Velocity Dispersion, Spin, and Viscosity in Planetary Rings

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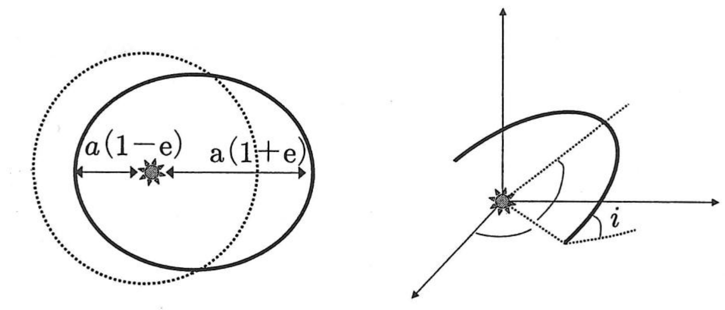
Introduction



- Velocity Dispersion

  deviation from coplanar, circular orbits

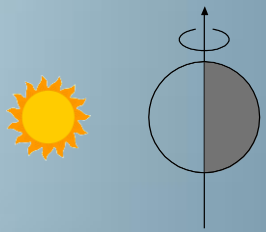
  - ➢ collisional outcome
    – accretion/rebound/fragmentation
  - ➢ ring thickness
  - ➢ formation of micro-structures (gravitational wakes)

- Spin of Ring Particles

  Not directly observable, but inferred from spacecraft and ground-based observations of rings' thermal emission

  - ◆ thermal modeling with results from dynamical study
  - ◆ comparison with observations

  ⟶ constraints on particles' physical property



- Ring Viscosity

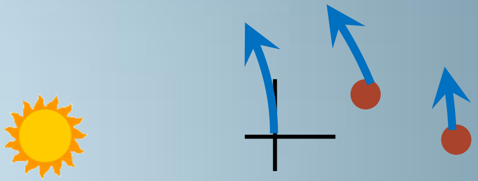
  Angular momentum is transferred through collisions and gravitational interactions

  ⟶ effects of particles' gravity and spins on ring viscosity

Introduction

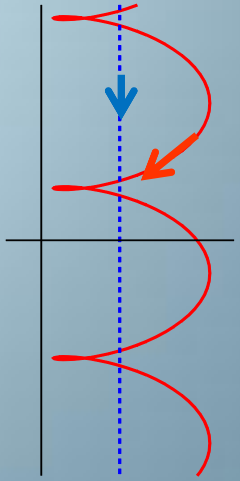
Ring particles orbiting a planet

Planetesimals orbiting the Sun



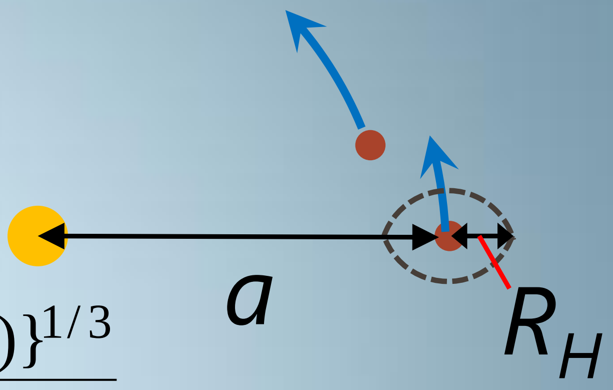
Particles: Kepler motion, perturbed by interactions (collisions, gravitational interaction) with other particles

Relative velocity between particles determines outcome/strength of interactions



Collision and Gravitational Interactions

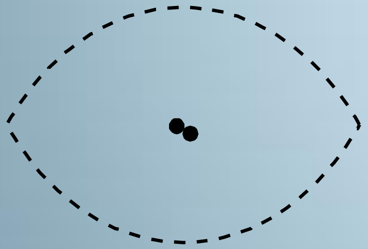
Physical Size vs. Hill Radius



Hill Radius: $R_H = a\left(\frac{m_1+m_2}{3M_c}\right)^{1/3}$

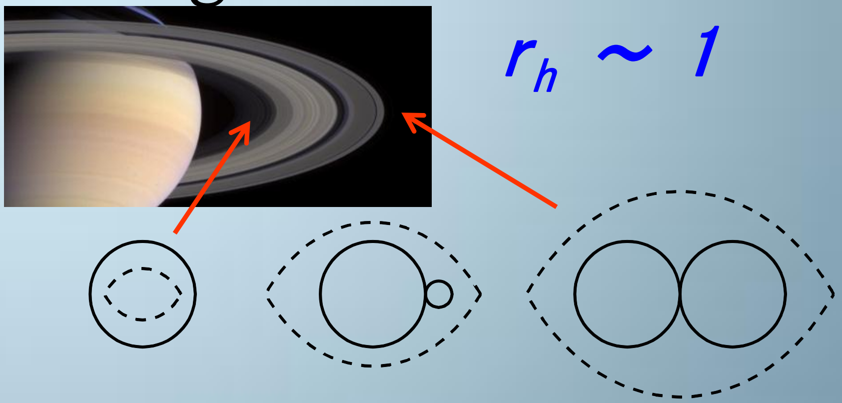
$$\rightarrow\; r_h \equiv \frac{R_H}{R_1+R_2} = \left(\frac{4\pi\rho}{9M_c}\right)^{1/3} a\,\frac{\{1+(m_1/m_2)\}^{1/3}}{1+(m_1/m_2)^{1/3}}$$

Planetesimals



$r_h \gg 1$

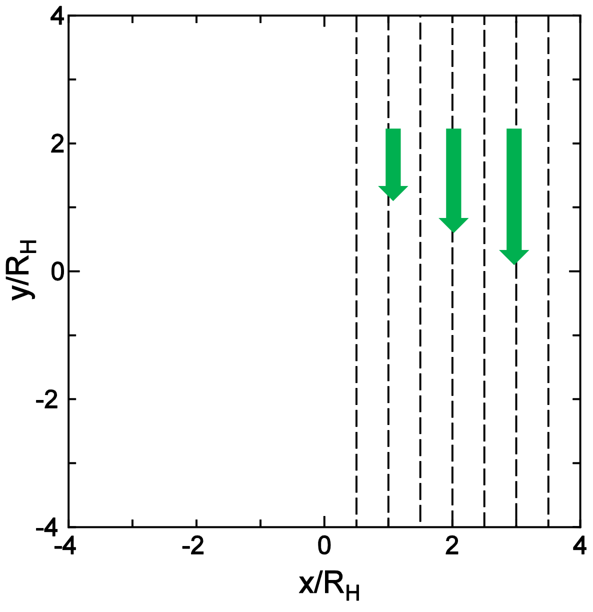
Rings

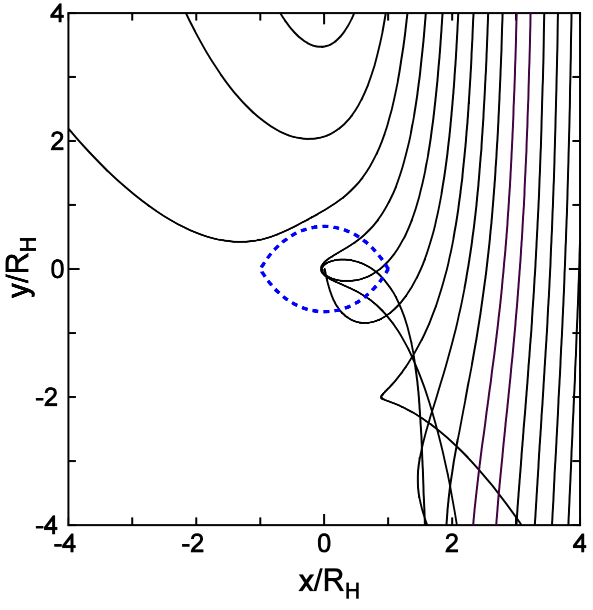


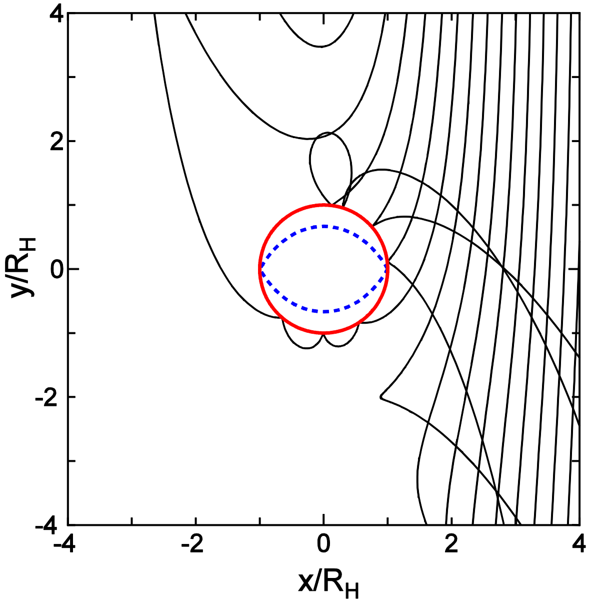
$r_h \sim 1$

→ Outcome of collision depends on r_h

Orbital Change


no col., no grav.


grav.


grav. + col.

