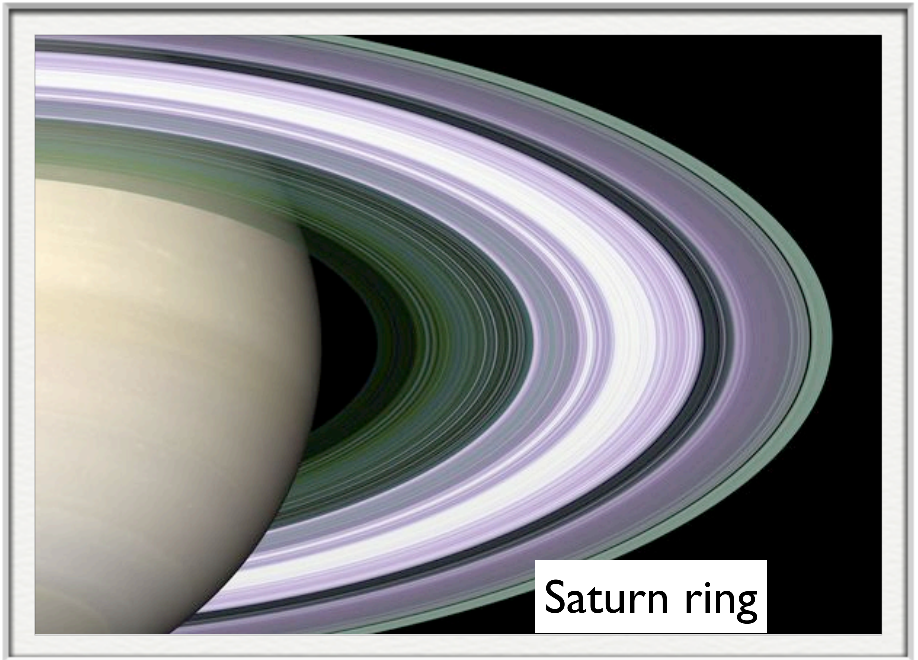
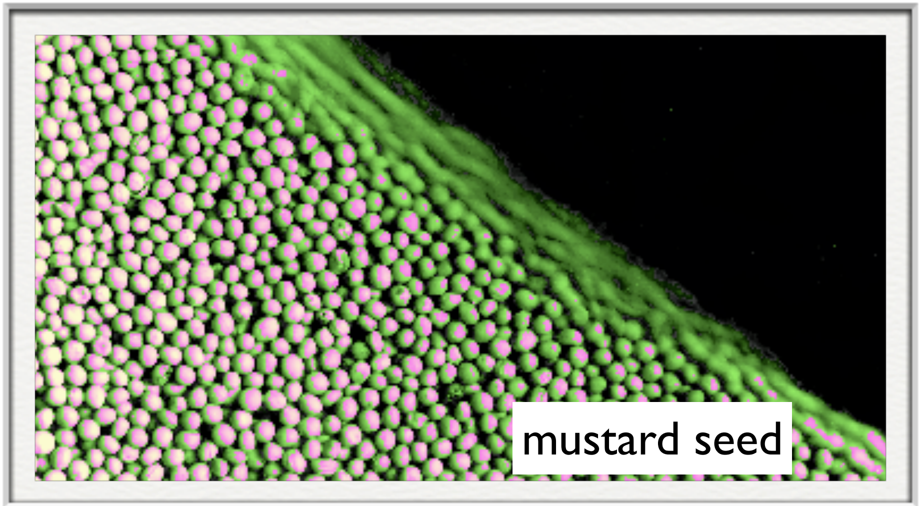
Critical scaling for the jamming transition of granular materials

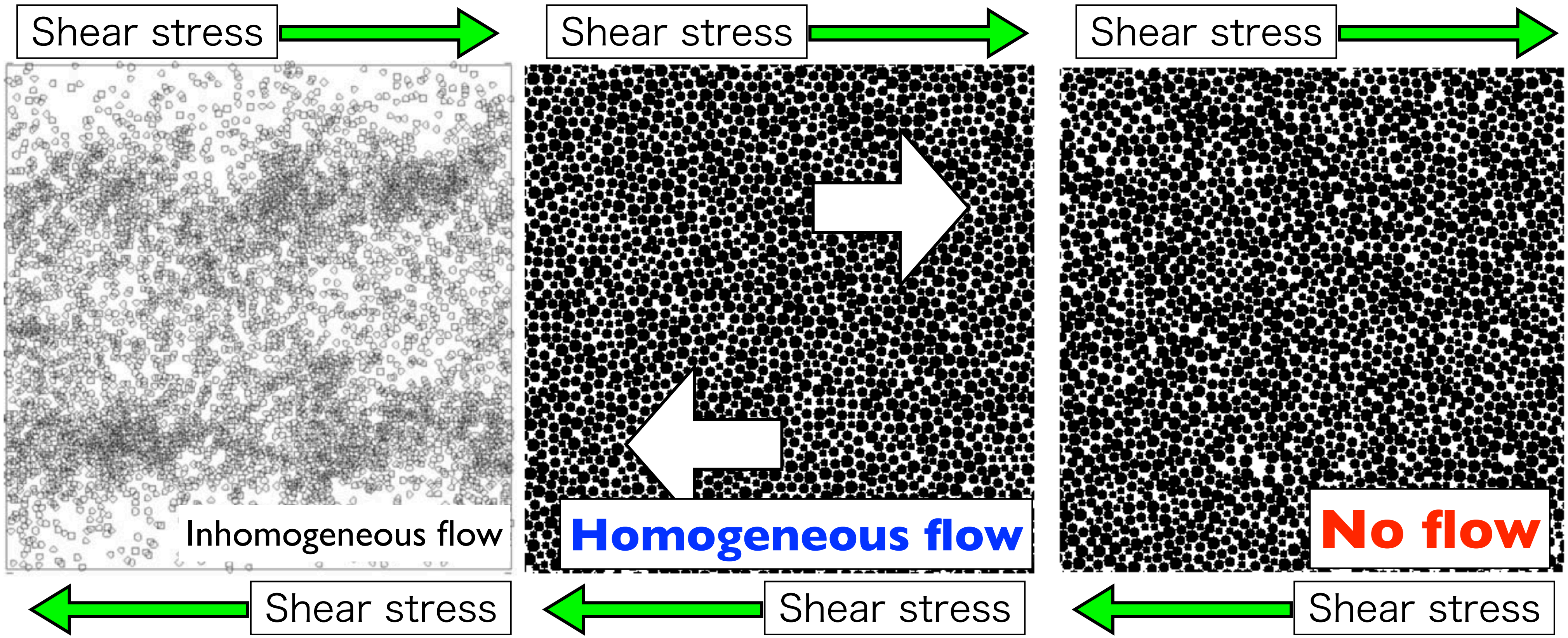
M. Otsuki (Aoyama Gakuin Univ.)
H. Hayakawa (Kyoto Univ.)

