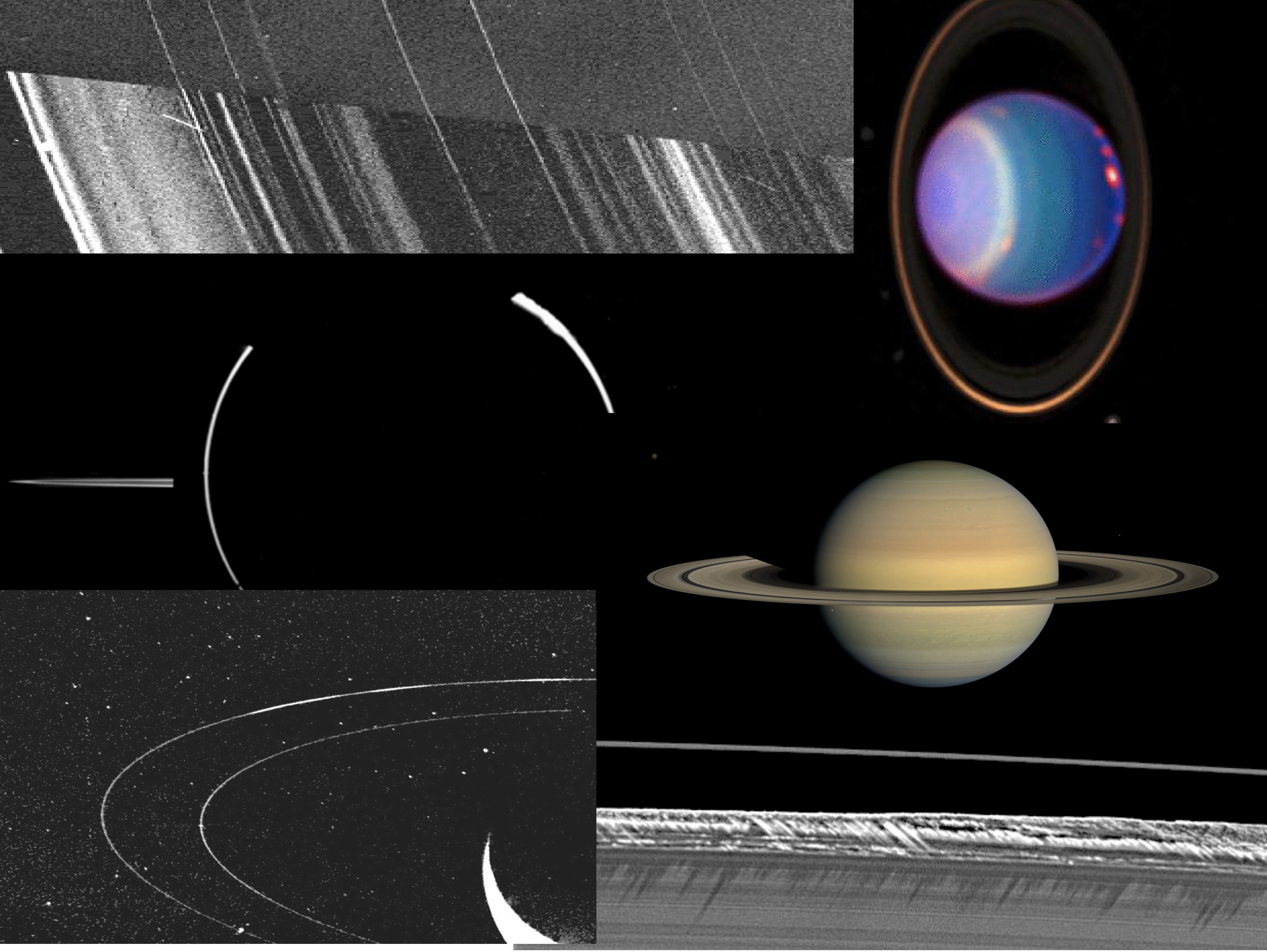
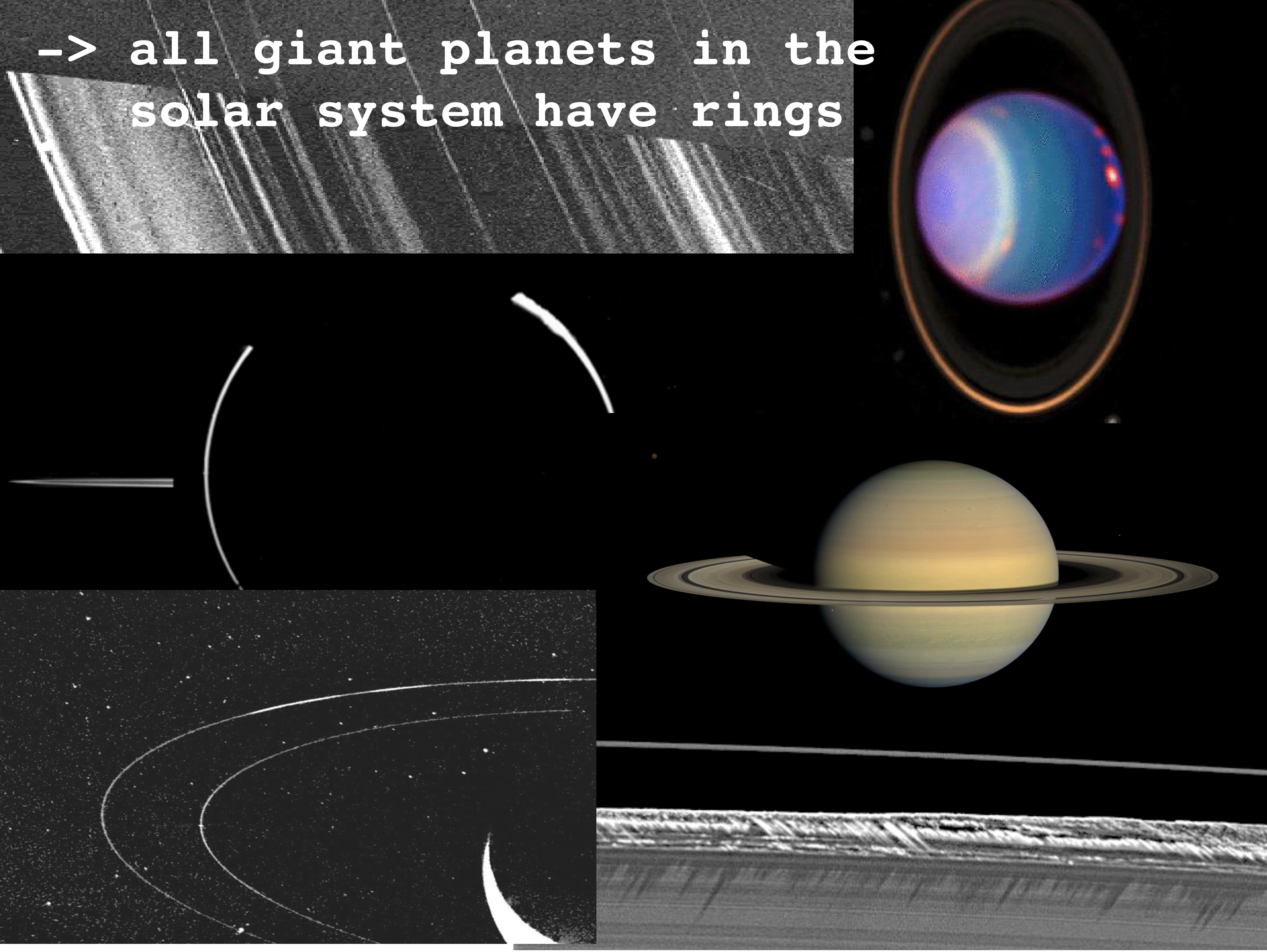
Dynamics of Planetary Rings

J Schmidt, H Salo
A Bodrova, N Brilliantov,
H Hayakawa, P Krapivsky,
F Spahn, M Sremcevic