Evolution of Velocity Dispersion

In dilute systems, evolution can be described by summation of successive two-body interactions

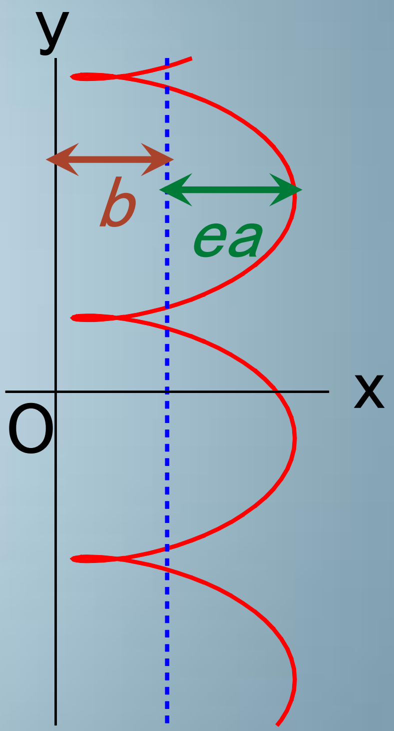
Collective effects are important in dense planetary rings

Equation of Motion

$$\begin{aligned}
\ddot{x}_i &= 2\Omega\dot{y}_i + 3\Omega^2 x_i - \frac{Gm_j}{r_{ij}^3}(x_i - x_j)\\
\ddot{y}_i &= -2\Omega\dot{x}_i - \frac{Gm_j}{r_{ij}^3}(y_i - y_j)\\
\ddot{z}_i &= -\Omega^2 z_i - \frac{Gm_j}{r_{ij}^3}(z_i - z_j)
\end{aligned}$$

Kepler Motion:

$$\begin{aligned}
x_i &= b_i - e_i a\cos(\Omega t - \tau_i)\\
y_i &= -\frac{3}{2}b_i\Omega(t - t_{0,i}) + 2e_i a\sin(\Omega t - \tau_i)\\
z_i &= i_i a\sin(\Omega t - \omega_i)
\end{aligned}$$

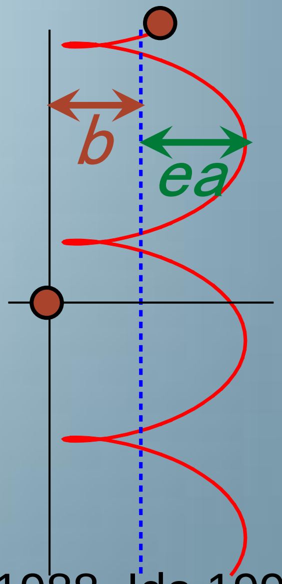


Evolution of Velocity Dispersion

$$\left(\begin{aligned}\Delta e_1^2 &= \Delta(\boldsymbol{E} - m_2'\boldsymbol{e})^2 \\ &= {m_2'}^2 \Delta e^2 - 2m_2' \boldsymbol{E}\cdot\Delta\boldsymbol{e}\end{aligned}\right)$$

$$\begin{aligned}\frac{d\langle e_{m_1}^2\rangle}{dt} &= \int n_s(m)\left\{\langle\Delta e_1^2\rangle \frac{3}{2}|b|\Omega db\right\} dm \\ &= a^2\Omega \int n_s(m) \frac{m h_{m_1,m}^4}{(m_1+m)^2}\left\{m\langle P_{VS}\rangle + \frac{m\langle e_m^2\rangle - m_1\langle e_{m_1}^2\rangle}{\langle e_{m_1}^2\rangle + \langle e_m^2\rangle}\langle P_{DF}\rangle\right\} dm\end{aligned}$$

$$h_{m_1,m} = \left(\frac{m_1+m}{3M_c}\right)^{1/3}$$



$<P_{VS}>$: Viscous Stirring rate

$<P_{DF}>$: Dynamical Friction rate

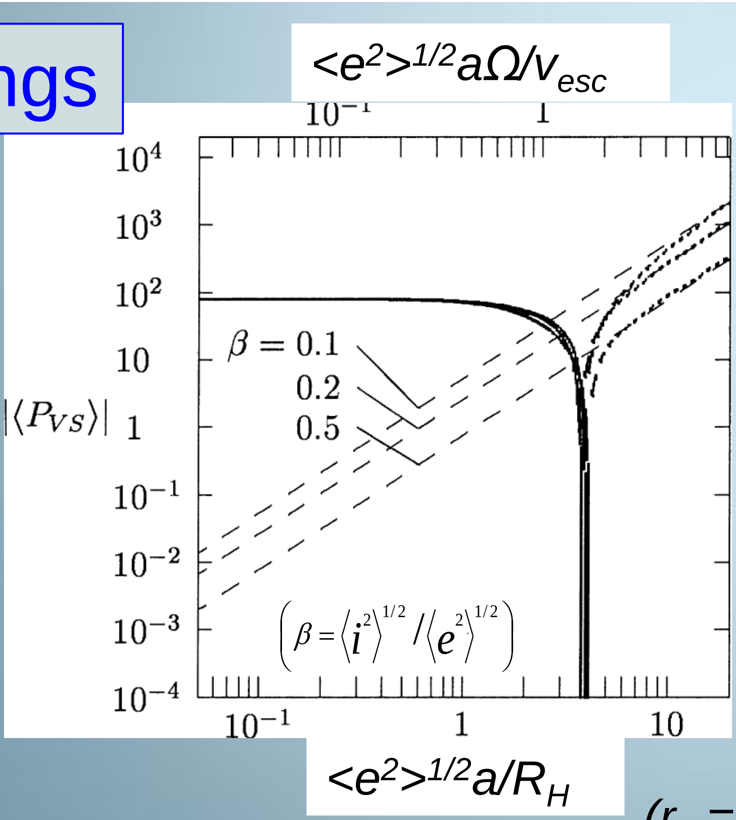
(Ohtsuki 1999; see also Stewart & Wetherill 1988, Ida 1990)

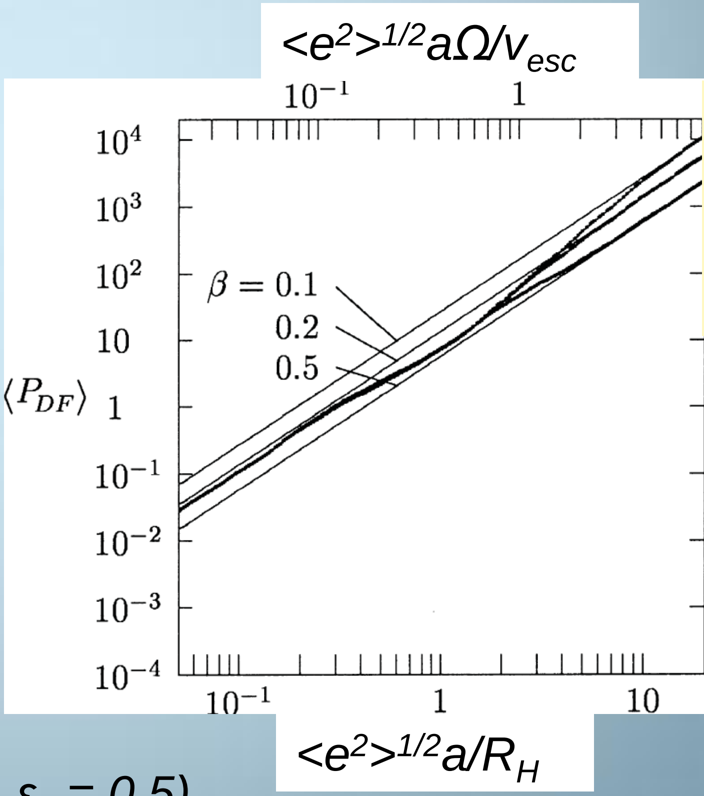
Stirring and Dynamical Friction Rates

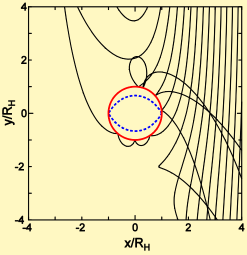
$$\frac{d\langle e_{m_1}^2\rangle}{dt} = a^2\Omega \int n_s(m)\frac{m h_{m_1,m}^4}{(m_1+m)^2}\left\{ m\langle P_{VS}\rangle + \frac{m\langle e_m^2\rangle - m_1\langle e_{m_1}^2\rangle}{\langle e_{m_1}^2\rangle + \langle e_m^2\rangle}\langle P_{DF}\rangle \right\} dm$$

$$\frac{d\langle i_{m_1}^2\rangle}{dt} = a^2\Omega \int n_s(m)\frac{m h_{m_1,m}^4}{(m_1+m)^2}\left\{ m\langle Q_{VS}\rangle + \frac{m\langle i_m^2\rangle - m_1\langle i_{m_1}^2\rangle}{\langle i_{m_1}^2\rangle + \langle i_m^2\rangle}\langle Q_{DF}\rangle \right\} dm$$

Rings







$(r_h = 1, \varepsilon_n = 0.5)$

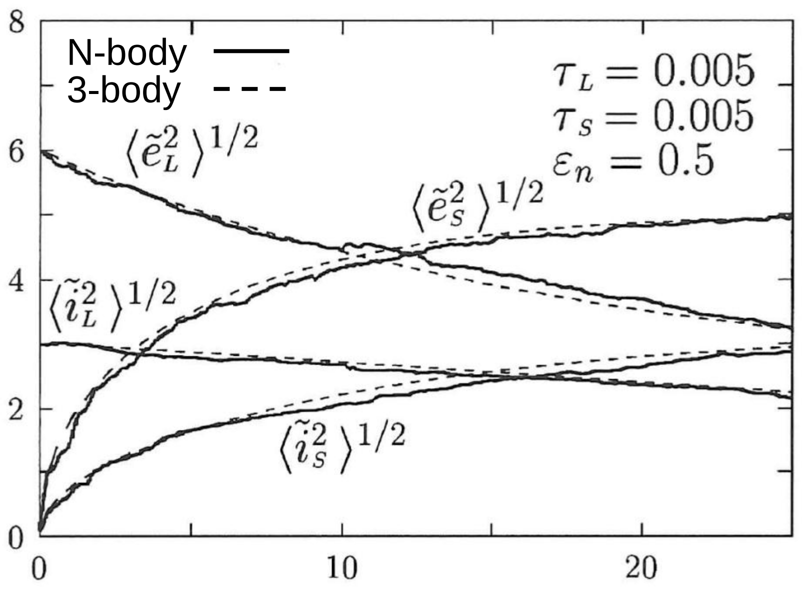
(Ohtsuki 2000)