Granular materials





Sheared granular materials

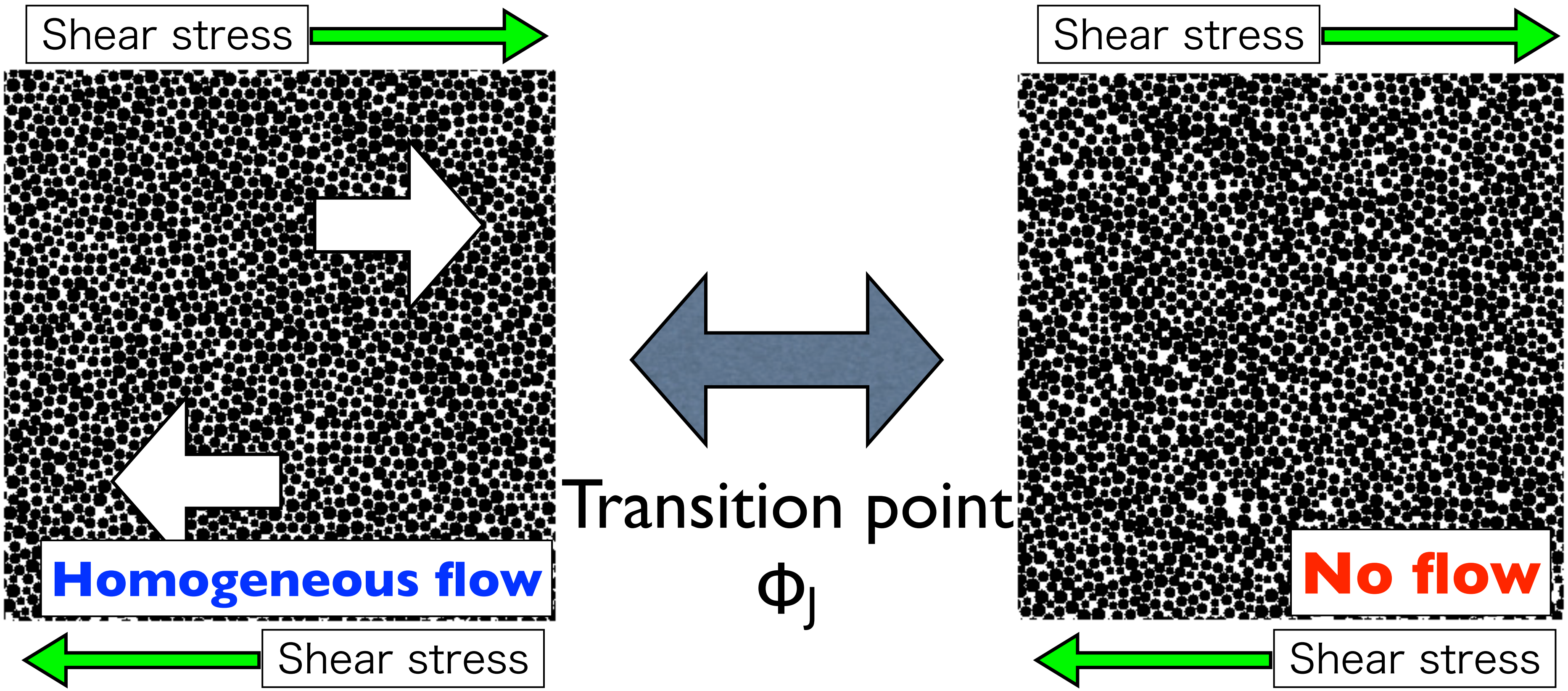


Gas
($\Phi = 0.12$)

Dense liquid
($\Phi = 0.8$)

Amorphous solid
($\Phi = 0.85$)

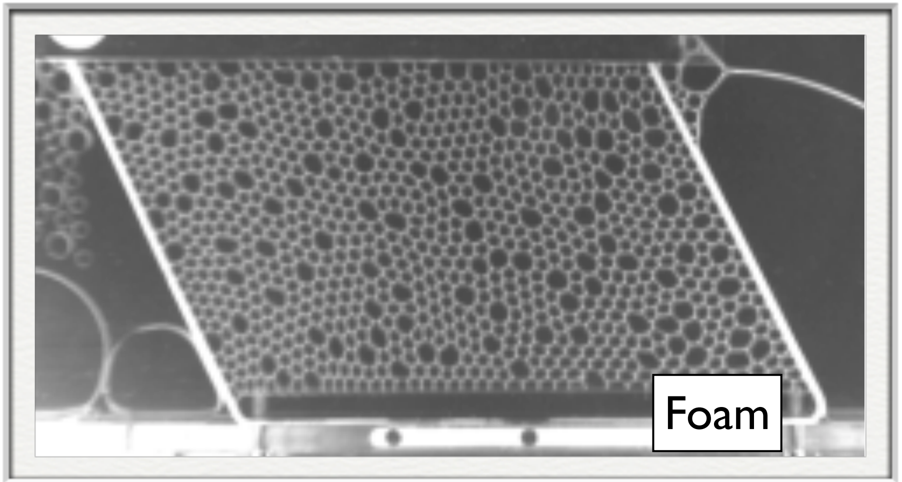
Jamming transition

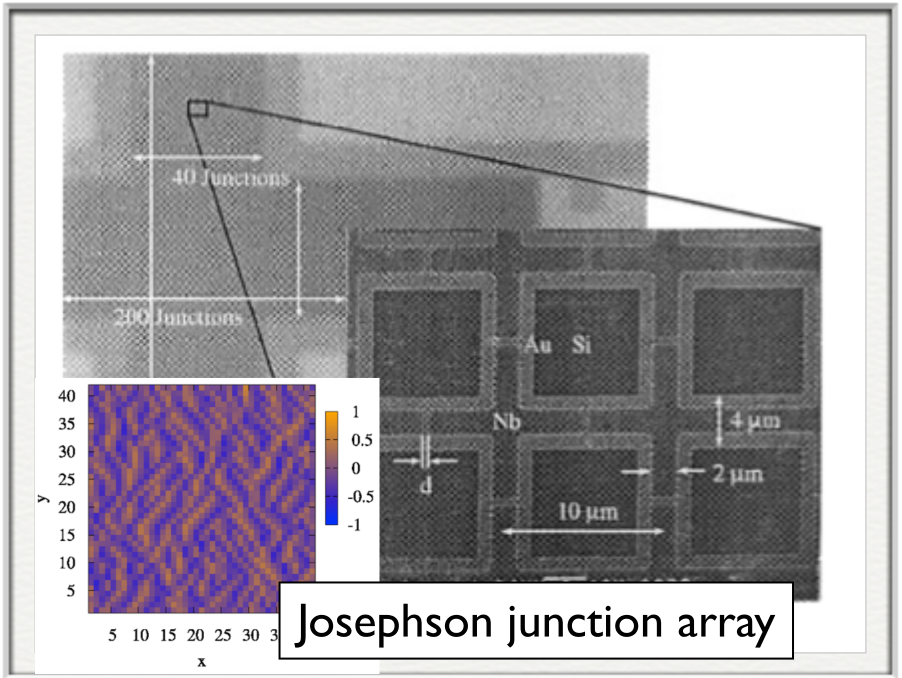


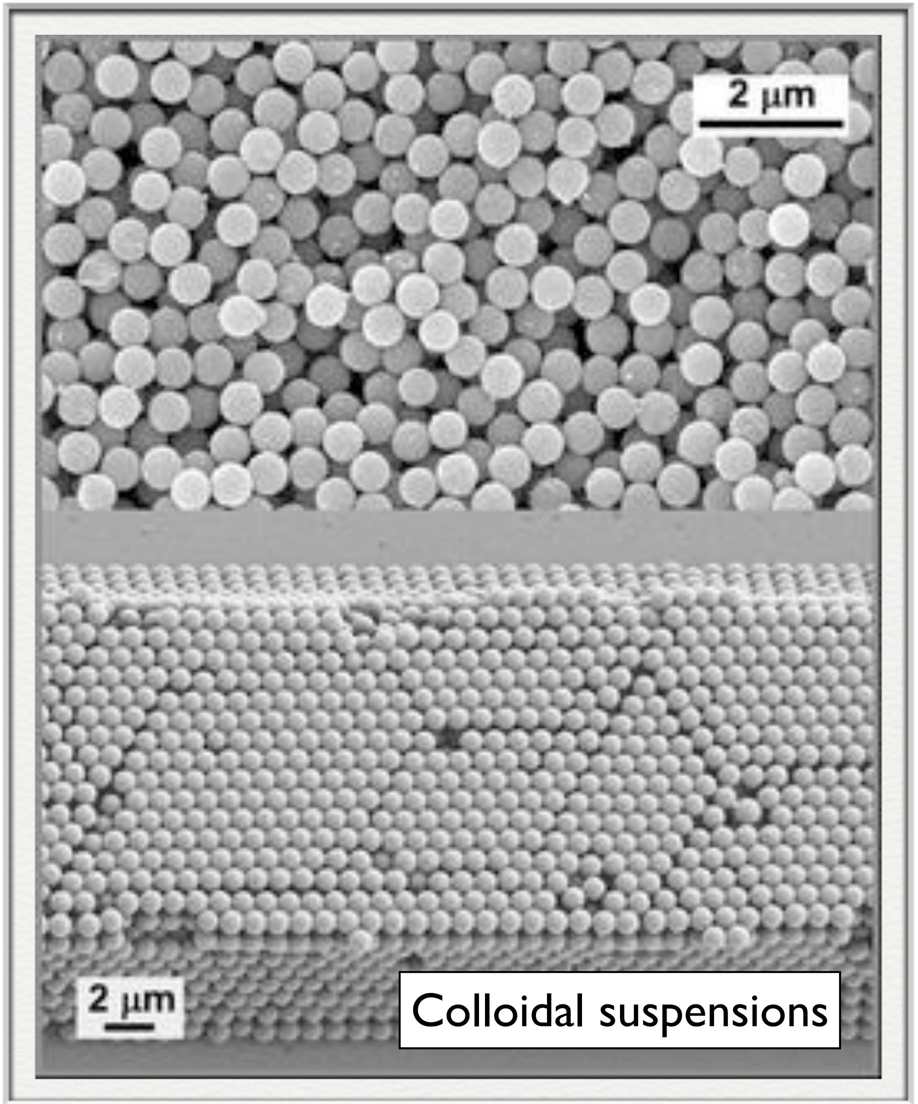
Dense liquid
($\Phi = 0.8$)

Amorphous solid
($\Phi = 0.85$)