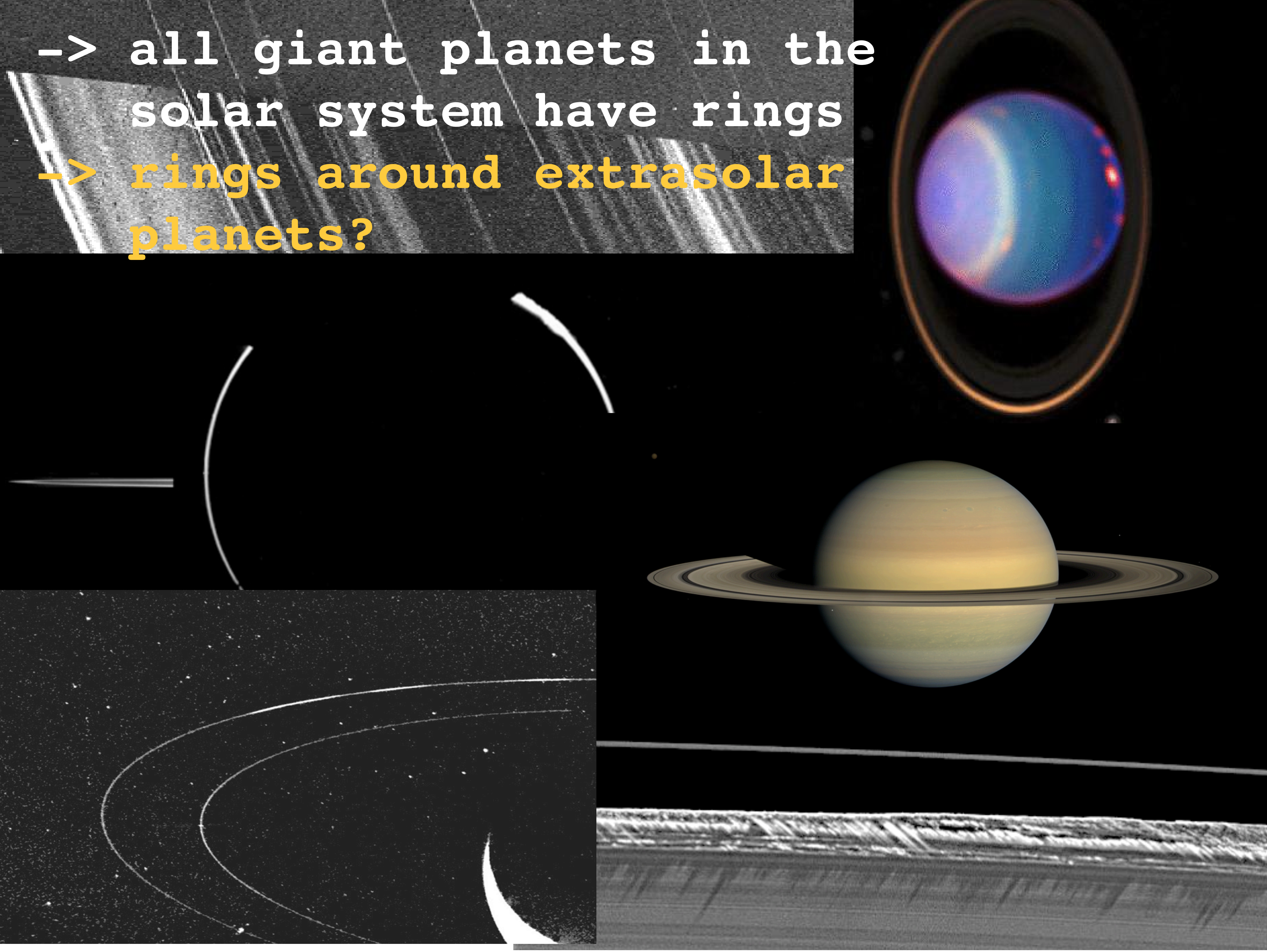


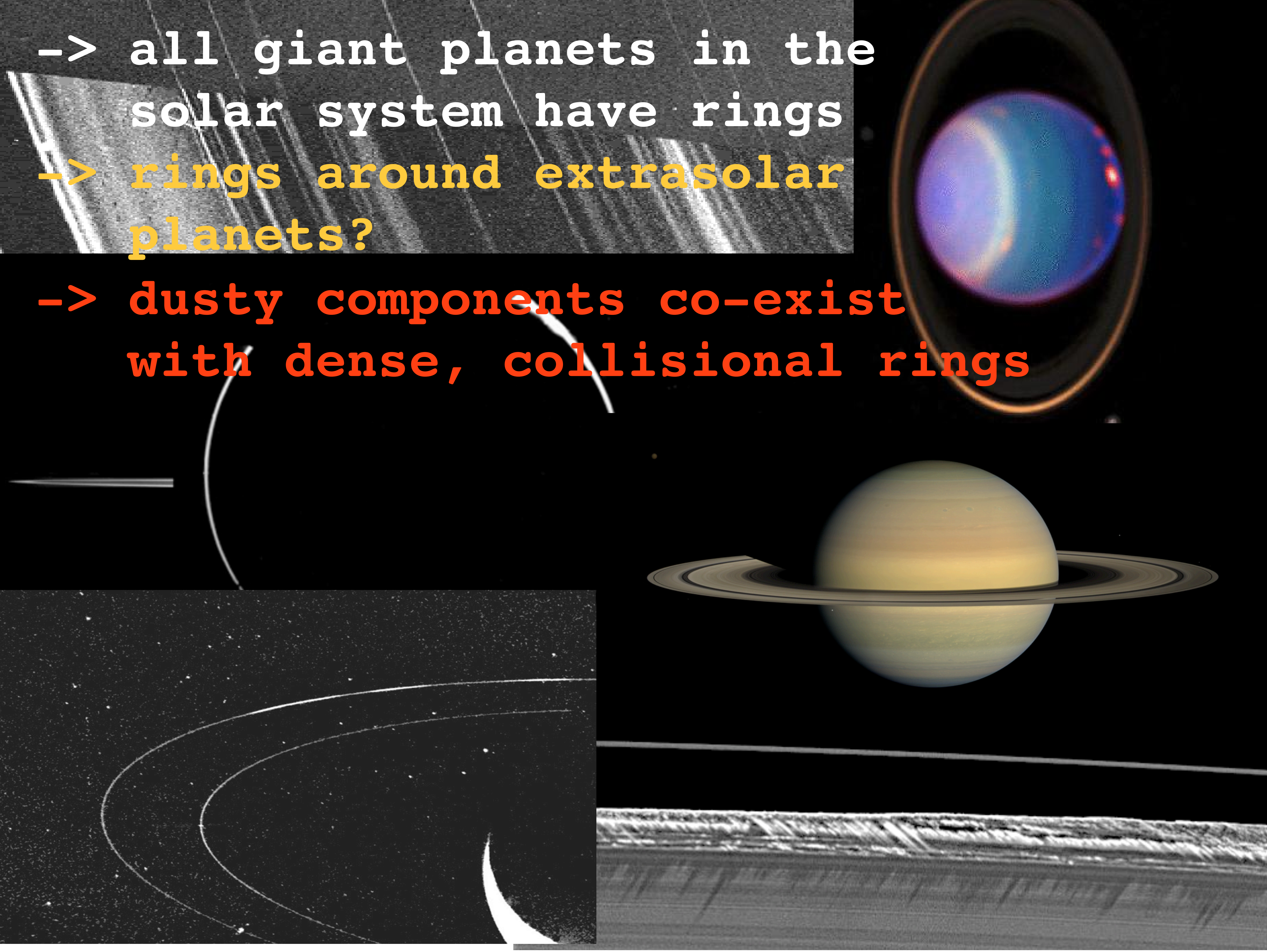
-> all giant planets in the solar system have rings



-> all giant planets in the solar system have rings

-> rings around extrasolar planets?