Comparison with N-body Simulation

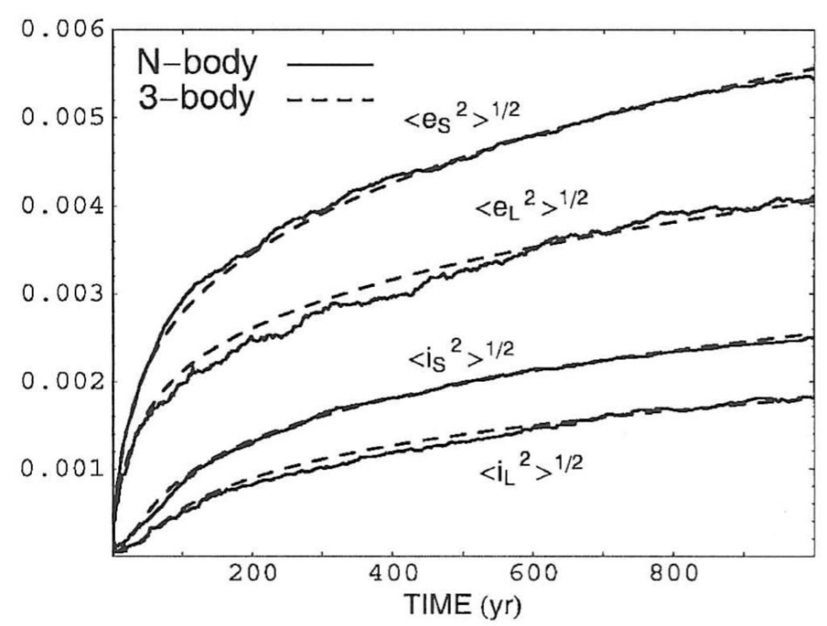
$$\frac{d\langle e_{m_1}^2\rangle}{dt} = a^2\Omega\int n_s(m)\frac{m h_{m_1,m}^4}{(m_1+m)^2}\left\{m\langle P_{VS}\rangle + \frac{m\langle e_m^2\rangle - m_1\langle e_{m_1}^2\rangle}{\langle e_{m_1}^2\rangle+\langle e_m^2\rangle}\langle P_{DF}\rangle\right\}dm$$

$$\frac{d\langle i_{m_1}^2\rangle}{dt} = a^2\Omega\int n_s(m)\frac{m h_{m_1,m}^4}{(m_1+m)^2}\left\{m\langle Q_{VS}\rangle + \frac{m\langle i_m^2\rangle - m_1\langle i_{m_1}^2\rangle}{\langle i_{m_1}^2\rangle+\langle i_m^2\rangle}\langle Q_{DF}\rangle\right\}dm$$

Dilute Rings



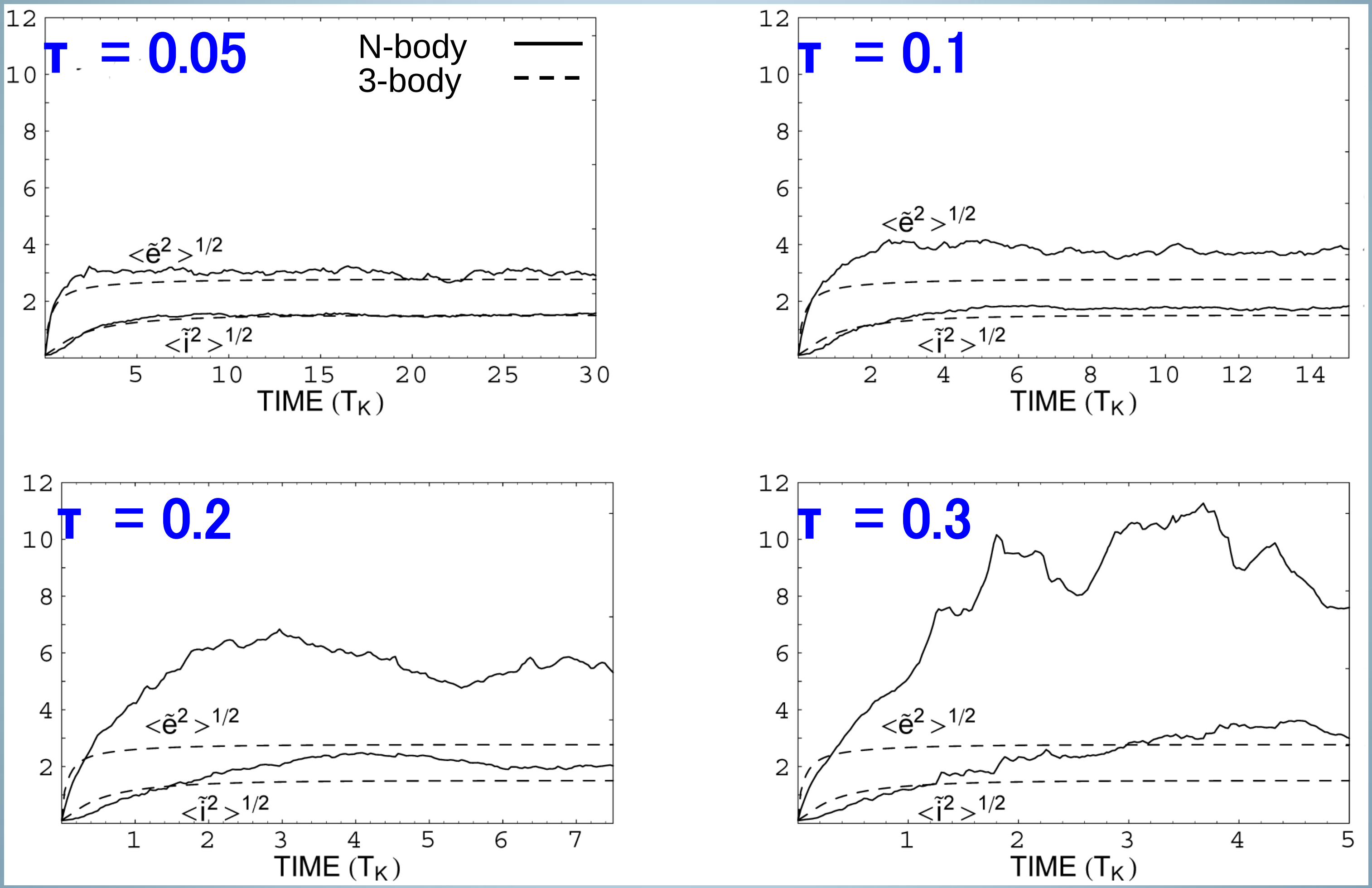
Planetesimals



(Ohtsuki 1999, Ohtsuki et al. 2002)

Comparison with N-body Simulation

Rings

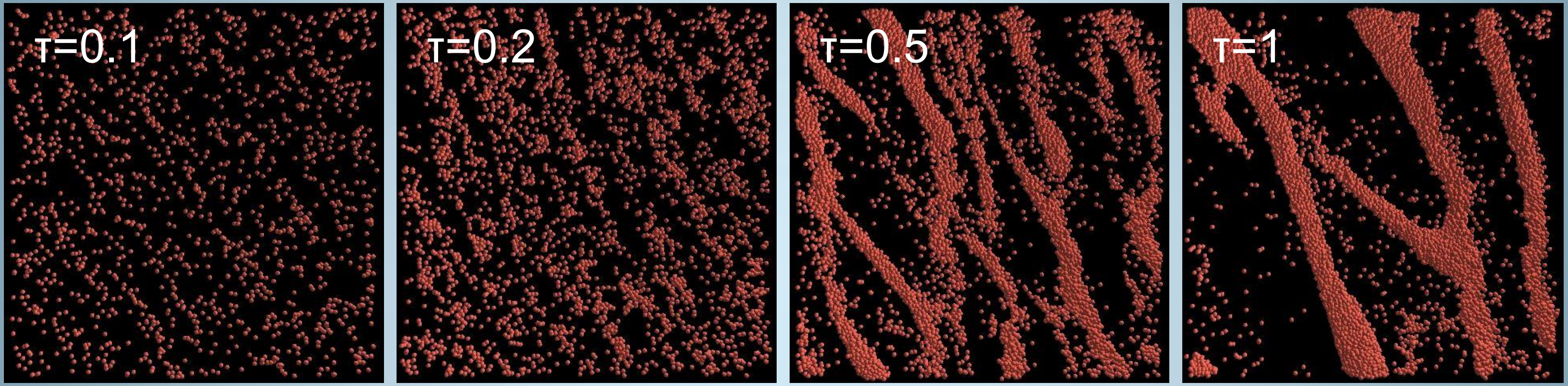


$(\tau = n_s \pi R^2)$

(Ohtsuki & Emori 2000)

Formation of Gravitational Wakes

(Salo 1992, 1995)



(from poster by Yuki Yasui)