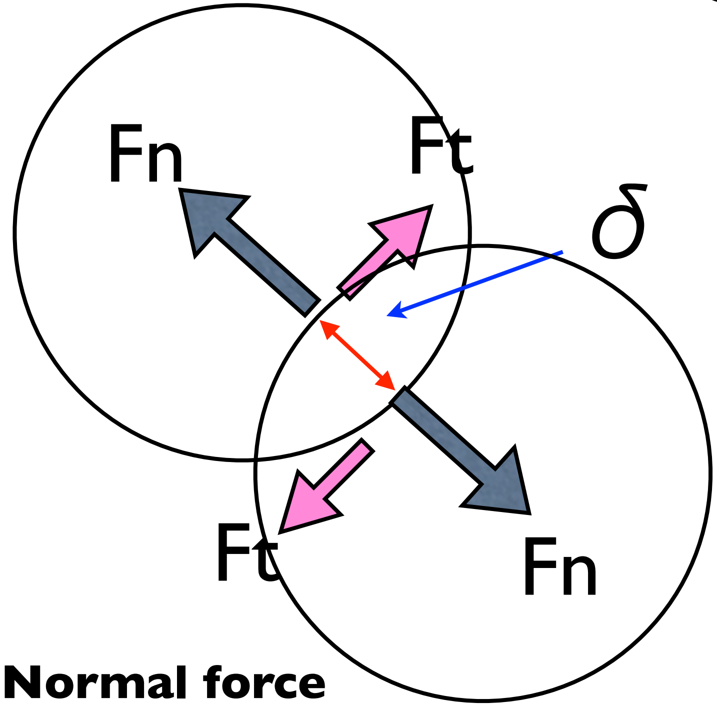
Jamming transition for athermal materials

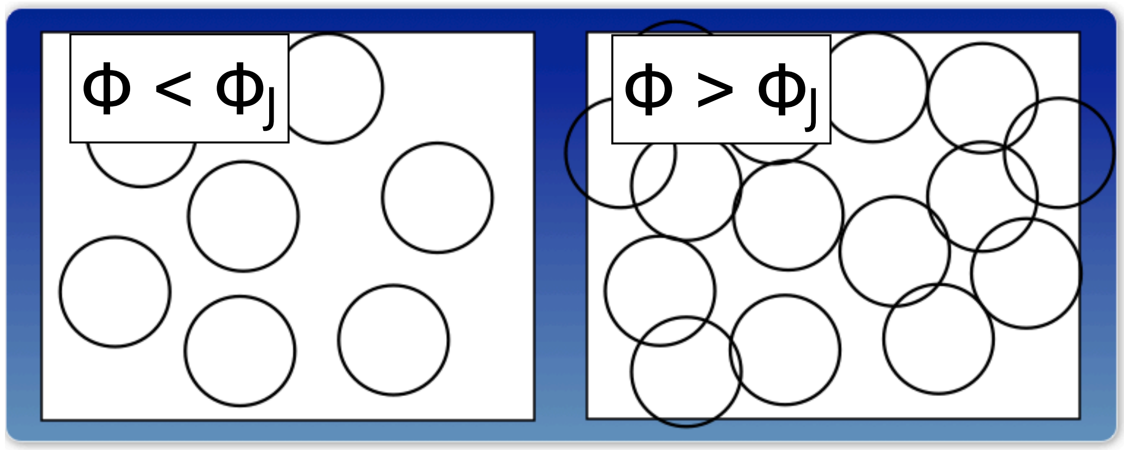






Model of granular materials





Normal force

- $F_n = k\,\delta^{\Delta} - \eta\, v_n$

  Elastic part — Dissipative part

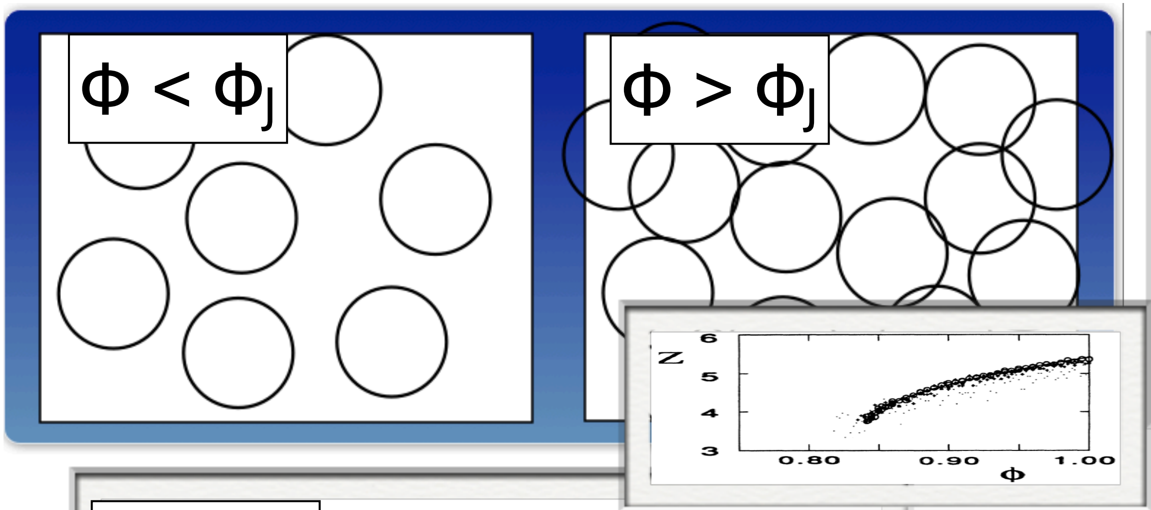
  - $\Delta = 1$ (Disk)
  - $\Delta = 3 / 2$ (Sphere)

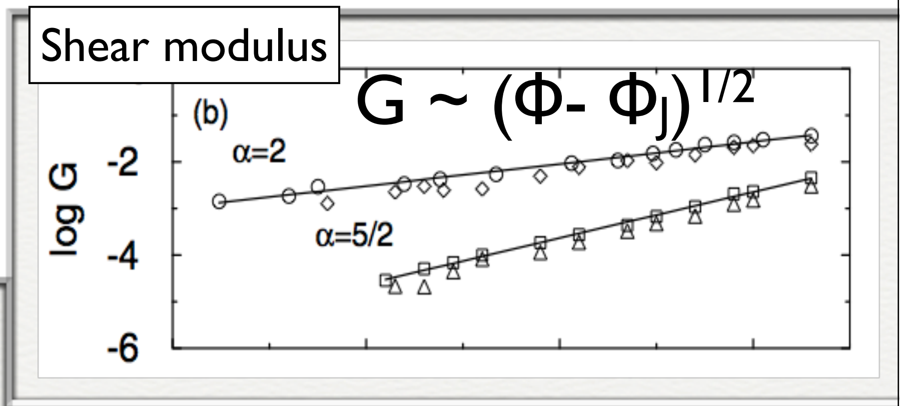
Tangential force

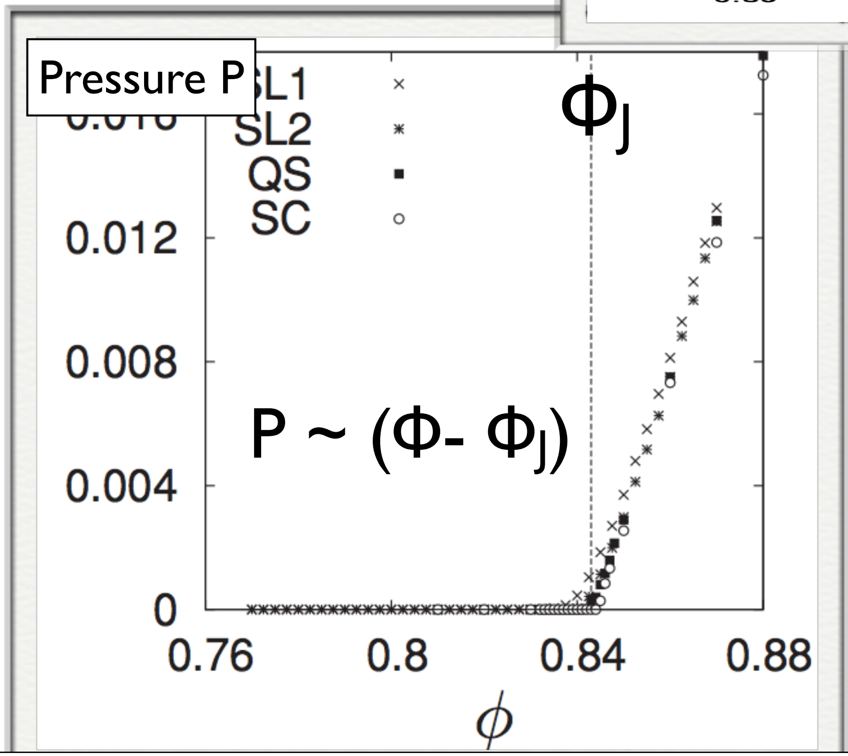
- Friction coefficient : μ
- $F_t < \mu F_n$ (Coulomb's friction)
- Frictionless : $\mu = 0$
- Frictional : $\mu > 0$