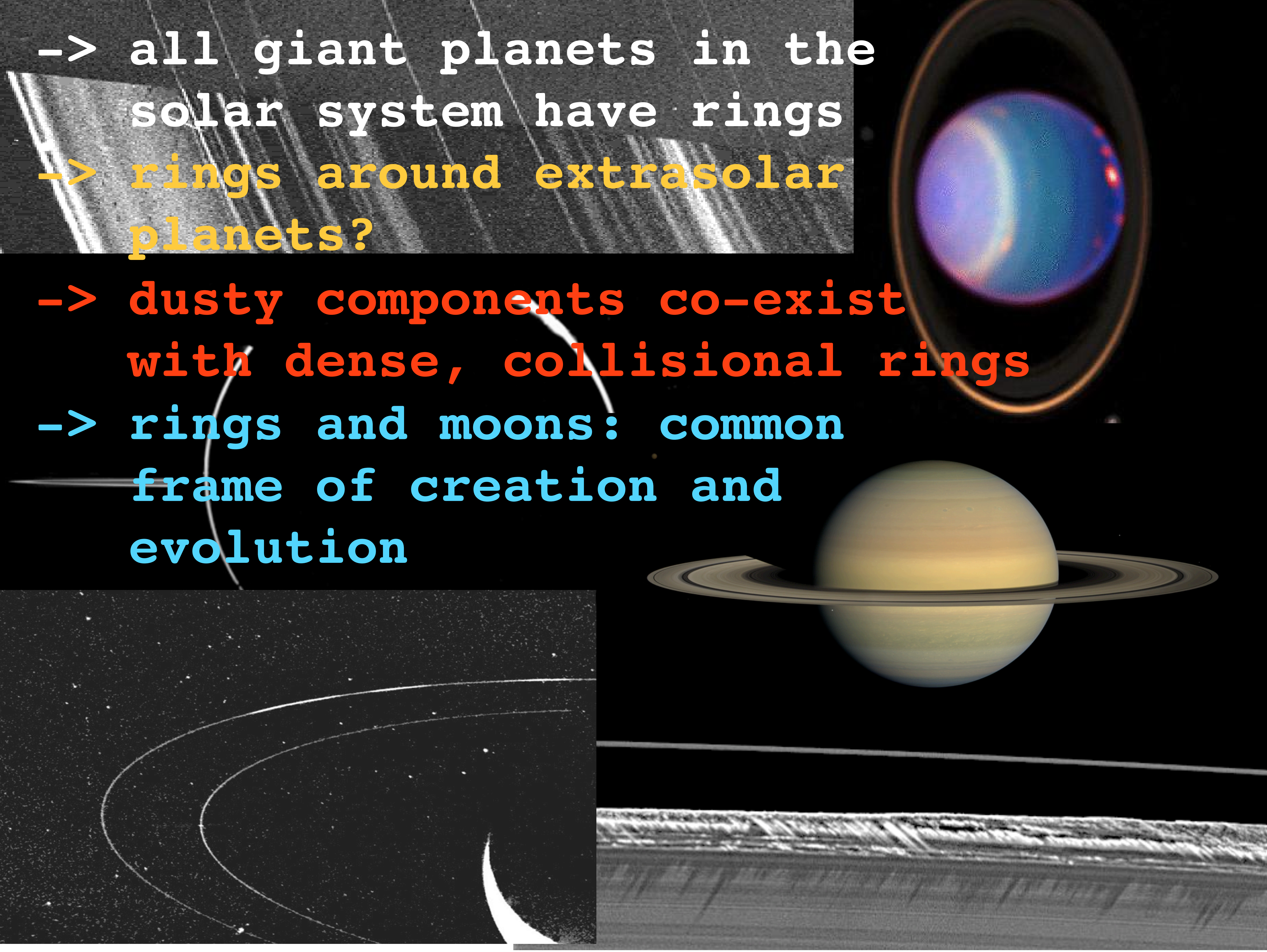




-> all giant planets in the solar system have rings

-> rings around extrasolar planets?

-> dusty components co-exist with dense, collisional rings



- -> all giant planets in the solar system have rings
- -> rings around extrasolar planets?
- -> dusty components co-exist with dense, collisional rings
- -> rings and moons: common frame of creation and evolution

- -> all giant planets in the solar system have rings
- -> rings around extrasolar planets?
- -> dusty components co-exist with dense, collisional rings
- -> rings and moons: common frame of creation and evolution
- -> collisional rings: structure on all length-scales

-> all giant planets in the solar system have rings

-> rings around extrasolar planets?

-> dusty components co-exist with dense, collisional rings

-> rings and moons: common frame of creation and evolution

-> collisional rings: structure on all length-scales

-> similar physics for proto-planetary disks, accretion disks, galaxies, ...

this talk:

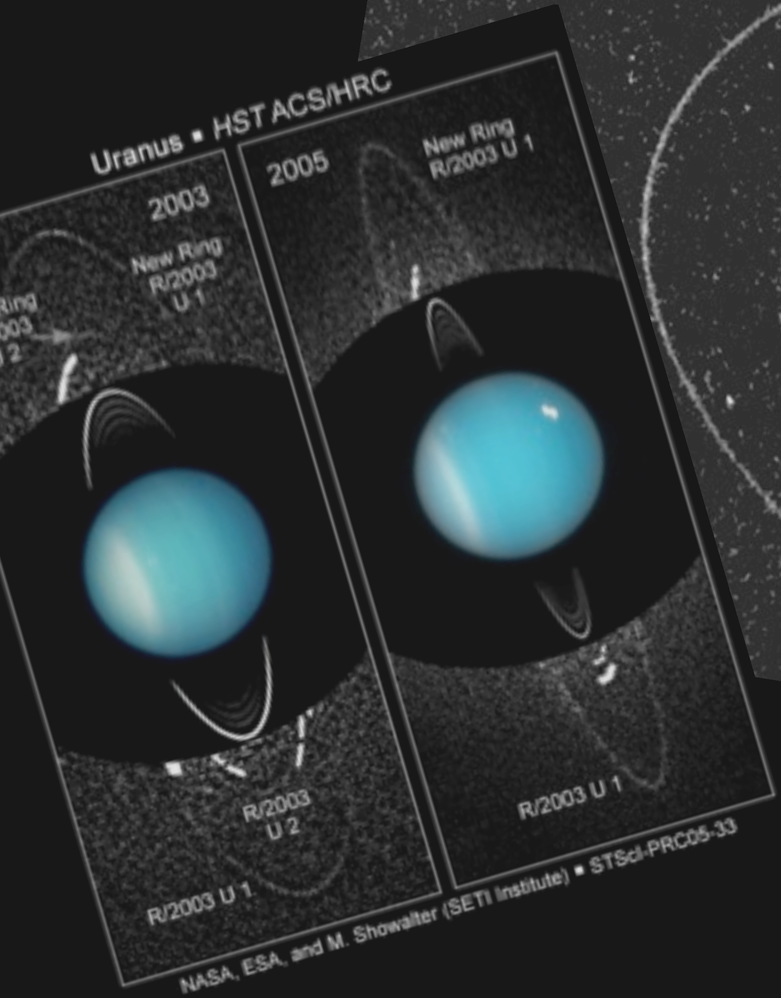
-> brief summary on dust rings

-> dense, collisional rings

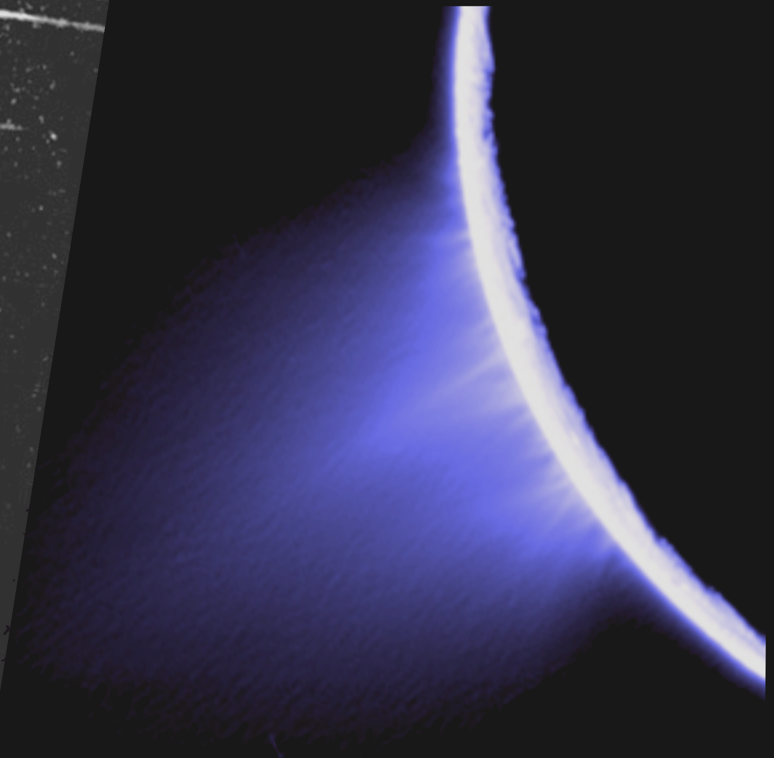
- basic physical properties and processes
- ring structure, instabilities
- kinetics of the size-distribution