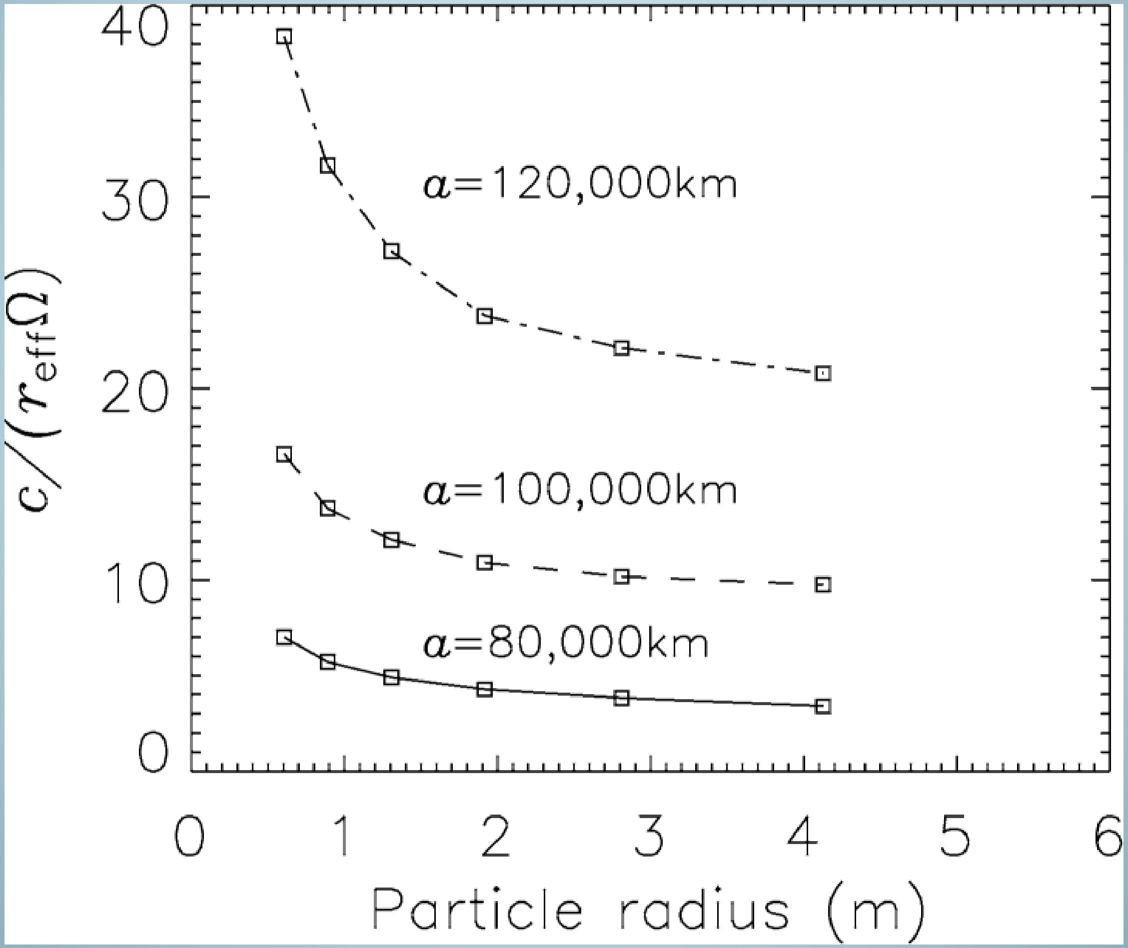
Velocity Dispersion

$$\begin{cases} \sim \max\{R^2\Omega, v_{esc}\} & \text{for dilute rings } \left(v_{esc} = \sqrt{2Gm/R}\right) \\ \sim G\Sigma / \Omega & \text{for dense, self-gravitating rings} \end{cases}$$

(Σ : ring's surface density)

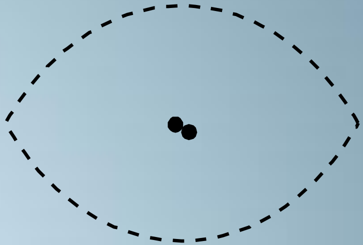
(Salo 1995)

Velocity Dispersion in Planetary Rings with Particle Size Distribution



(Morishima & Salo 2006)

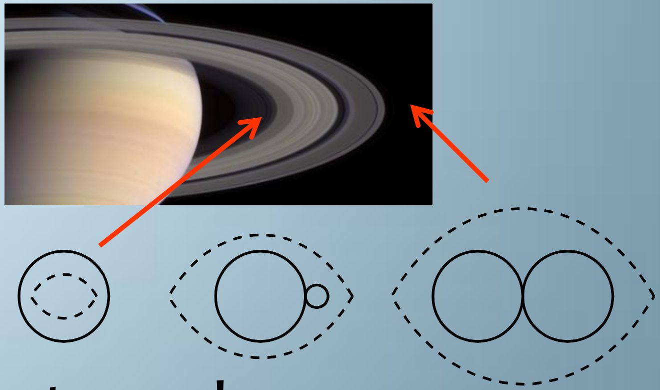
Collisional Outcome



Planetesimals: $r_h \gg 1$

accretion, if $v_{\mathrm{imp}} \lesssim v_{\mathrm{esc}}$

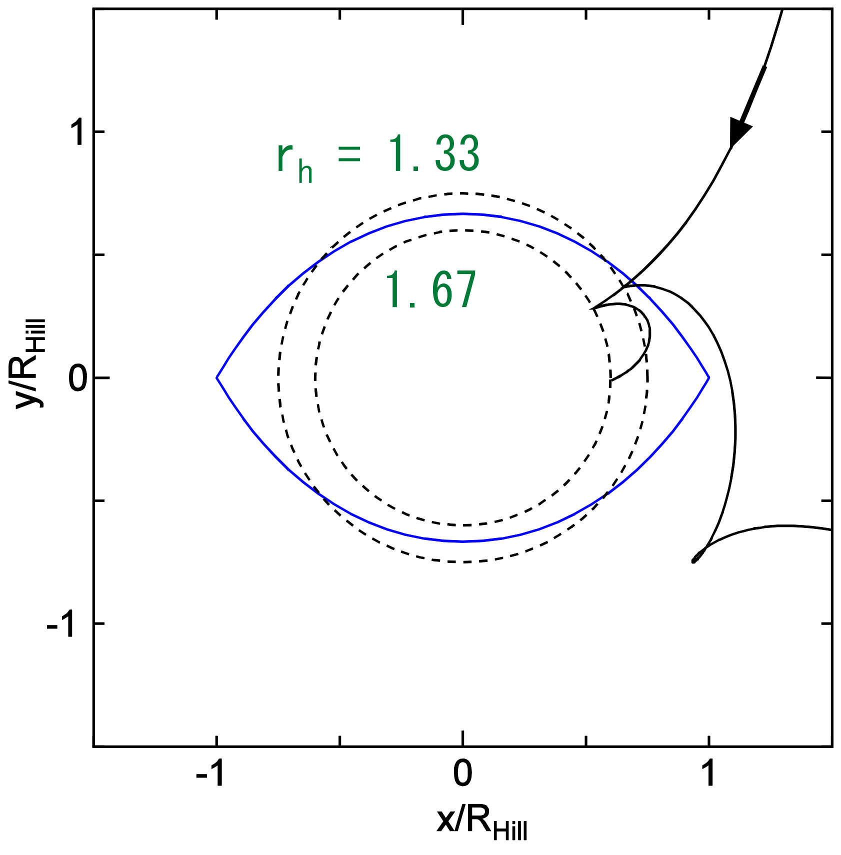
(fragmentation is important for high-velocity impacts; Agnor & Asphaug 2004, Kokubo & Genda 2010)

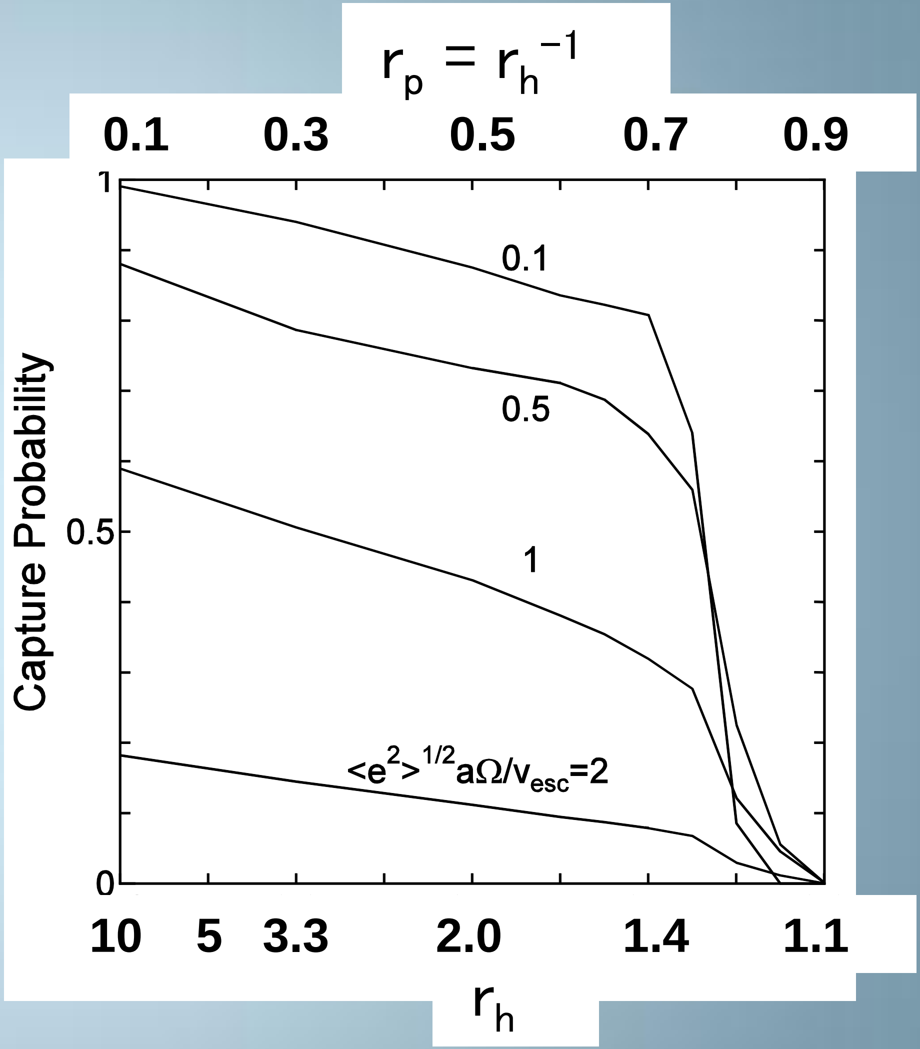


Rings: $r_h \sim 1$

- rebound in most regions
- accretion is possible near the outer edge, depending on density