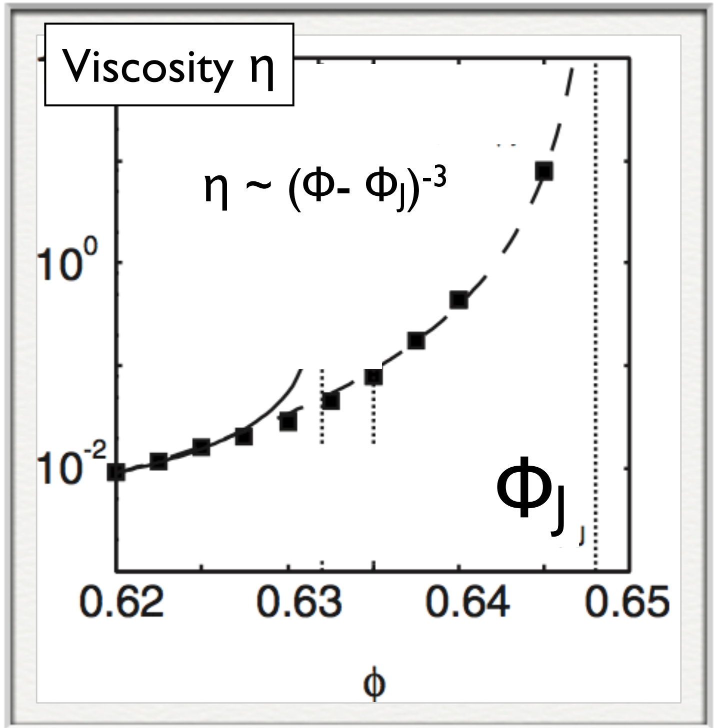
Critical properties

Frictionless case, $\Delta = 1$



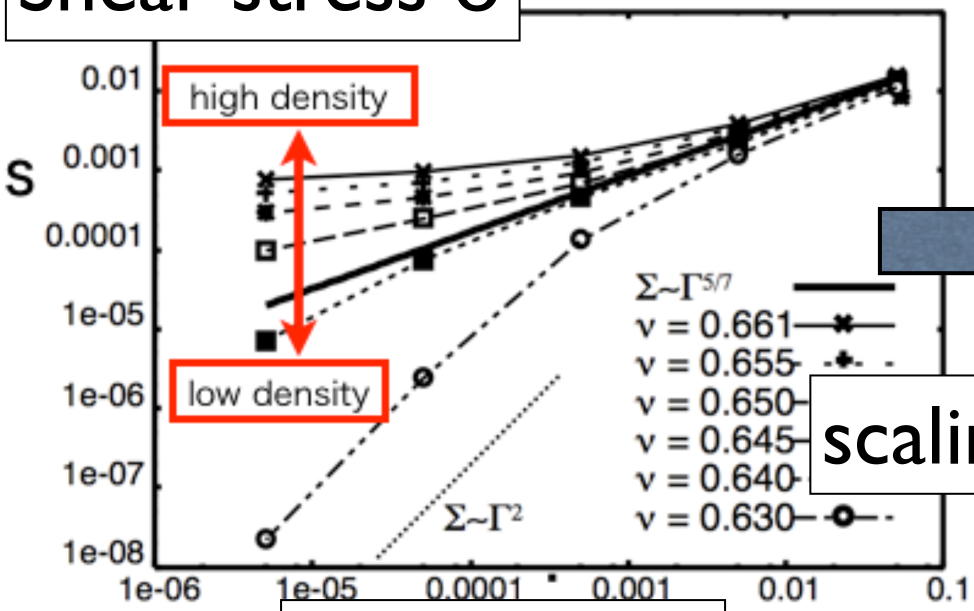






Rheological property

Shear stress σ

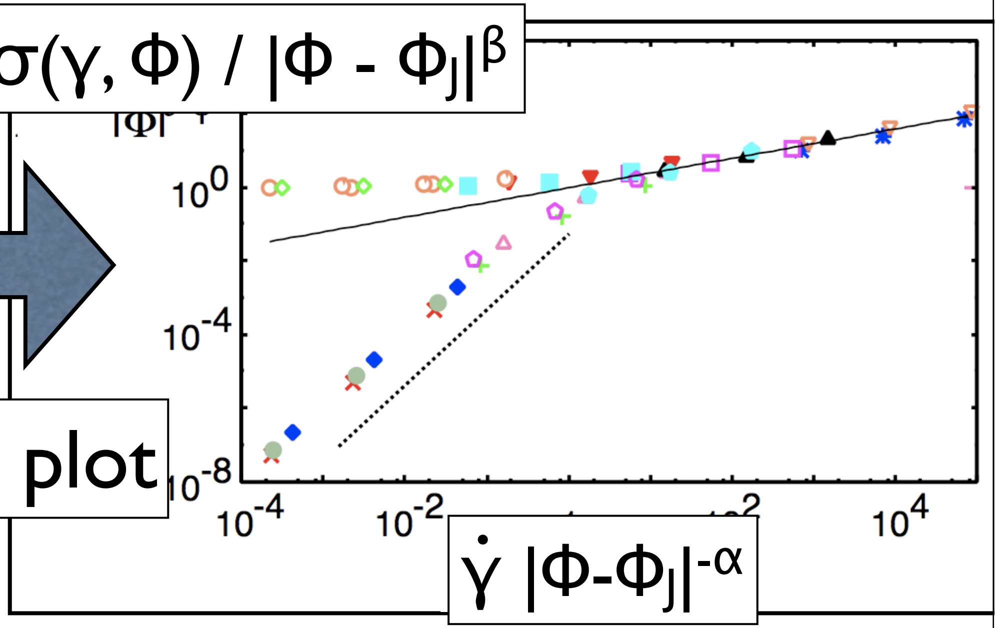


Shear rate $\dot{\gamma}$

scaling plot

Frictionless case, $\Delta = 1$

$\sigma(\gamma, \Phi) / |\Phi - \Phi_J|^\beta$



$\dot{\gamma}\,|\Phi-\Phi_J|^{-\alpha}$

Hatano, 2008

non-linear transport property

For $\Phi < \Phi_J$, $\sigma \propto \dot{\gamma}^2$ (liquid)

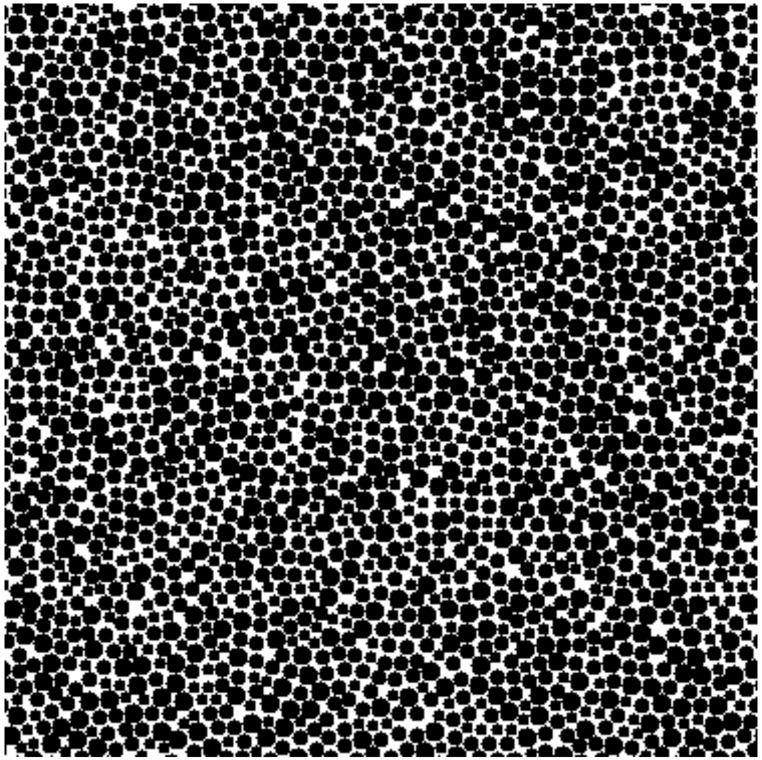
For $\Phi > \Phi_J$, $\sigma \simeq$ const (solid)

