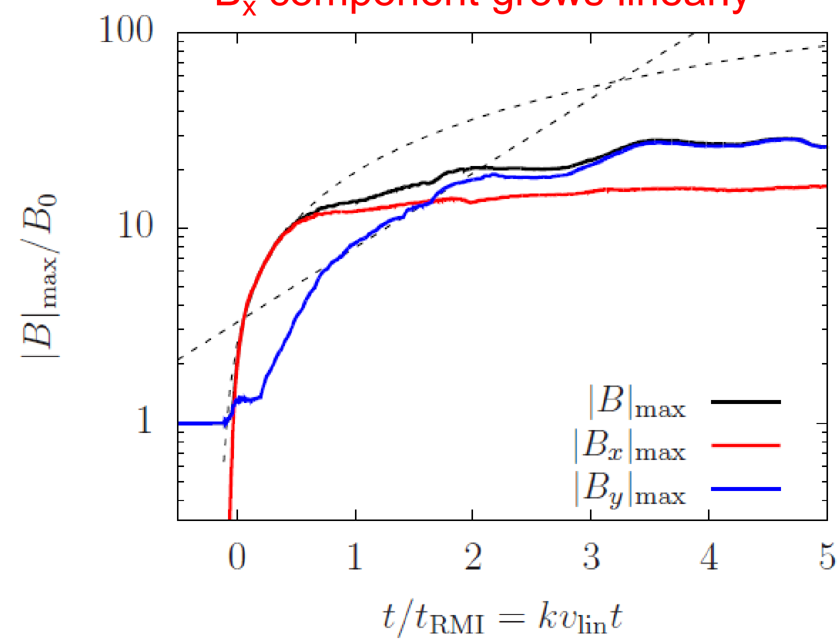


B-field amplification rate by stretching

(at $kv_{lin}t$=0.5)

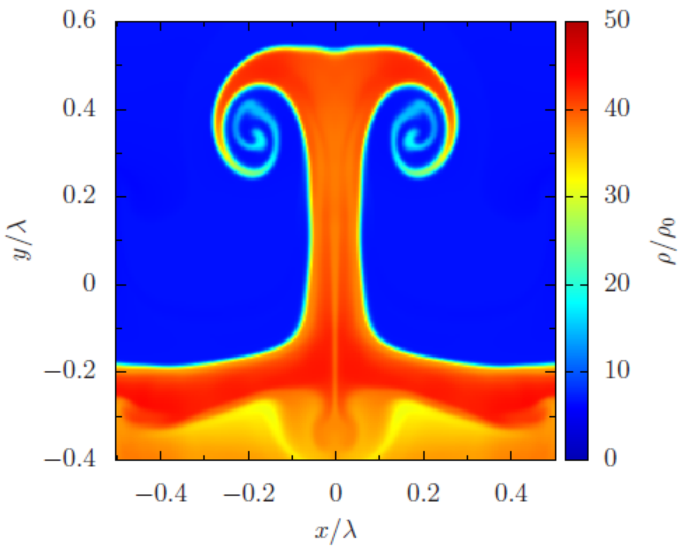


B_x component grows linearly

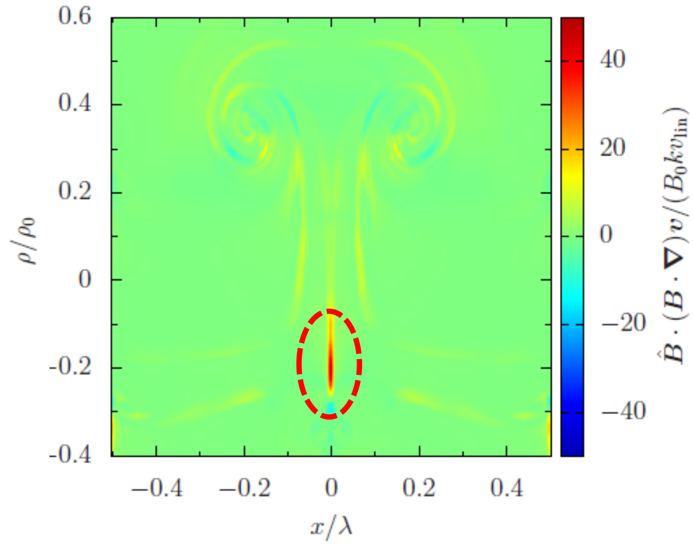


B-field amplification at root of stem appears later in parallel shock which corresponds to its stretching due to interaction with bulk vorticity.

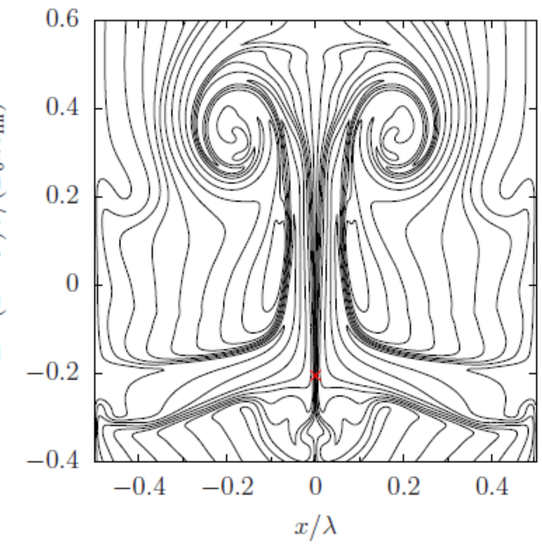
density (at $kv_{lin}t=6.0$)



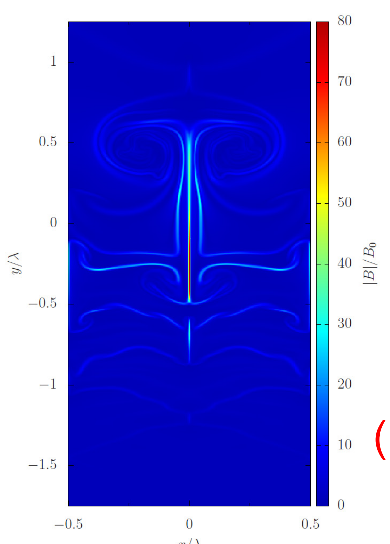
B-field amplification rate by stretching



B field lines



B ⊥ interface



(at $kv_{lin}t=10$)

Due to the interaction between the vortex sheet and bulk vorticity left behind a rippled strong shock, the stretching of the root of the stems appears later for a case of parallel shock (B-field lines ⊥ interface)

Conclusion and Acknowledgement

1. We showed strong amplification of magnetic field by the Richtmyer-Meshkov instability for three different shock propagation directions to magnetic field.
2. The amplification factor obtained from 2-d MHD simulations were greater than 100 for every cases, which agrees with the observations in SNR. We also discussed various parameter dependences of the amplification factors.
3. It is shown that the stretching of the interface and spike due to RMI mainly causes the magnetic field amplification.

I thank to Dr. Takeshi Inoue at Aoyama Gakuin Univ.
for introduction of B-field amplification in SNR and valuable discussions.