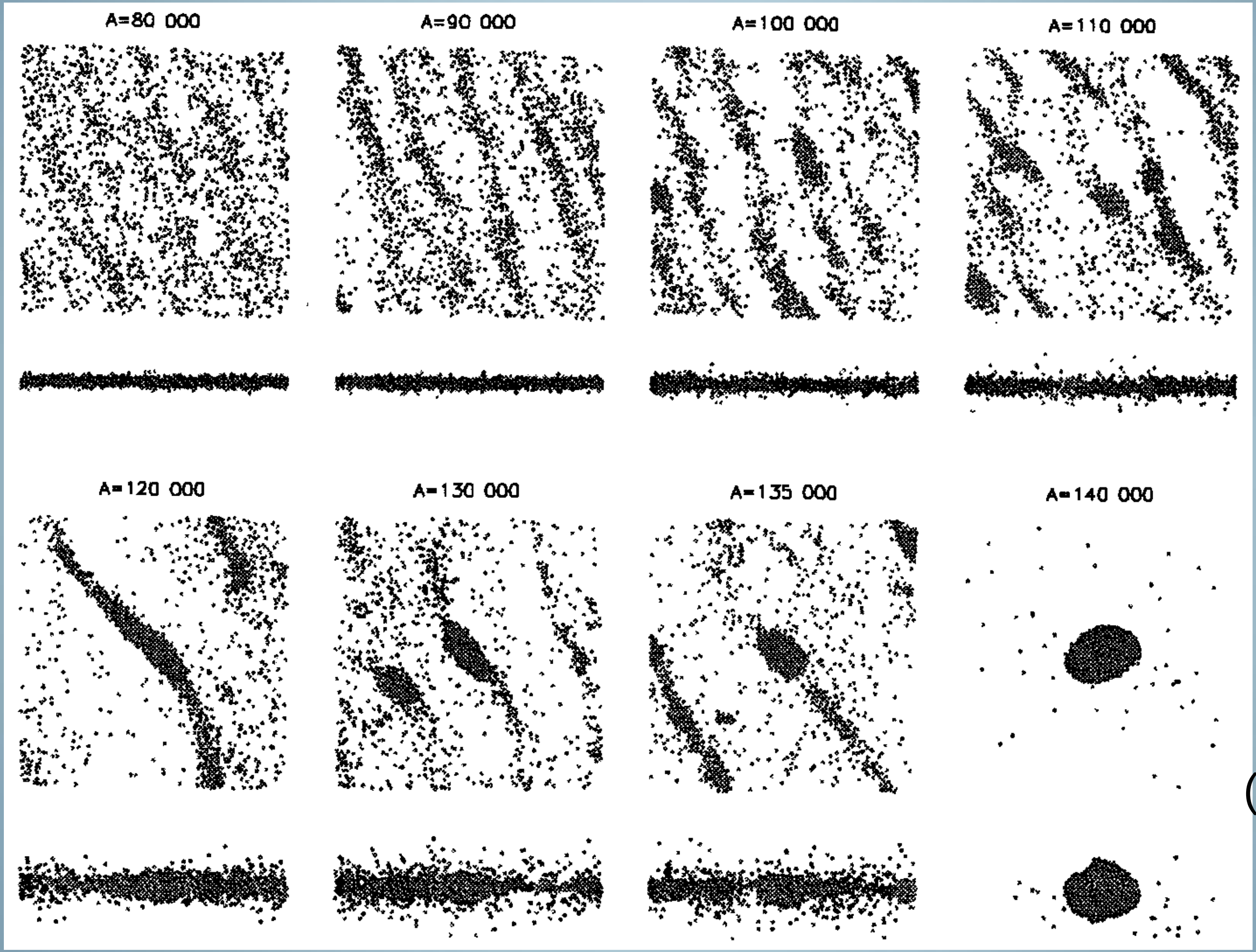
Collisional Outcome





(e.g. Ohtsuki 1993)

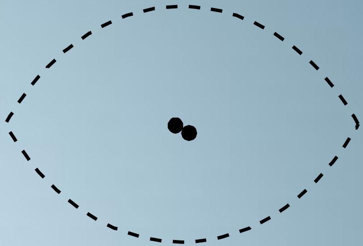
Gravitational Accretion in Rings



(Salo 1995)

(Salo 1992, Karjalainen & Salo 2004, Porco et al. 2007, Charnoz et al. 2007)

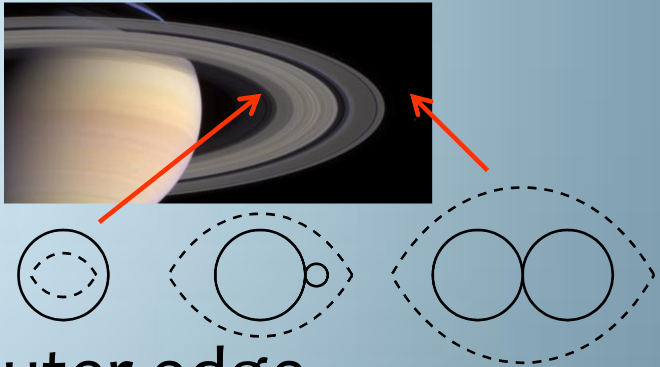
Collisional Outcome



Planetesimals: $r_h \gg 1$

➡ ***accretion***, if $v_{imp} \lesssim v_{esc}$

(fragmentation is important for high-velocity impacts; Agnor & Asphaug 2004, Kokubo & Genda 2010)



Rings: $r_h \sim 1$

- ***rebound*** in most regions
- ➡ accretion is possible near the outer edge, depending on density

➡ *Spins caused by collisions*

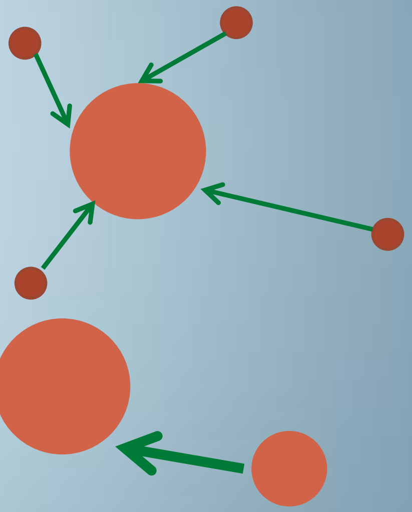
Planetary Rotation by Accretion of Planetesimals

$$\mathbf{L} = \sum_{i=1}^{N} m_i \mathbf{l}_i$$

$$\langle \mathbf{L}^2 \rangle \approx \langle \mathbf{L} \rangle^2 + N \langle m^2 \rangle \langle \mathbf{l}^2 \rangle$$

1st term: “systematic component”

2nd term: “random component”

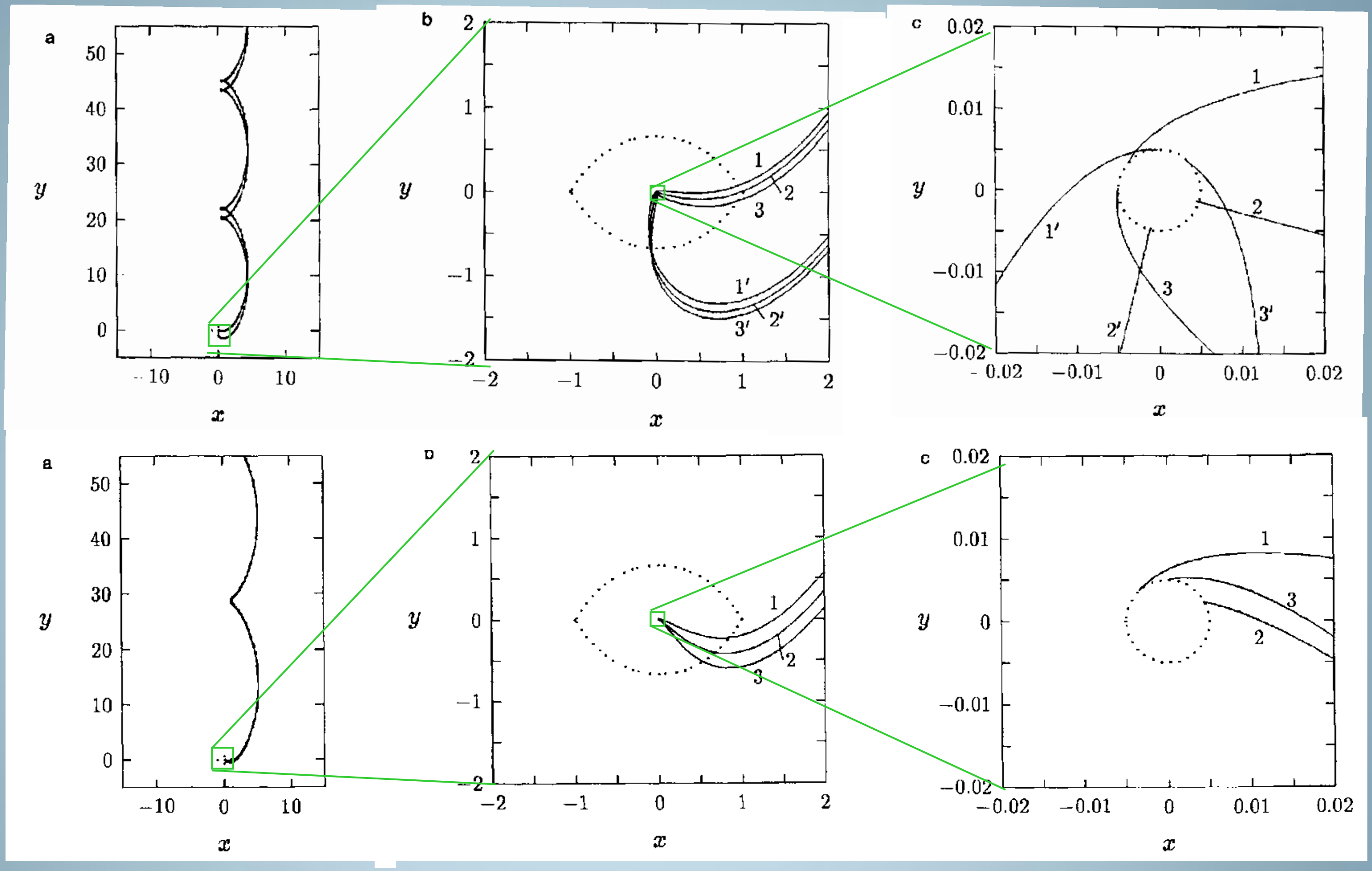


Relative importance of each component depends on distribution of mass and angular momentum of impactors

(Dones & Tremaine 1993)

Systematic Component of Planetary Rotation

Sun ←



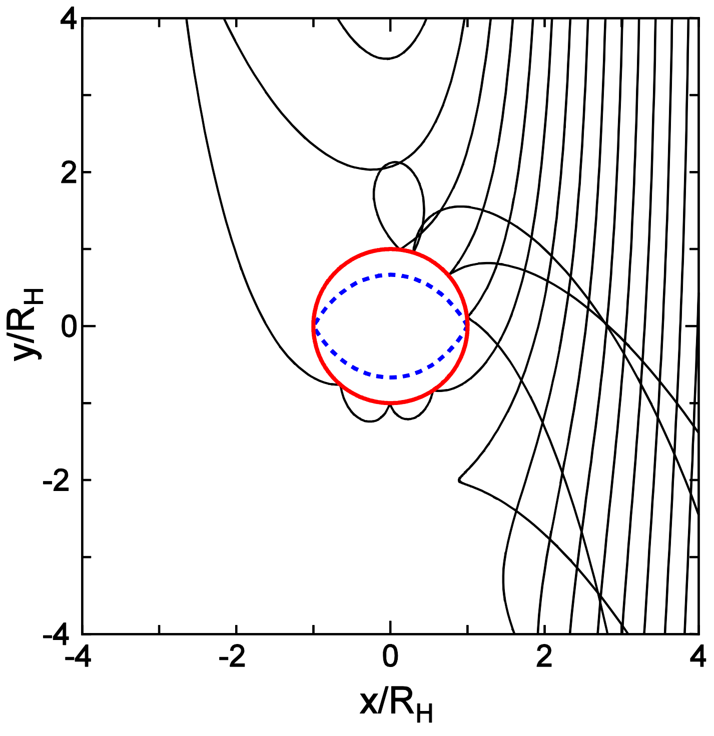
(Ohtsuki & Ida 1998)