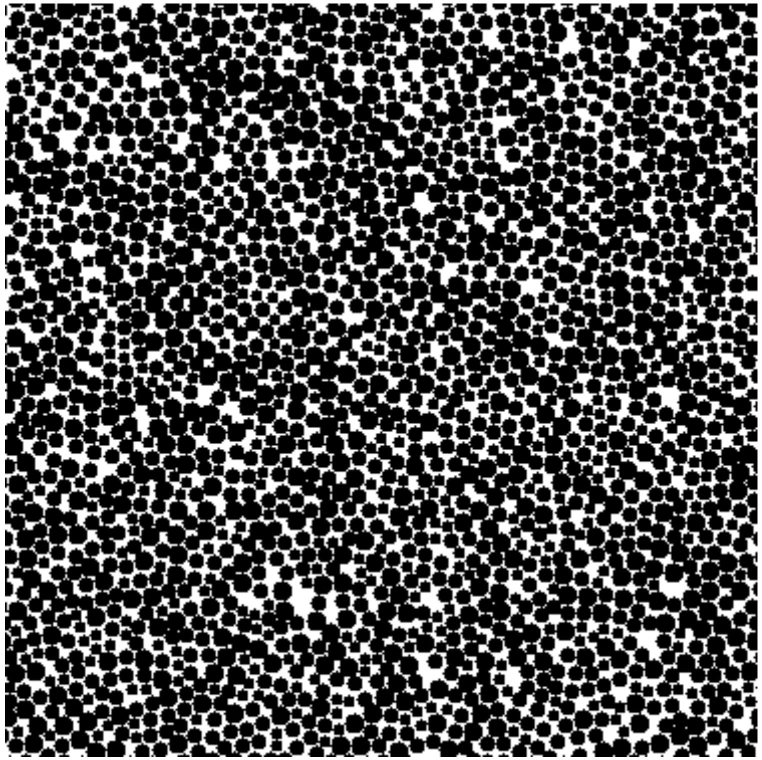
For $\Phi \simeq \Phi_J$, $\sigma \propto \dot{\gamma}^{y_\gamma}$

$$\sigma(\gamma, \Phi) = |\Phi - \Phi_J|^\beta S_\pm(\dot{\gamma}\,|\Phi-\Phi_J|^{-\alpha})$$

α, β : Critical exponents

Dynamics (constant shear rate)

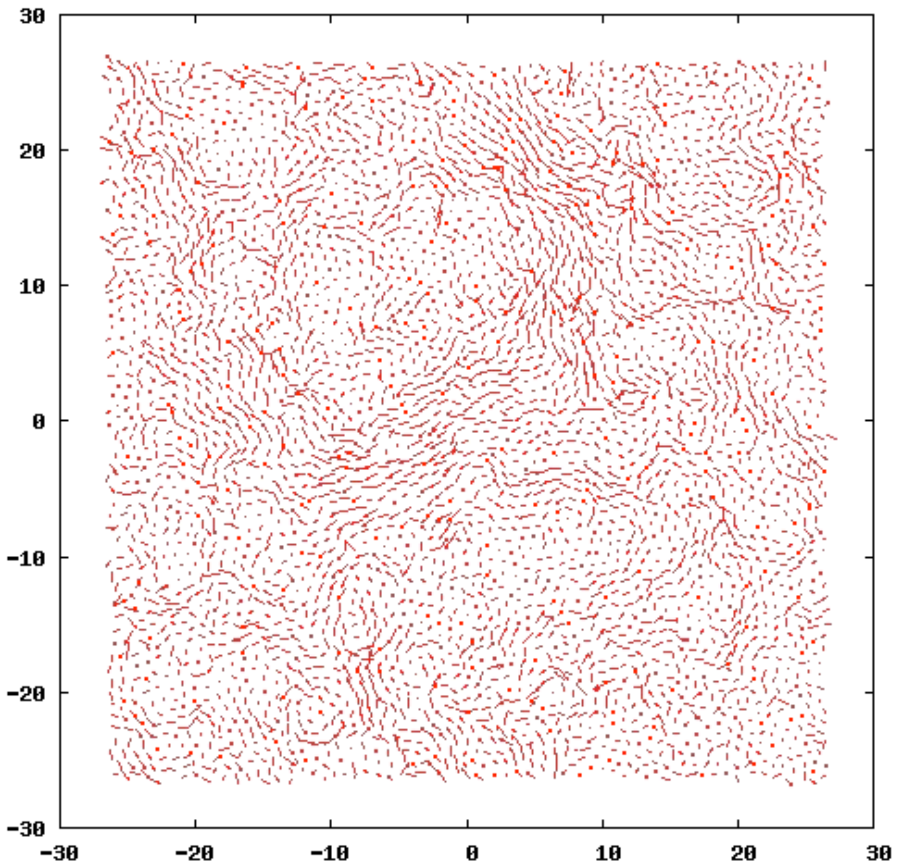




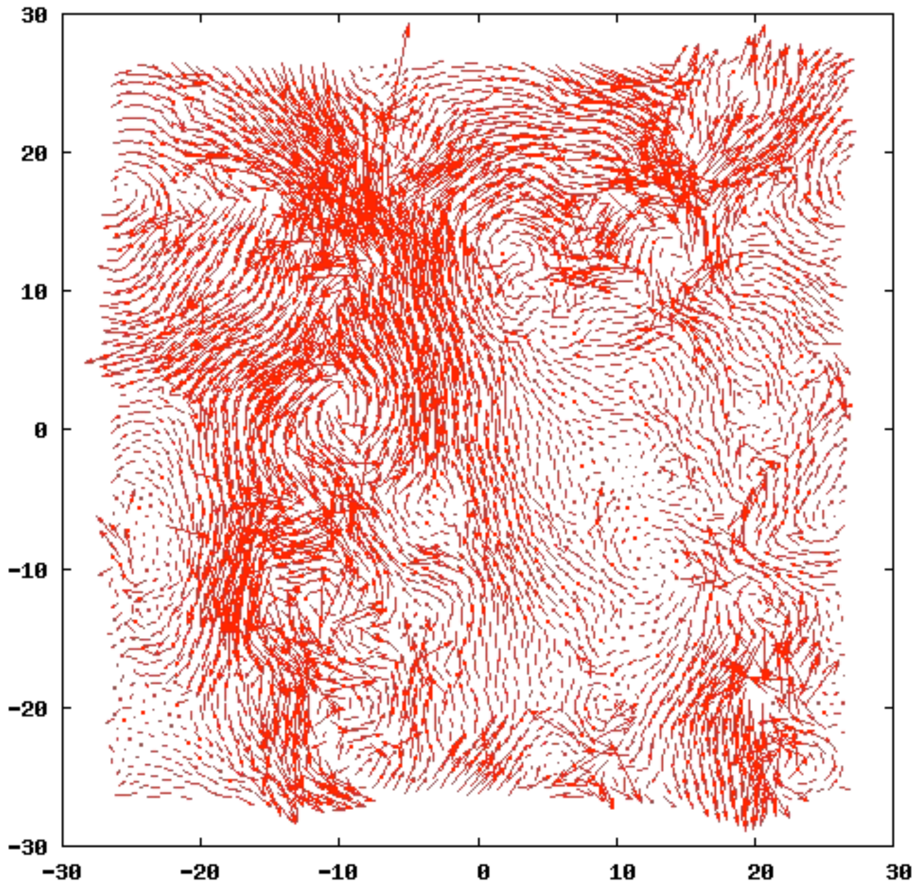
$\Phi = 0.80 < \Phi_J$

$\Phi = 0.85 > \Phi_J$

Dynamics (velocity fluctuation)