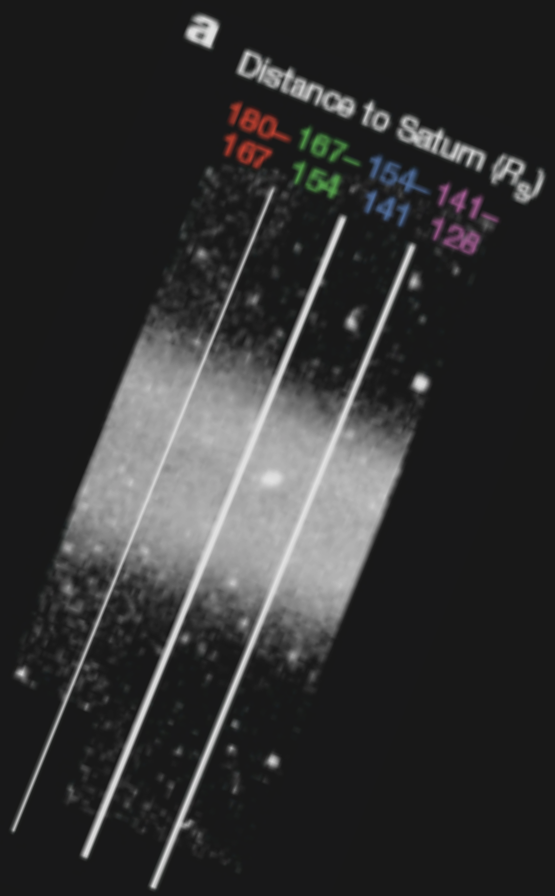
dust rings

dust rings



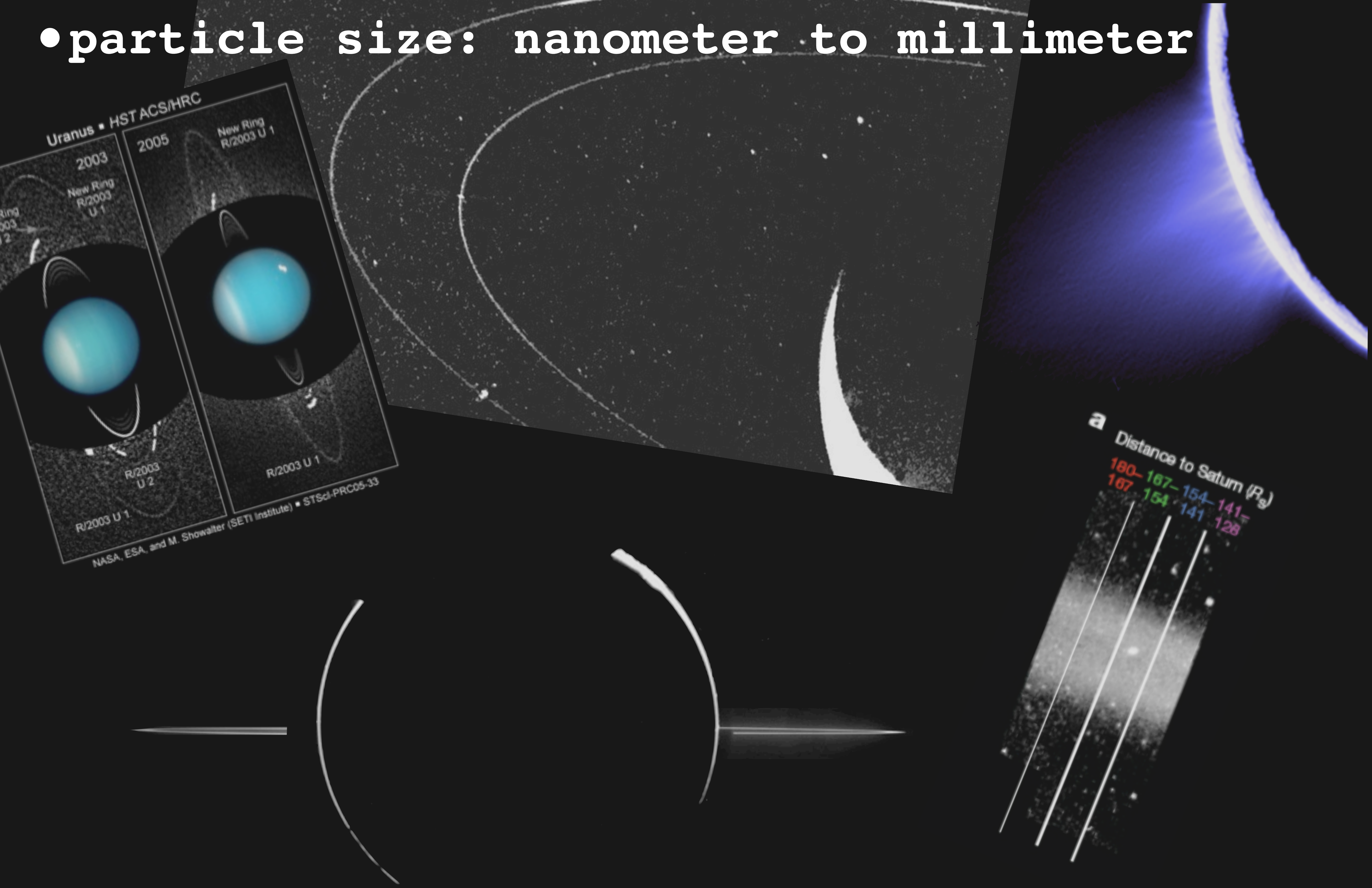
NASA, ESA, and M. Showalter (SETI Institute) ▪ STScI-PRC05-33





dust rings

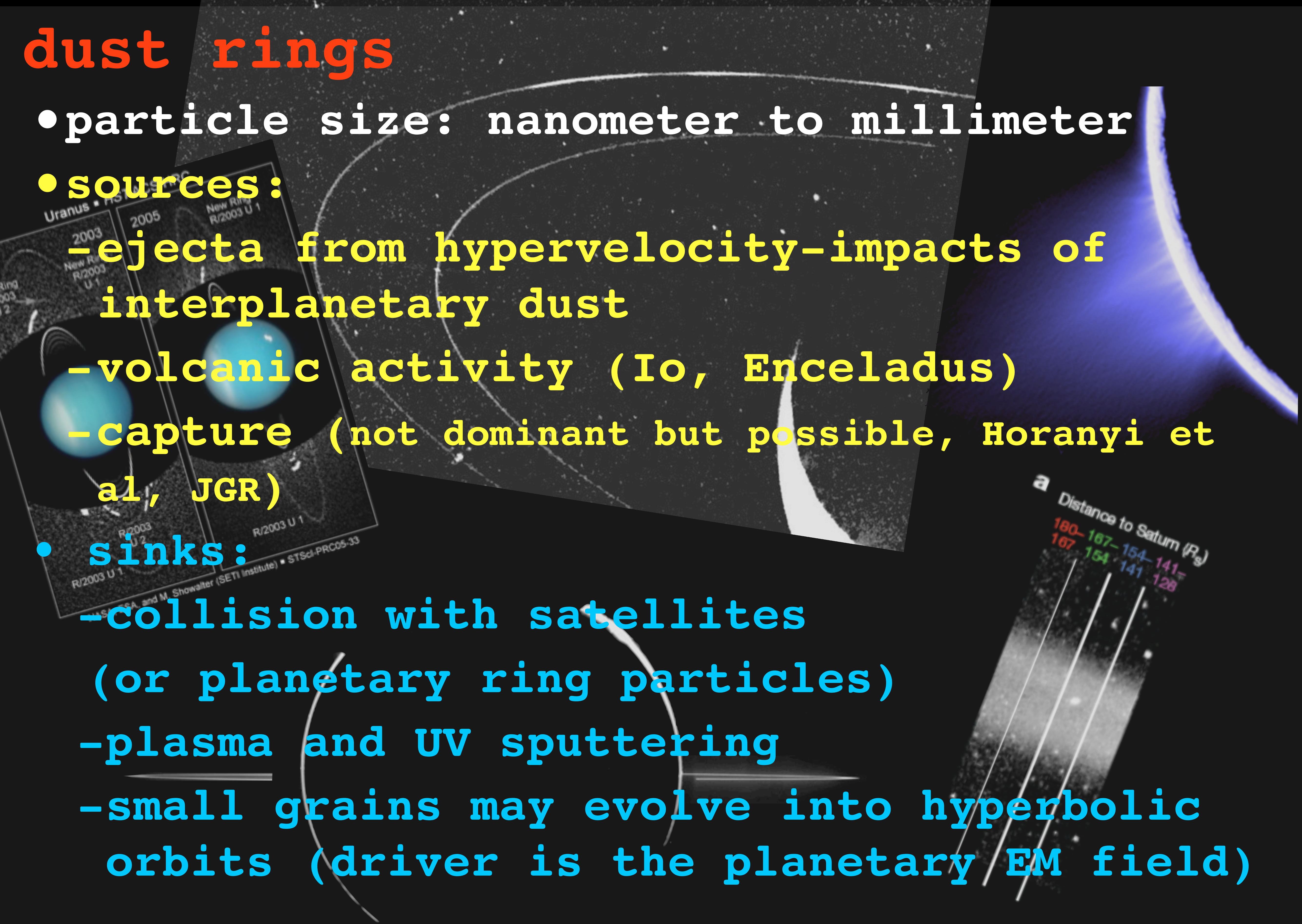
- particle size: nanometer to millimeter