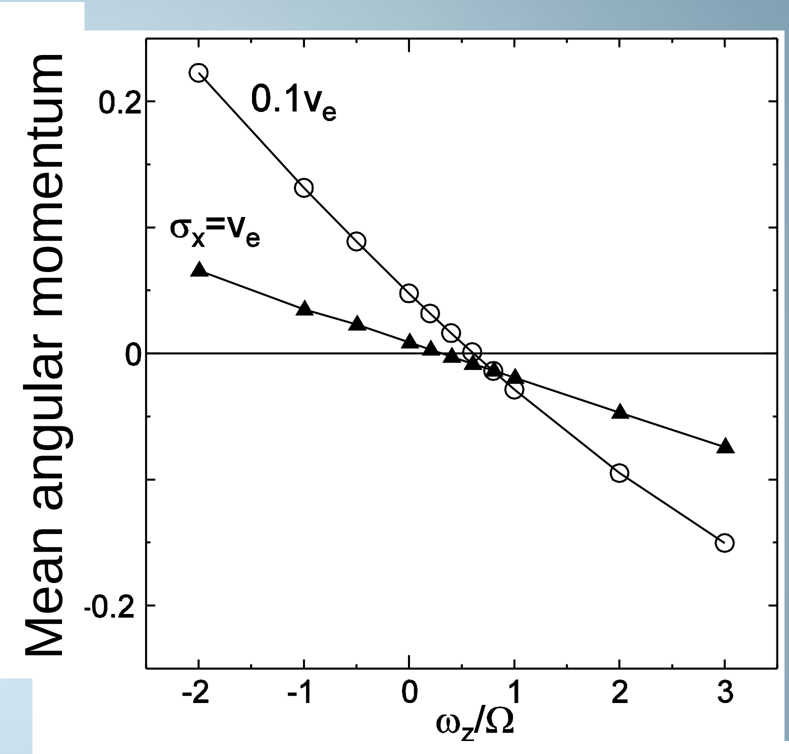
Systematic component is too small to account for terrestrial planet rotation (Earth-Moon system, Mars)

➡ Large impacts played a major role (Dones & Tremaine 1993)

Moonlet Rotation

(Morishima & Salo 2004, Ohtsuki 2004a, b)





Moonlet's initial rotation rate

$$\boxed{\omega_{eq} \approx 0.3 - 0.7\Omega}$$

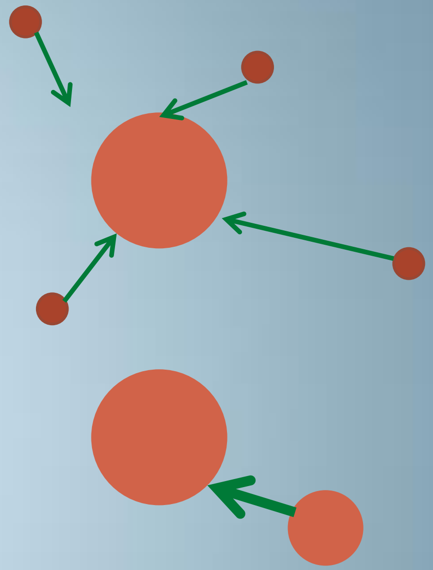
Slow prograde rotation

Moonlet Rotation

$$\mathbf{L} = \sum_{i=1}^{N} m_i \mathbf{l}_i$$

$$\langle \mathbf{L}^2 \rangle \approx \underline{\langle \mathbf{L}_{eq} \rangle^2} + \underline{N \langle m^2 \rangle \langle \mathbf{l}^2 \rangle}$$

$$\approx \langle \mathbf{L}_{eq} \rangle^2 \left\{ 1 + S_m^2 S_l^2 \right\}$$



$$\left(\mathbf{L}_{eq} = I\boldsymbol{\omega}_{eq},\quad S_m^2 = \frac{<m^2>}{M<m>},\quad S_l^2 \approx \frac{(1-\varepsilon_t)\sigma_x^2}{R^2 \omega_{eq}^2} \right)$$

ε_t : tangential coefficient of restitution

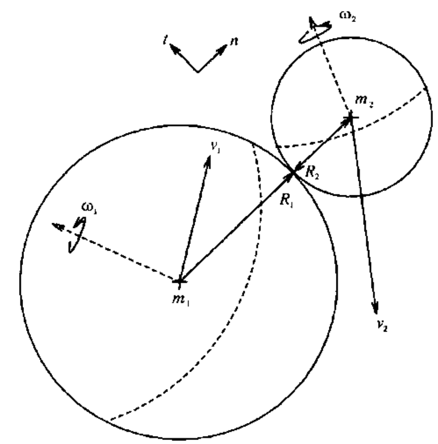
σ_x : particles' velocity dispersion

⇒ Random component is dominant for $m/M \gtrsim 0.3$

(Ohtsuki 2004)

Spin of Ring Particles with Size Distribution

$$\begin{aligned} E_{rot} &= \frac{1}{2} I \omega^2 \quad \left(I = \frac{2}{5} m R^2\right) \\ &= \frac{1}{5} m R^2 \omega^2 \end{aligned}$$



$$\begin{aligned} \frac{d\langle E^2_{rot,m_1}\rangle}{dt} &= \frac{m_1 R_1^2}{5} \int n_s(m) \left\{ \langle \Delta\omega_1^2 \rangle \frac{3}{2} |b| \Omega db \right\} dm \\ &= \frac{m_1}{5} a^4 \Omega^3 \int n_s(m) \frac{m h^4_{m_1,m}}{(m_1+m)^2} \left\{ m \langle S_{CS} \rangle + \frac{m\langle s_m^2\rangle - m_1 \langle s_{m_1}^2 \rangle}{\langle s_{m_1}^2 \rangle + \langle s_m^2 \rangle} \langle S_{RF} \rangle \right\} dm \\ &= a^2 \Omega \int n_s(m) \left\{ \underline{C_{CS}} + \underline{\left(\langle E_{rot,m_2}\rangle - \langle E_{rot,m_1}\rangle\right) C_{RF}} \right\} dm \end{aligned}$$

Collisional stirring

Rotational friction

($s_m = R\omega$: spin velocity)

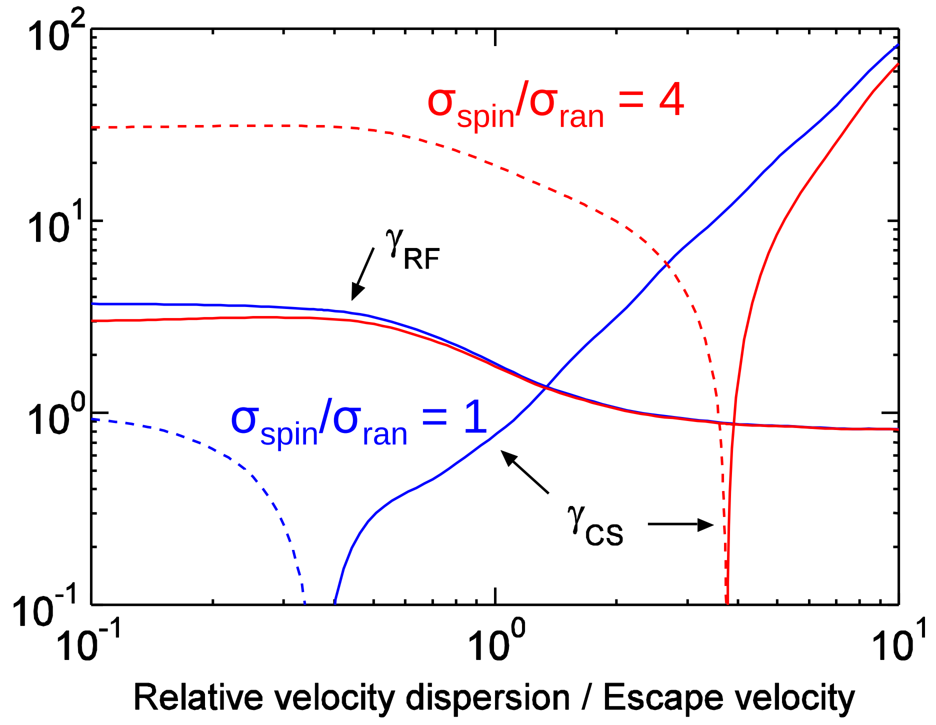
(Ohtsuki 2005, 2006)