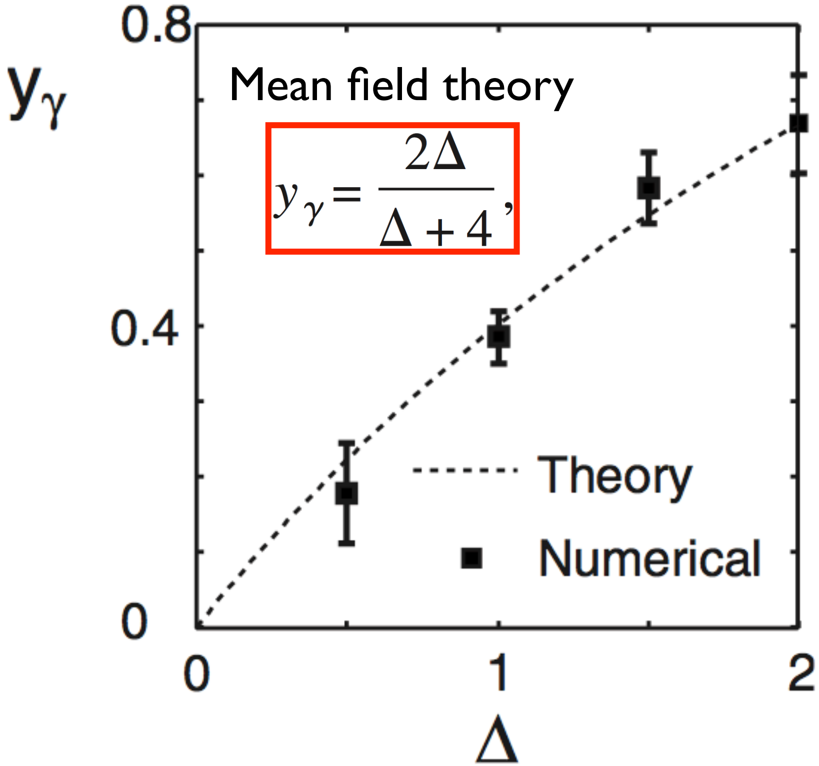


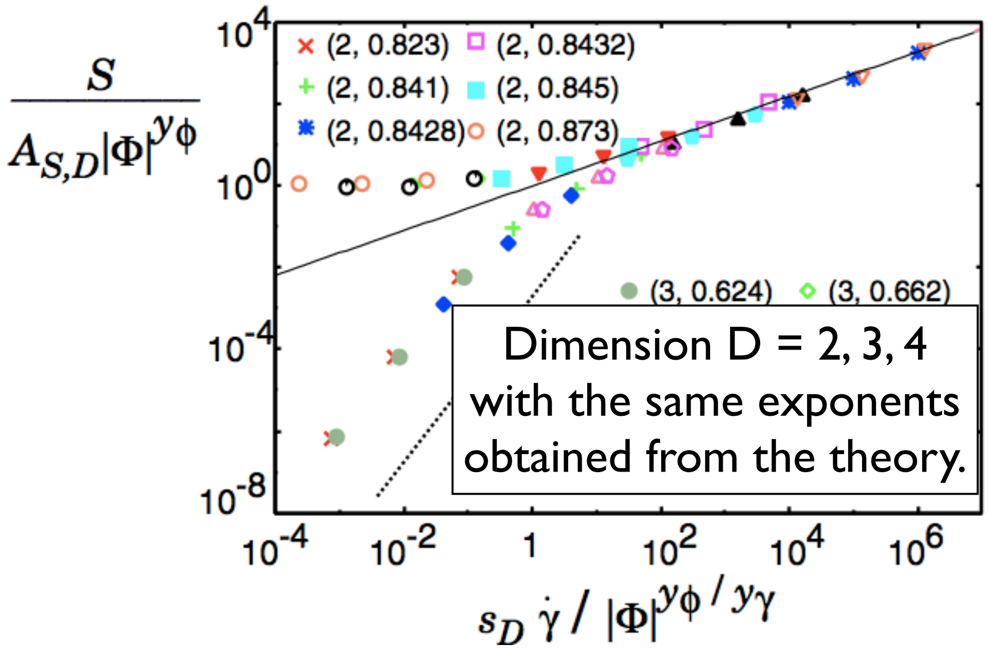
$\Phi = 0.80 < \Phi_J$



$\Phi = 0.85 > \Phi_J$

Characteristic features



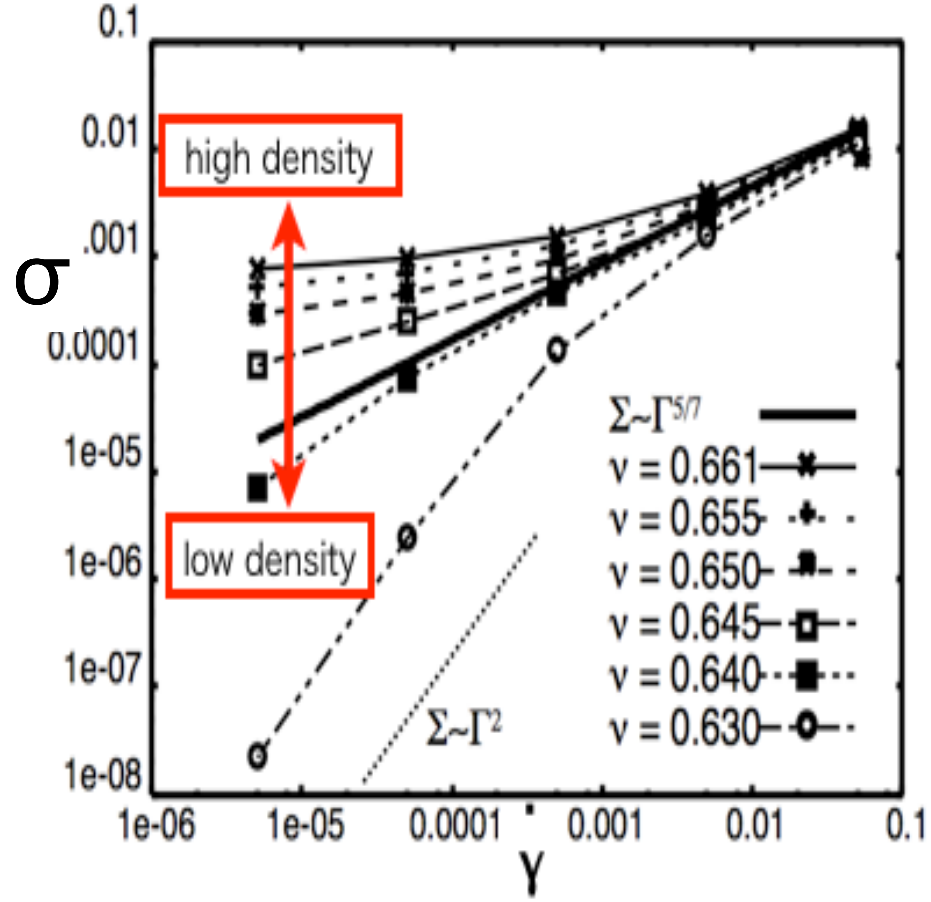


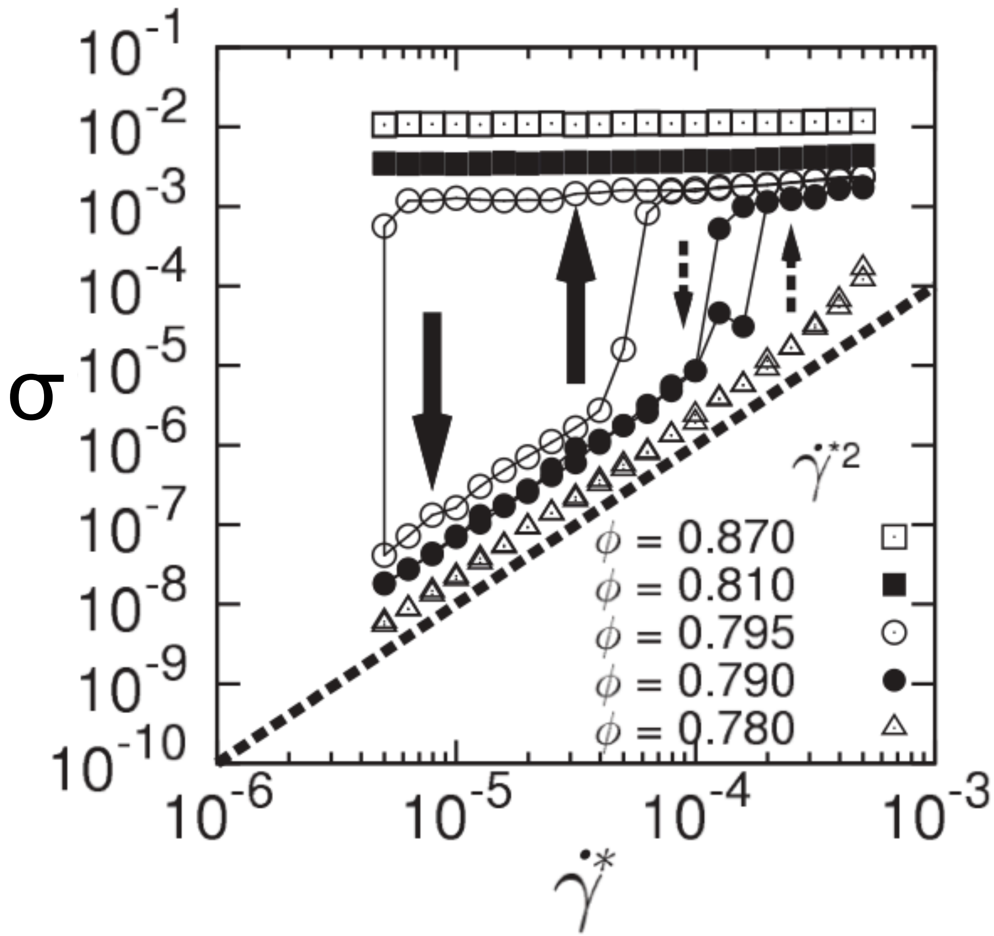
The critical exponents depend on the type of the contact force.

The critical exponents are independent of the dimension.