dust rings

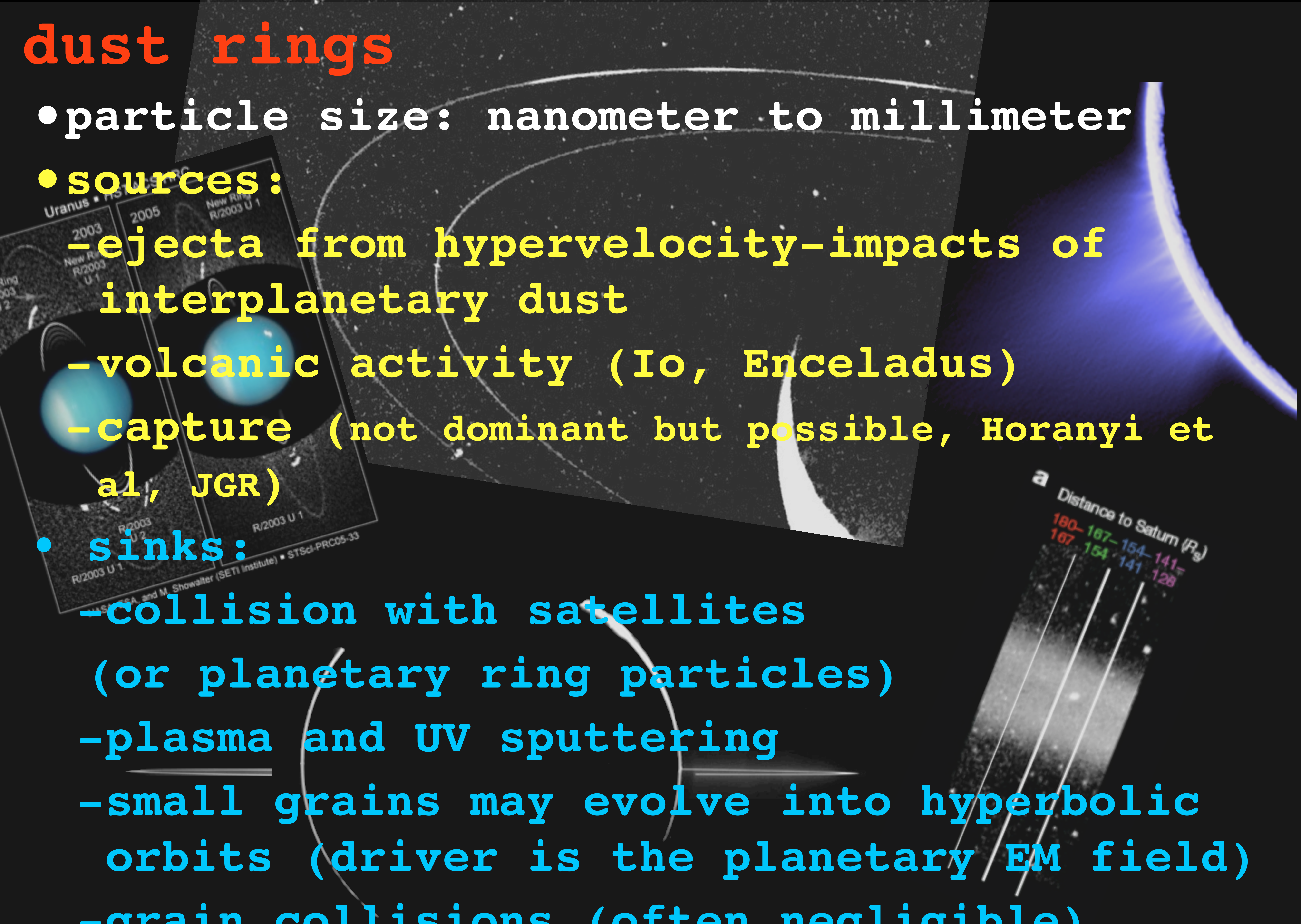
- particle size: nanometer to millimeter
- sources:
  - -ejecta from hypervelocity-impacts of interplanetary dust
  - -volcanic activity (Io, Enceladus)
  - -capture (not dominant but possible, Horanyi et al, JGR)

dust rings



- particle size: nanometer to millimeter
- sources:
  - -ejecta from hypervelocity-impacts of interplanetary dust
  - -volcanic activity (Io, Enceladus)
  - -capture (not dominant but possible, Horanyi et al, JGR)
- sinks:
  - -collision with satellites (or planetary ring particles)
  - -plasma and UV sputtering
  - -small grains may evolve into hyperbolic orbits (driver is the planetary EM field)

dust rings



- particle size: nanometer to millimeter
- sources:
  - -ejecta from hypervelocity-impacts of interplanetary dust
  - -volcanic activity (Io, Enceladus)
  - -capture (not dominant but possible, Horanyi et al, JGR)
- sinks:
  - -collision with satellites (or planetary ring particles)
  - -plasma and UV sputtering
  - -small grains may evolve into hyperbolic orbits (driver is the planetary EM field)
  - -grain collisions (often negligible)

non-gravitational forces

acceleration by planetary (electro-)magnetic fields:

$$\dot{\vec{v}} \propto \frac{q}{m} \propto \frac{1}{r^2} \qquad \left(\Phi_{equ} \propto \frac{q}{r}\right)$$

grain charging:
solar UV, plasma currents,
secondary electron emission

non-gravitational forces

acceleration by planetary (electro-)magnetic fields:

$$\dot{\vec{v}} \propto \frac{q}{m} \propto \frac{1}{r^2} \qquad \left(\Phi_{equ} \propto \frac{q}{r}\right)$$

grain charging:
solar UV, plasma currents,
secondary electron emission

acceleration by solar radiation:

$$\dot{\vec{v}} \propto \frac{\sigma}{m} \propto \frac{1}{r}$$

direct radiation pressure and Poynting-Robertson drag

non-gravitational forces

acceleration by planetary (electro-)magnetic fields:

$$\dot{\vec{v}} \propto \frac{q}{m} \propto \frac{1}{r^2} \qquad \left(\Phi_{equ} \propto \frac{q}{r}\right)$$

grain charging:
solar UV, plasma
currents,
secondary electron
emission

acceleration by solar radiation:

$$\dot{\vec{v}} \propto \frac{\sigma}{m} \propto \frac{1}{r}$$

direct radiation pressure and
Poynting-Robertson drag

drag exerted by planetary plasma

direct drag force and
coulomb drag

further perturbation forces

- higher gravity moments of the planet
- gravity of satellites
- solar gravity

further perturbation forces

- higher gravity moments of the planet
- gravity of satellites
- solar gravity

perturbation forces depend differently on
-> grain size
-> planetary distance
-> solar distance
-> magnetospheric conditions
and may vary stochastically
(e.g. Schaffer & Burns, 1987)