Spin of Ring Particles with Size Distribution

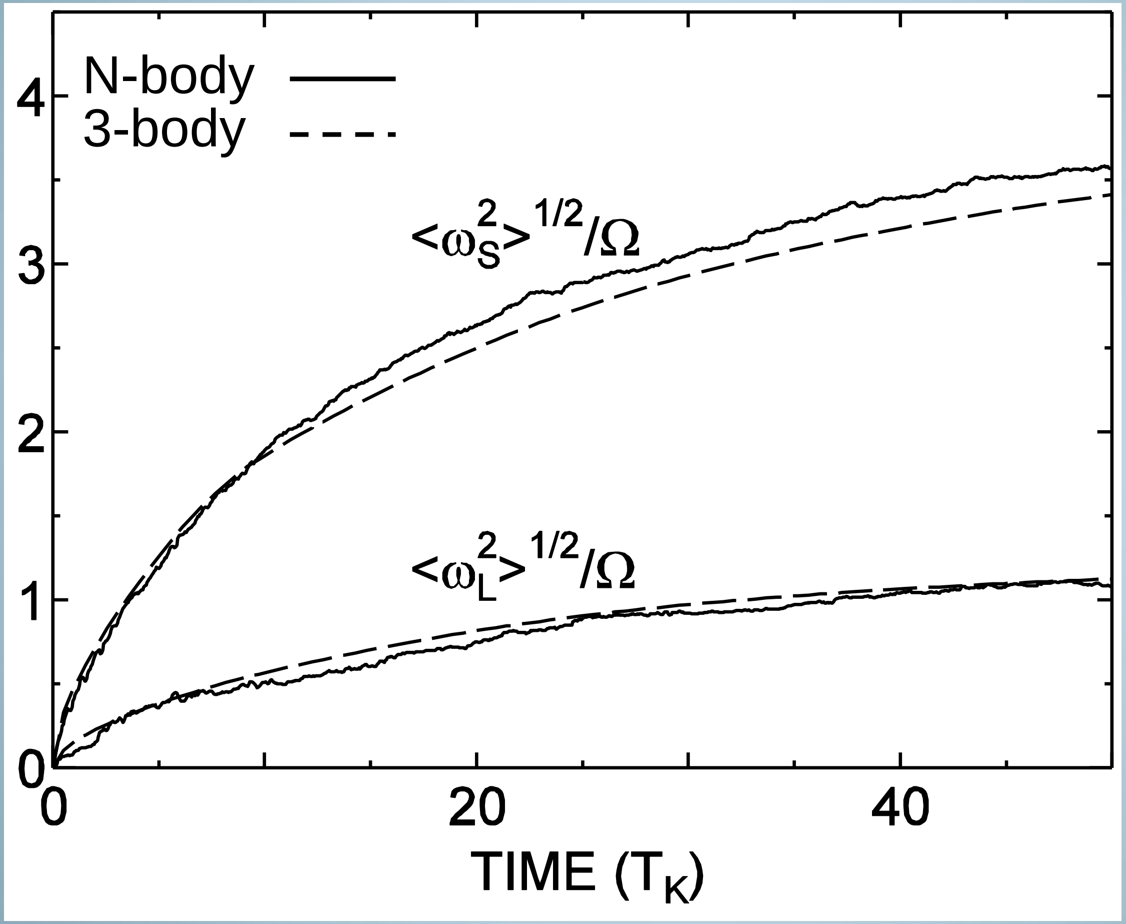
$$\boxed{\frac{d\langle E_{rot}(m_1)\rangle}{dt} = \int n_S(m_2)\left\{C_{CS} + \left(\langle E_{rot}(m_2)\rangle - \langle E_{rot}(m_1)\rangle\right)C_{RF}\right\}dm_2}$$

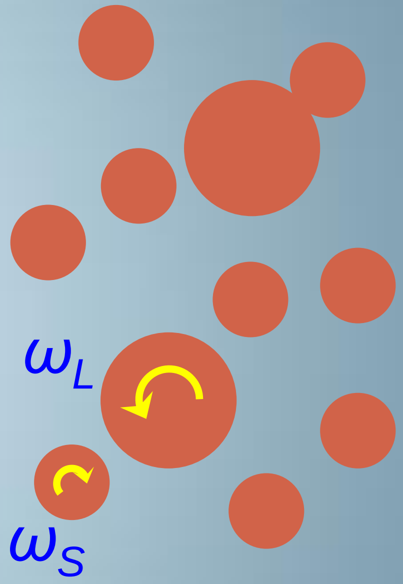
$$C_{CS} = \frac{m_1 m_2^2 \Omega^3 R_H^4 \gamma_{CS}}{5(m_1+m_2)^2}, \qquad C_{RF} = \frac{m_1 m_2 \Omega R_H^2 \gamma_{RF}}{(m_1+m_2)^2}$$

$$\left(\begin{array}{c} \sigma_{ran} = \left(<e^2>/2\right)^{1/2} a\Omega \\ \sigma_{spin} = R<\omega^2>^{1/2} \end{array}\right)$$



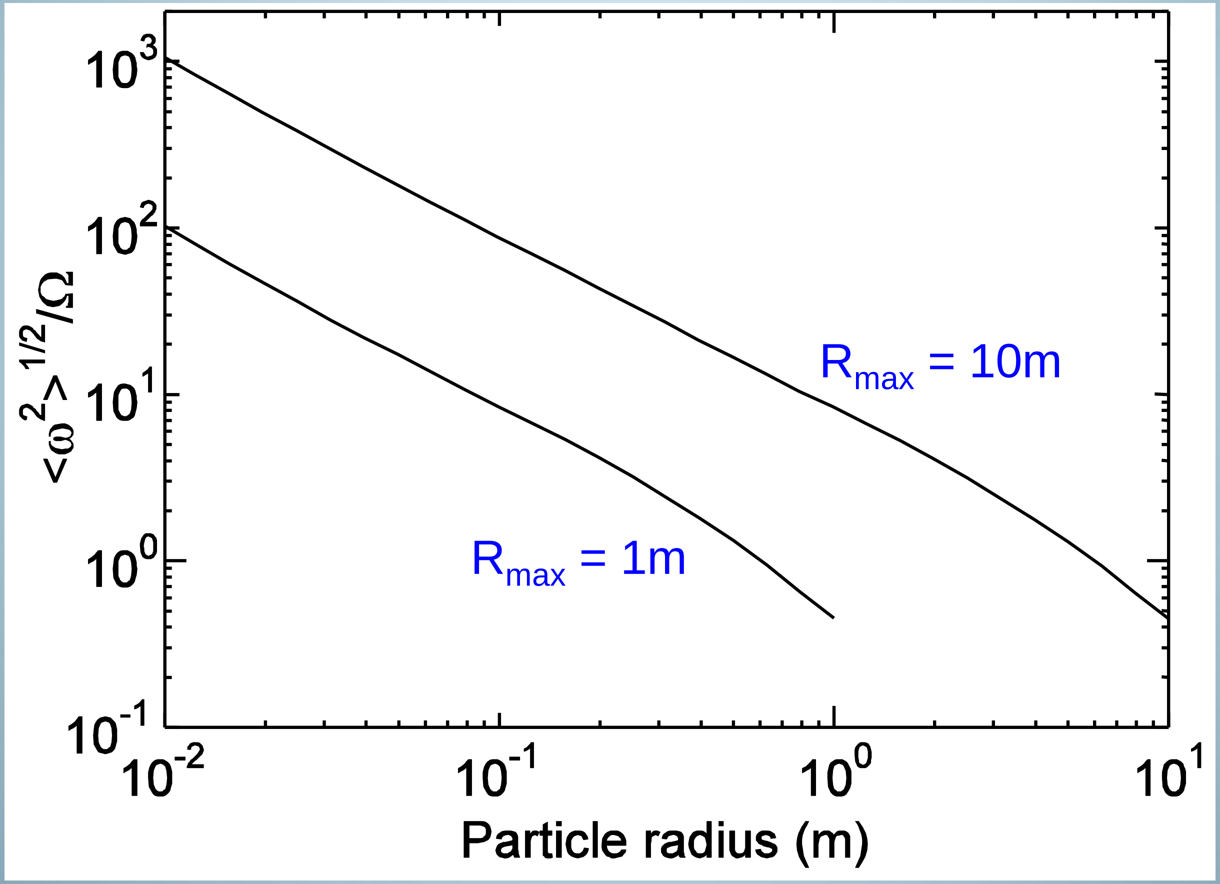
Comparison with N-body Simulation





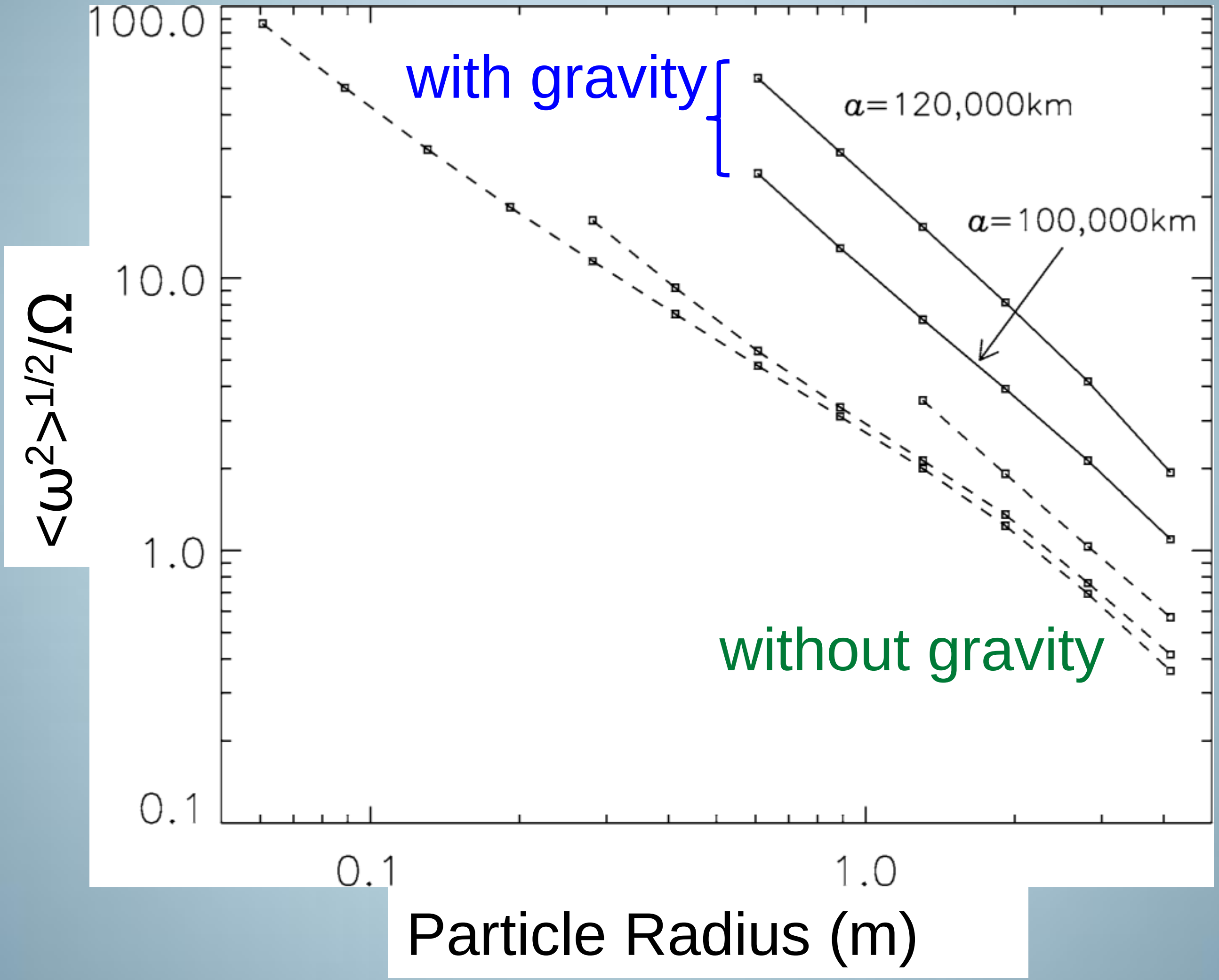
Rotation Rates of Ring Particles with a Broad Size Distribution