$F_n = k\, \delta^{\Delta}$

Effect of Friction



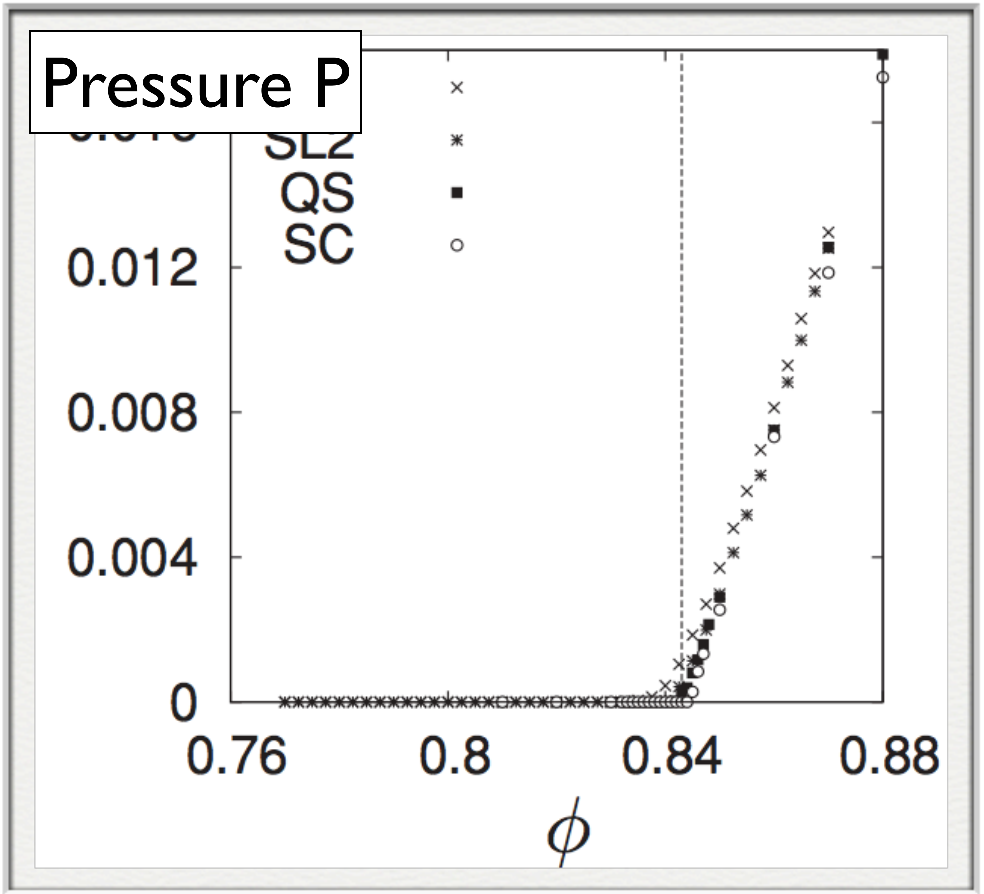


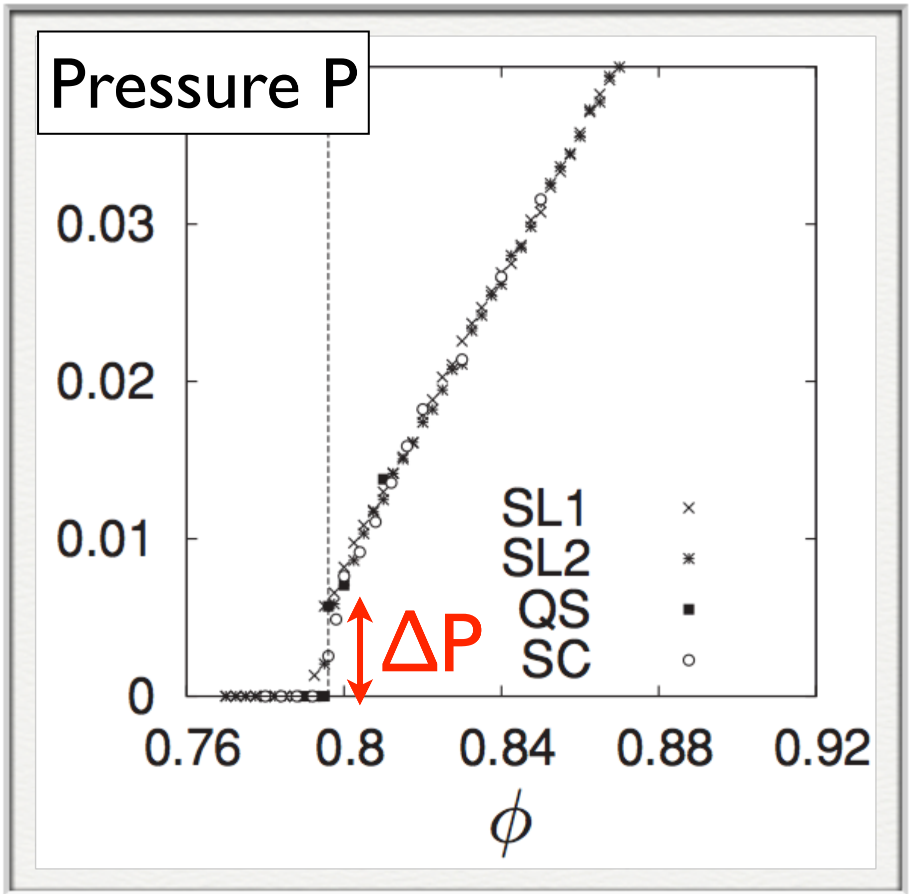
Frictionless (μ = 0.0)

Frictional (μ = 2.0)

Hysteresis loop for frictional case

Effect of friction (pressure)