further perturbation forces

- higher gravity moments of the planet
- gravity of satellites
- solar gravity

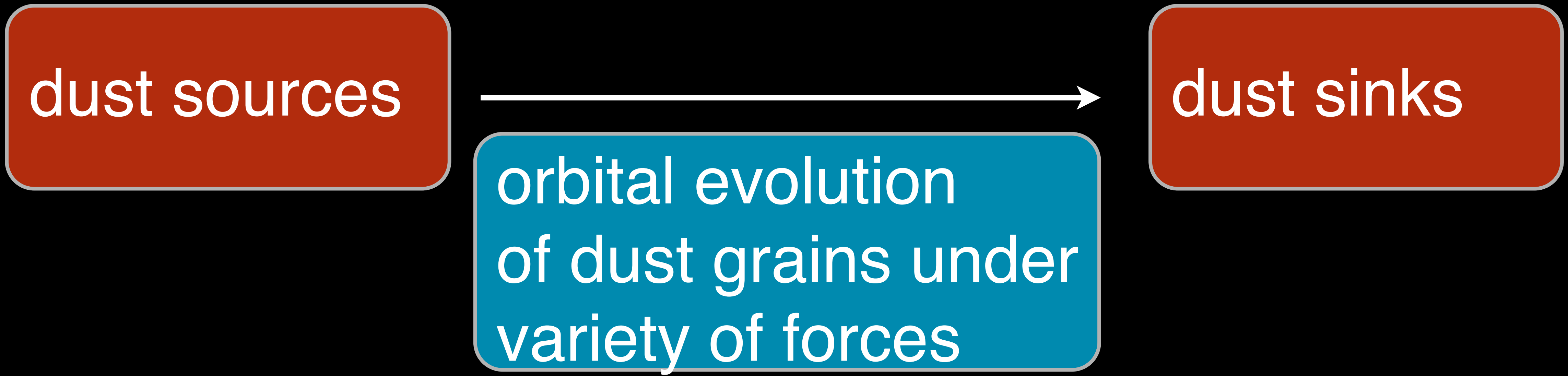
perturbation forces depend differently on
-> grain size
-> planetary distance
-> solar distance
-> magnetospheric conditions
and may vary stochastically
(e.g. Schaffer & Burns, 1987)