$(n(R) \propto R^{-3})$



(Ohtsuki 2005, 2006)

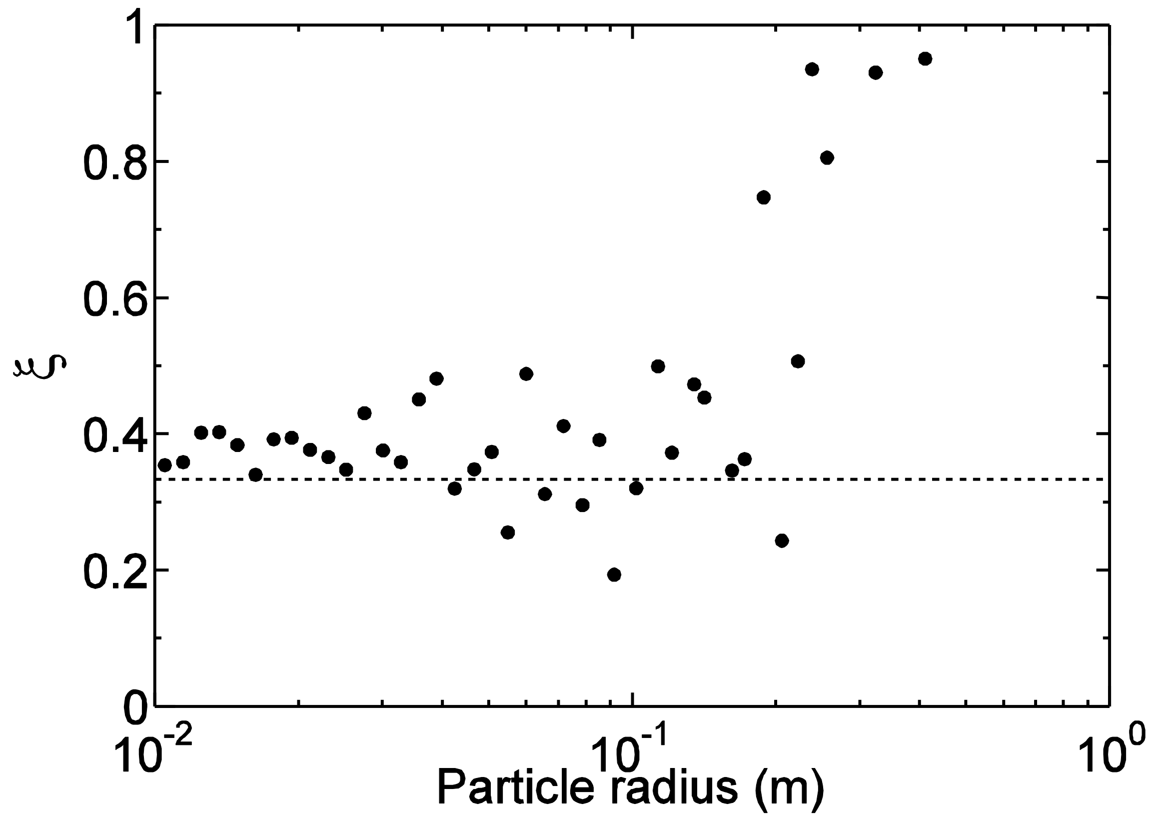
Rotation Rates of Ring Particles with a Broad Size Distribution



(Morishima & Salo 2006)

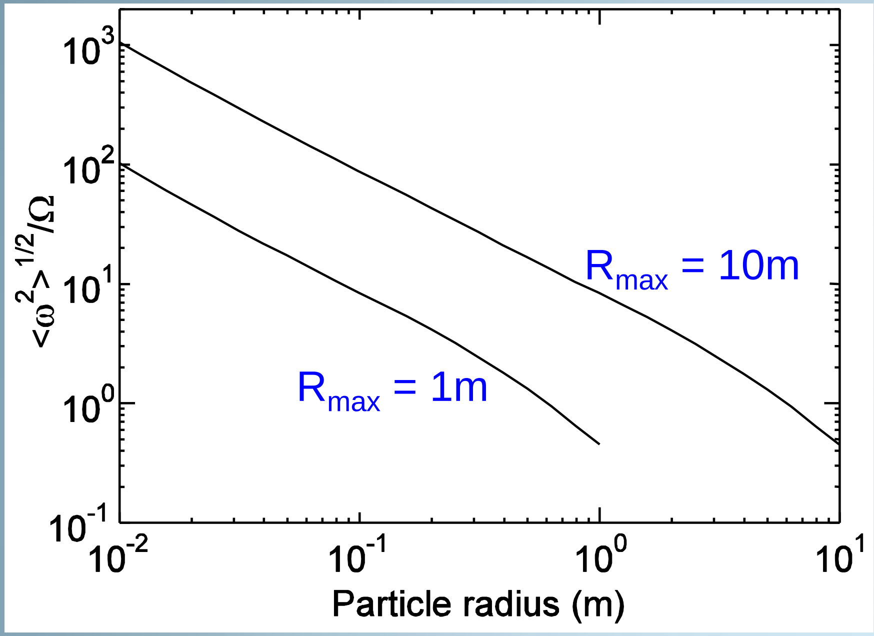
Orientation of Spin Axes

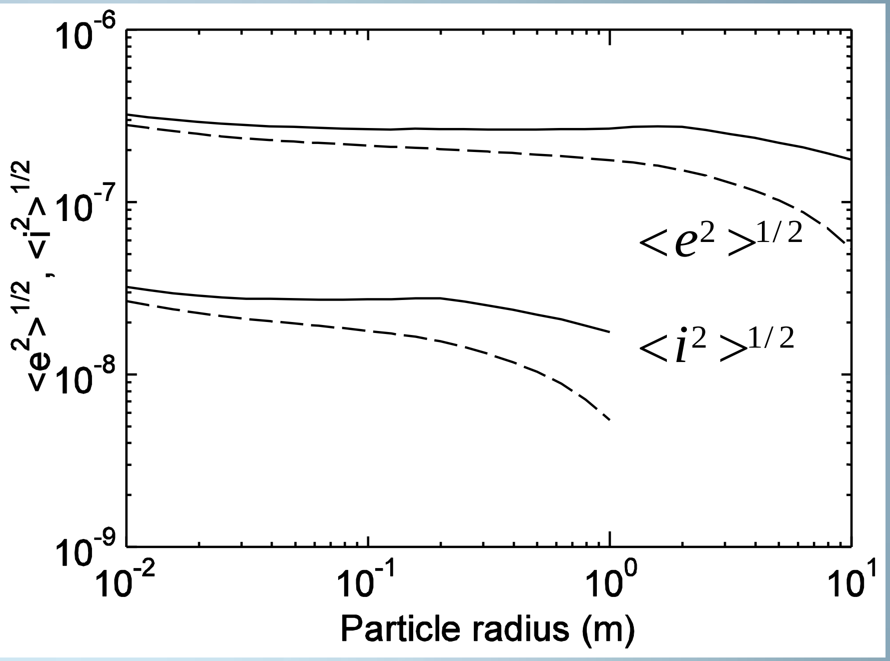
$$\xi \equiv \frac{\langle \omega_z^2 \rangle}{\langle \omega_x^2 \rangle + \langle \omega_y^2 \rangle + \langle \omega_z^2 \rangle} \begin{cases} \simeq 1: & \text{vertically aligned} \\ \simeq 1/3: & \text{random orientation} \end{cases}$$



(Ohtsuki & Toyama 2005)

Spins and Vertical Structure





- Small particles: large scale height, fast spin
- Large particles: small scale height, slow spin

➡ particles’ spin states have vertical heterogeneity
(important for thermal modeling)

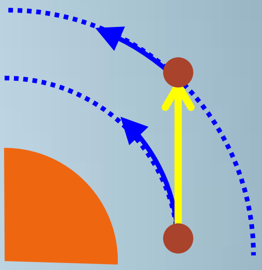
Viscosity in Planetary Rings

Angular momentum is transported through collisions and gravitational interactions

- Dilute rings:

  particles' radial random motion due to collisions and gravitational encounters

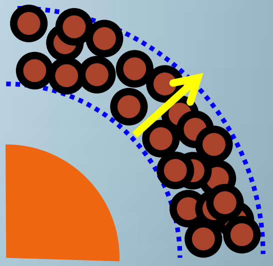
  $$\nu \approx \frac{c^2\tau}{\Omega}$$



- Dense, non-gravitating rings:

  collisions

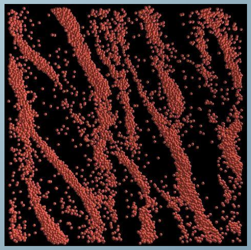
  $$\nu \approx R^2\Omega\tau$$



- Dense, self-gravitating rings:

  gravitational wakes

  $$\nu = C(r_h)\frac{G^2\Sigma^2}{\Omega^3}$$



(Goldreich & Tremaine 1978, Araki & Tremaine 1986, Daisaka et al. 2001)

Summary

- Velocity Dispersion
  - determined by collisions, gravitational encounters, or rings' self-gravity
  - small particles with large scale height, and large particles with small scale height
  - determines collisional outcome
- Spin of Ring Particles
  - small particles fast rotation + random orientation, and large particles with slow rotation with aligned spin axes
  - important for thermal modeling
- Ring Viscosity
  - determined by particles' random motion in dilute rings
  - determined by self-gravity in dense rings