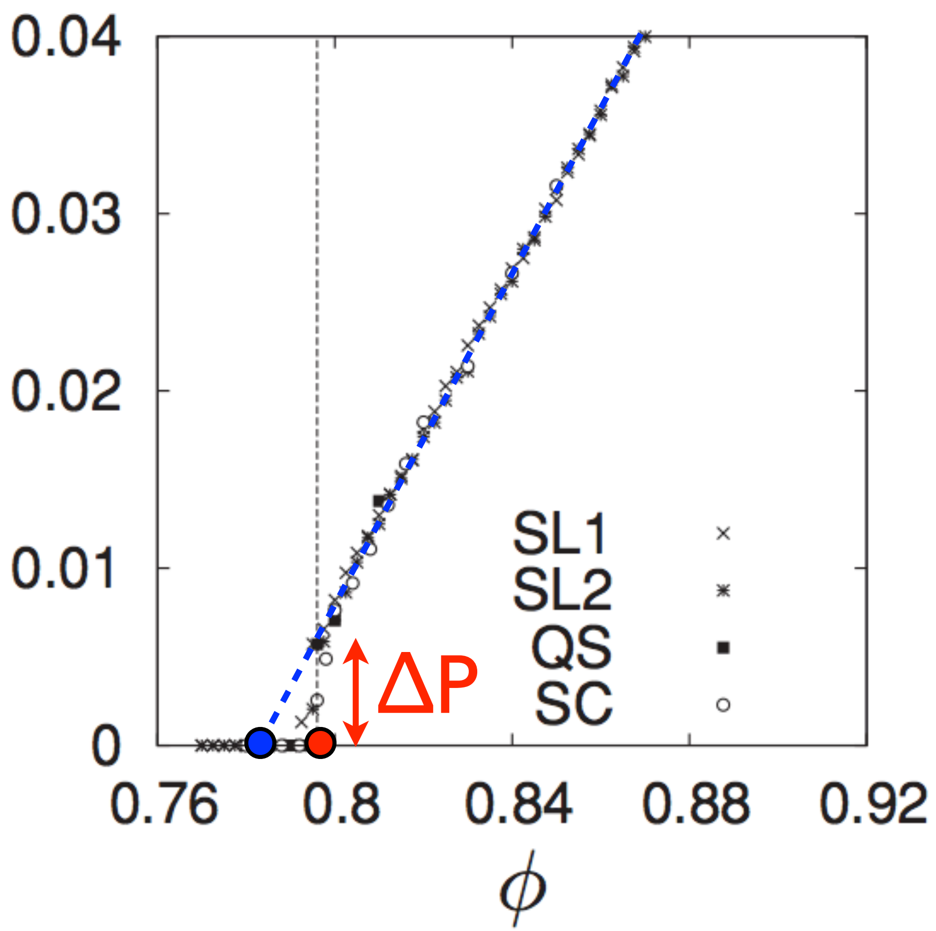


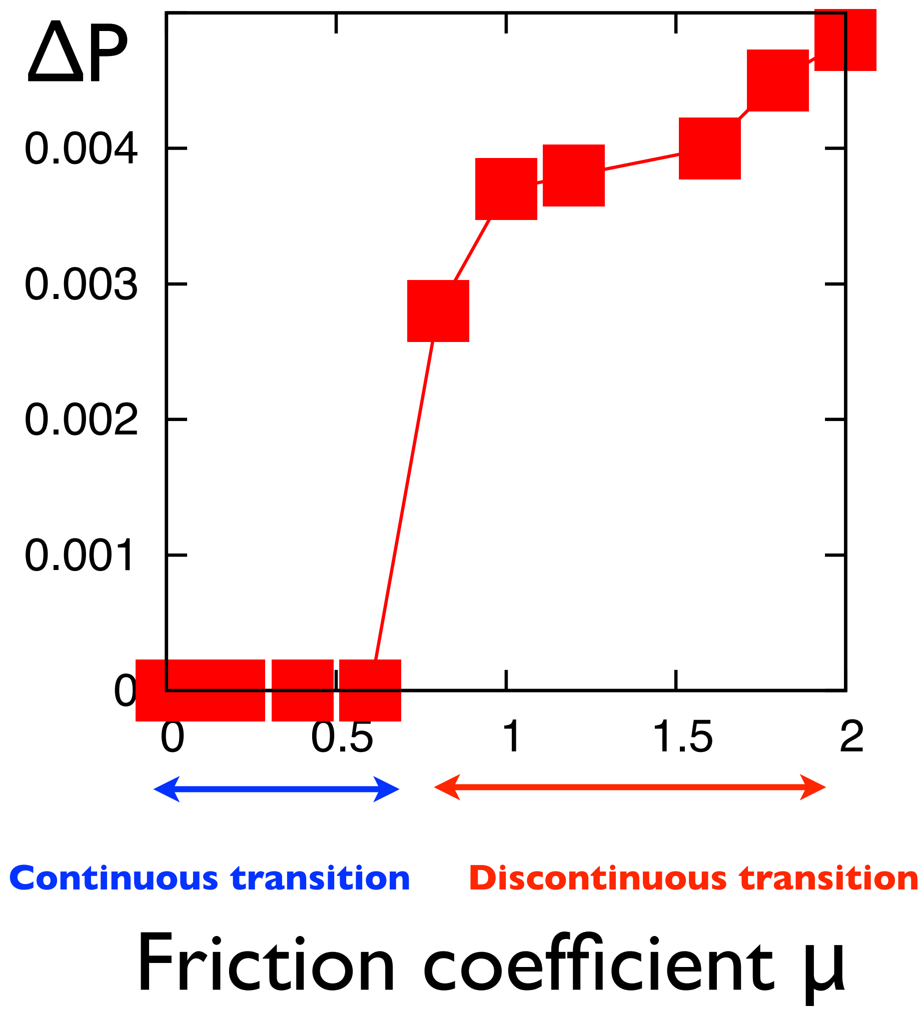


Frictionless (μ = 0.0)
Continuous transition

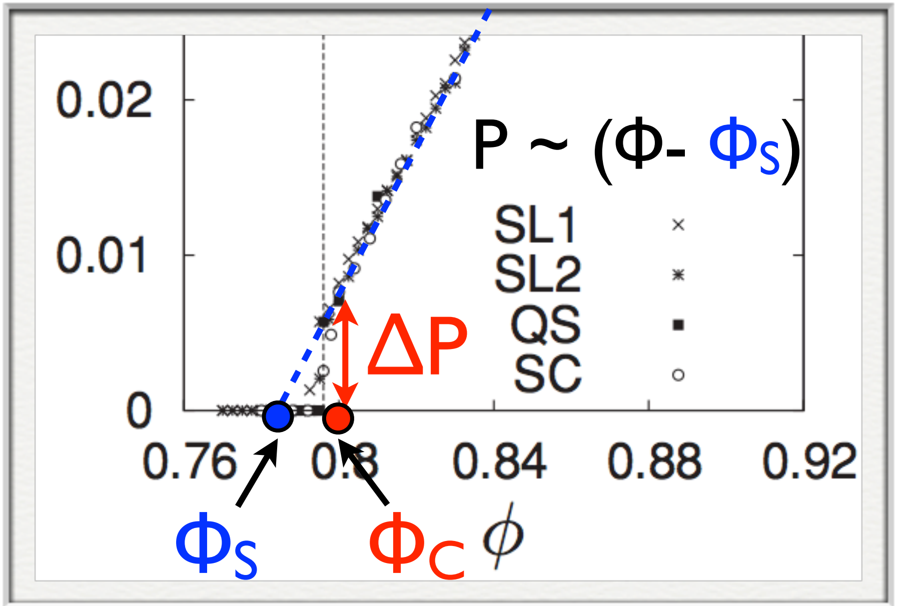
Frictional (μ = 2.0)
Discontinuous transition

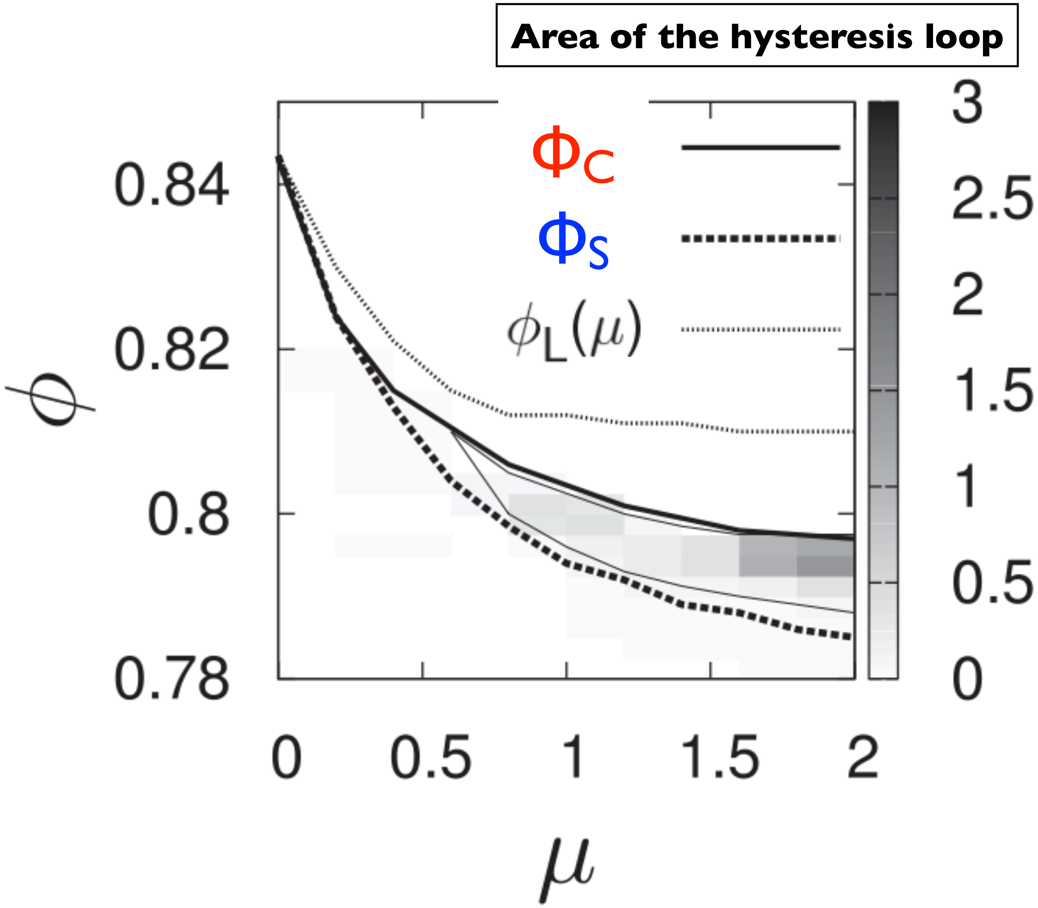
Effect of friction (type of the transition)

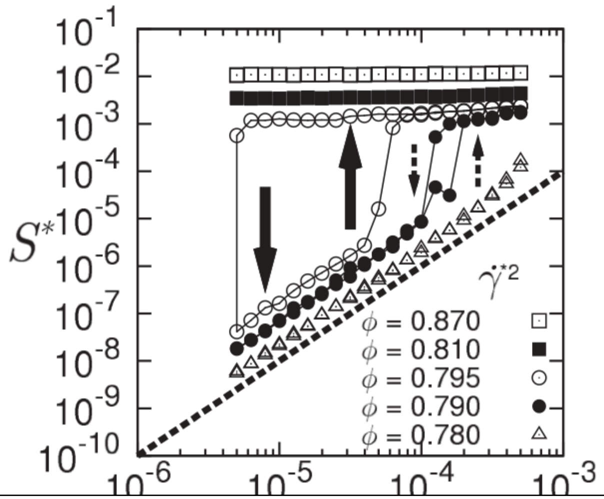




Phase diagram







Summary & Discussion

- **Jamming transition** : Athermal transition from liquid-like states to solid-like states.
- Critical exponents depend on the interaction.
- Continuous transition for frictionless case, discontinuous transition for frictional case.
- Hysteresis loop, many critical densities.
- Our result may provide a better understanding of dynamics and non-linear transport properties of dense matters.