=> rich dynamics

circumplanetary dust dynamics

dust sources → dust sinks

circumplanetary dust dynamics

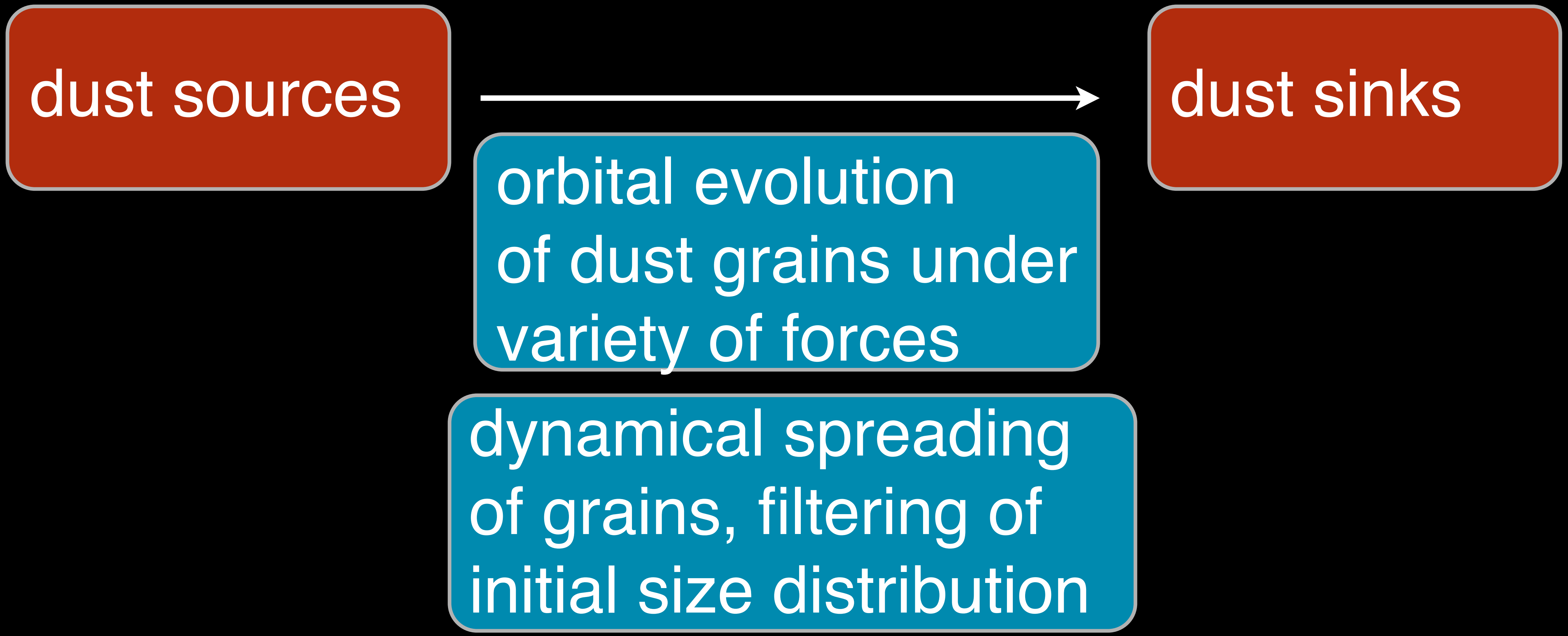


dust sources

dust sinks

orbital evolution of dust grains under variety of forces

circumplanetary dust dynamics



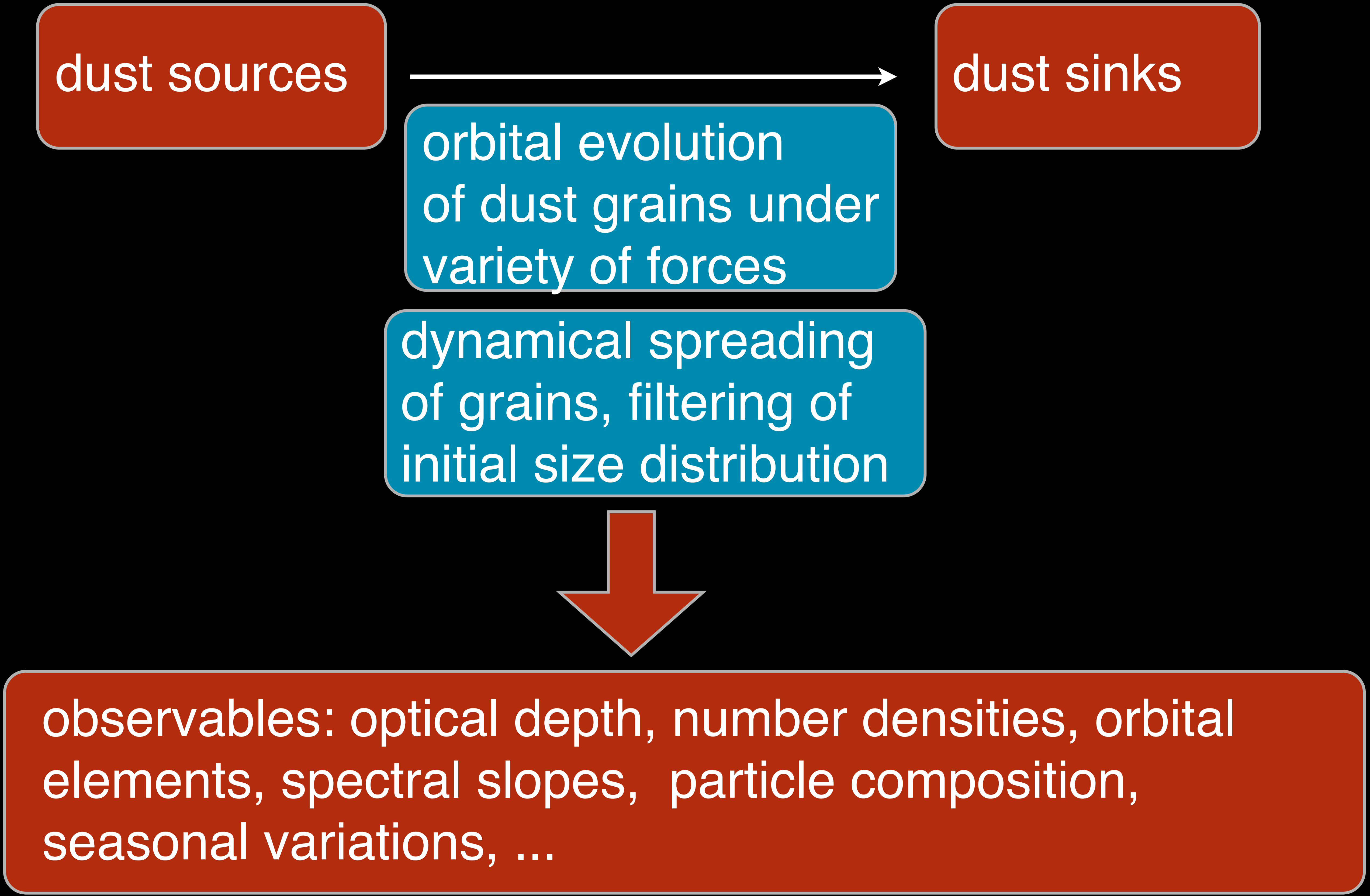
dust sources

dust sinks

orbital evolution of dust grains under variety of forces

dynamical spreading of grains, filtering of initial size distribution

circumplanetary dust dynamics



dust sources

dust sinks

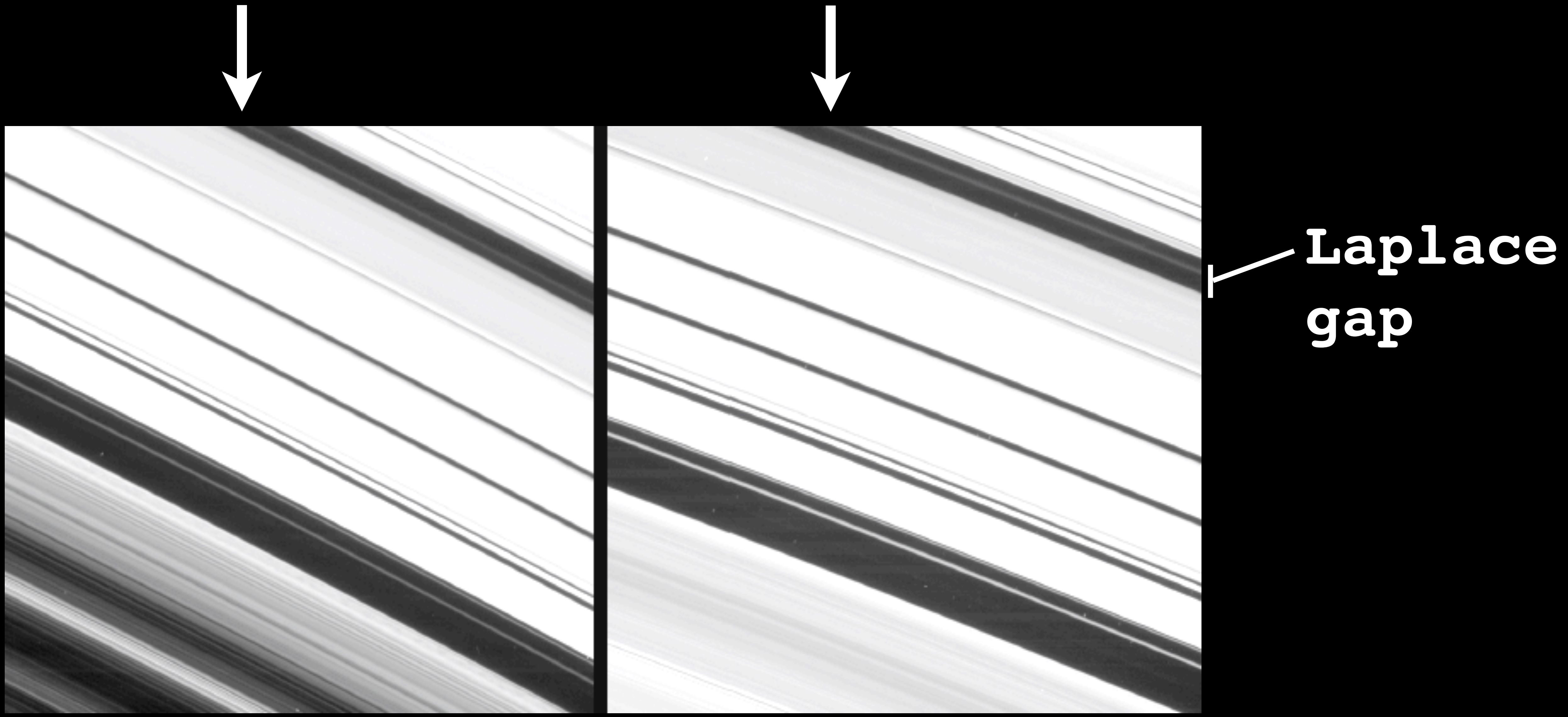
orbital evolution of dust grains under variety of forces

dynamical spreading of grains, filtering of initial size distribution

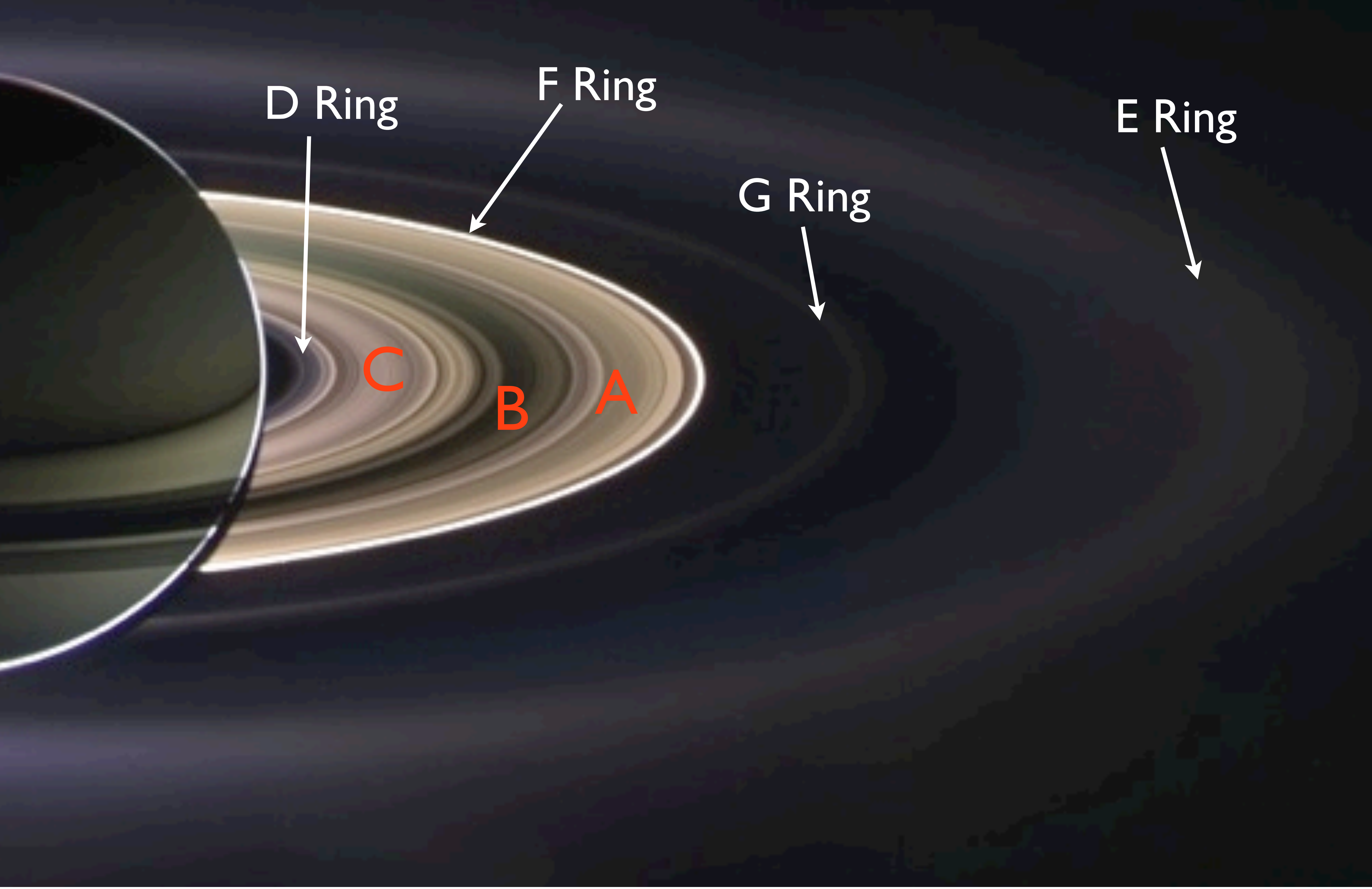
observables: optical depth, number densities, orbital elements, spectral slopes, particle composition, seasonal variations, ...

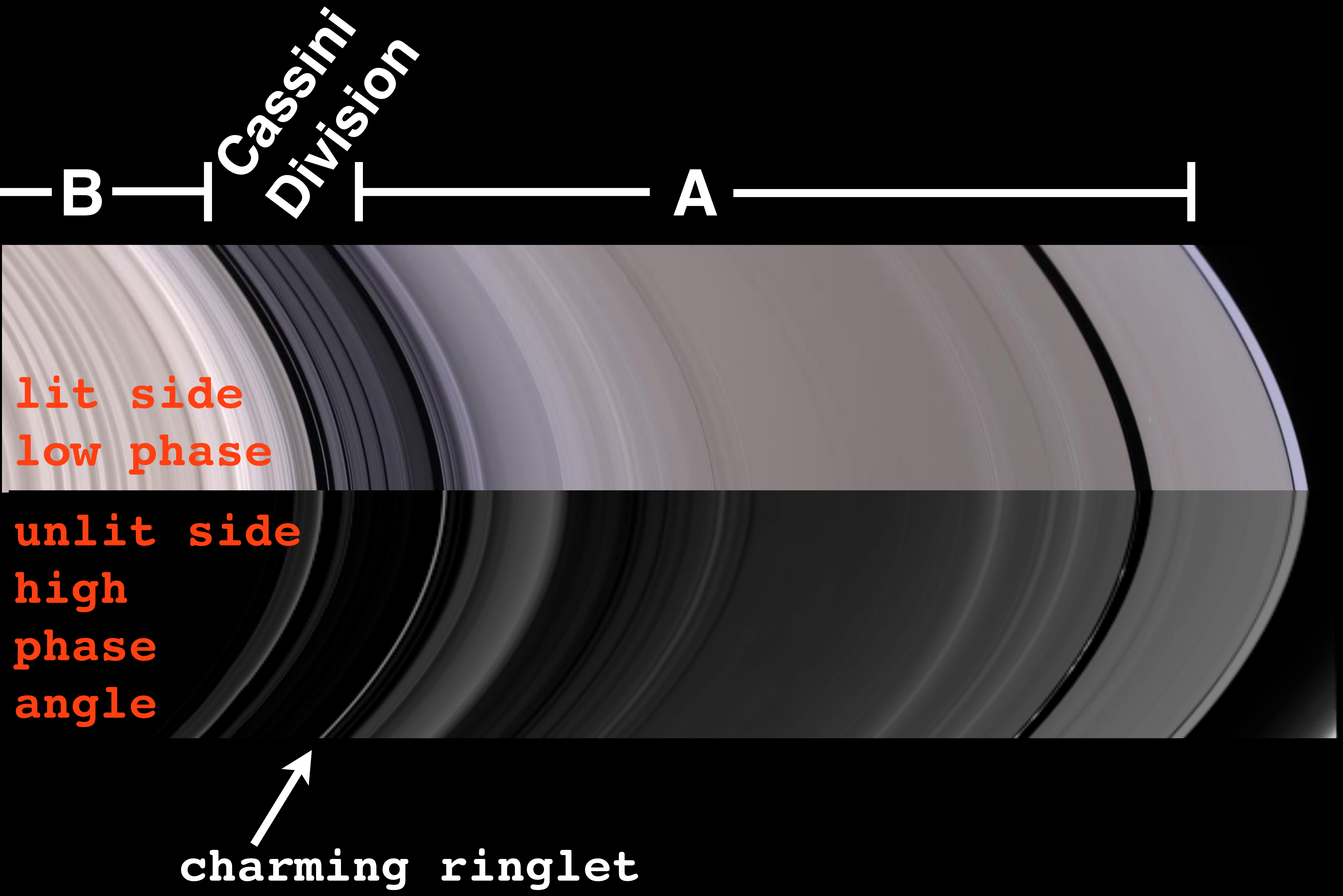
example

Saturn’s charming ringlet is perturbed by sunlight:
on the anti-sun side the ringlet
is always found closer to the planet
(Hedman et al., 2010)

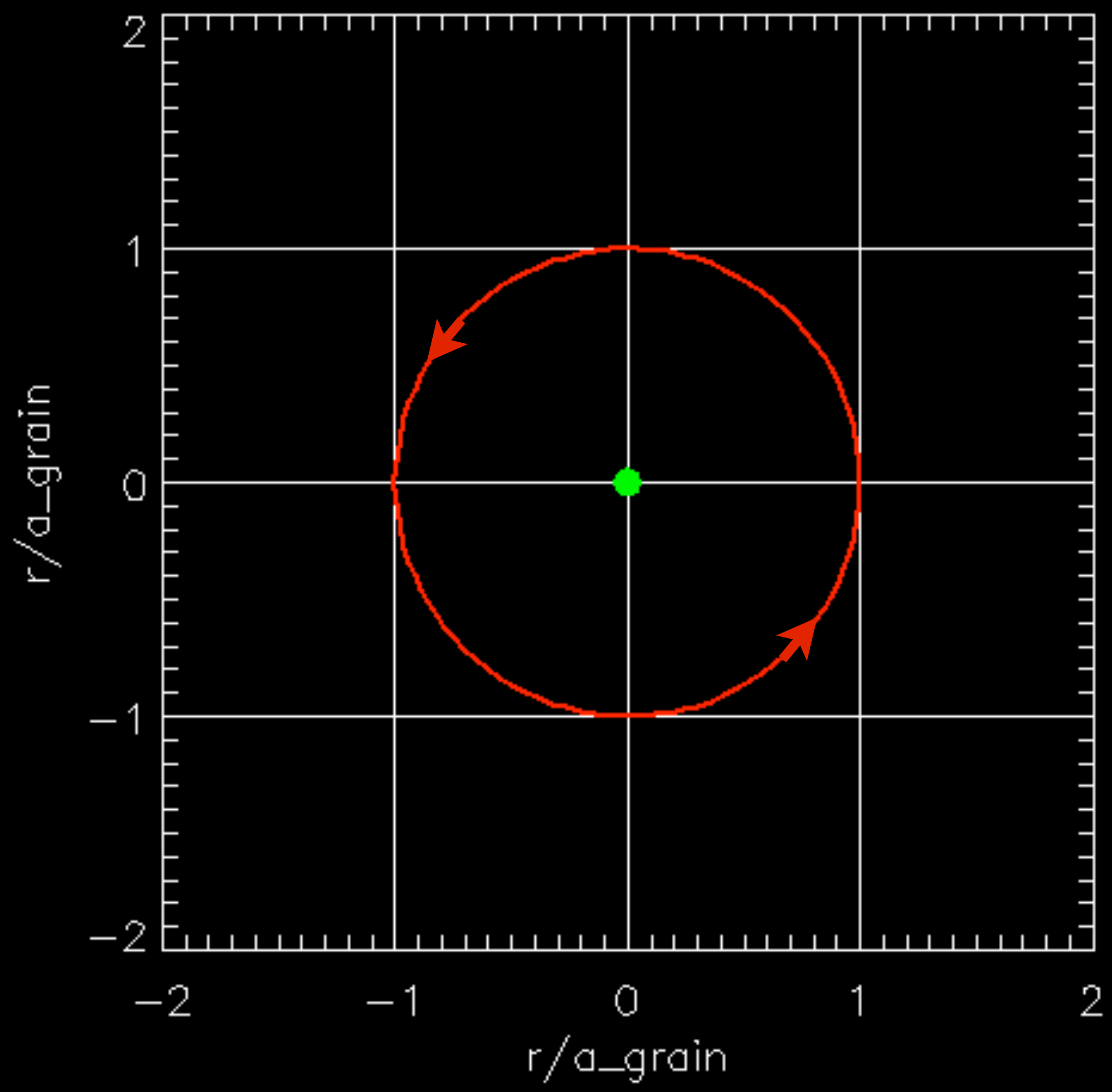


dust becomes visible at high phase angles (sun - object - observer)

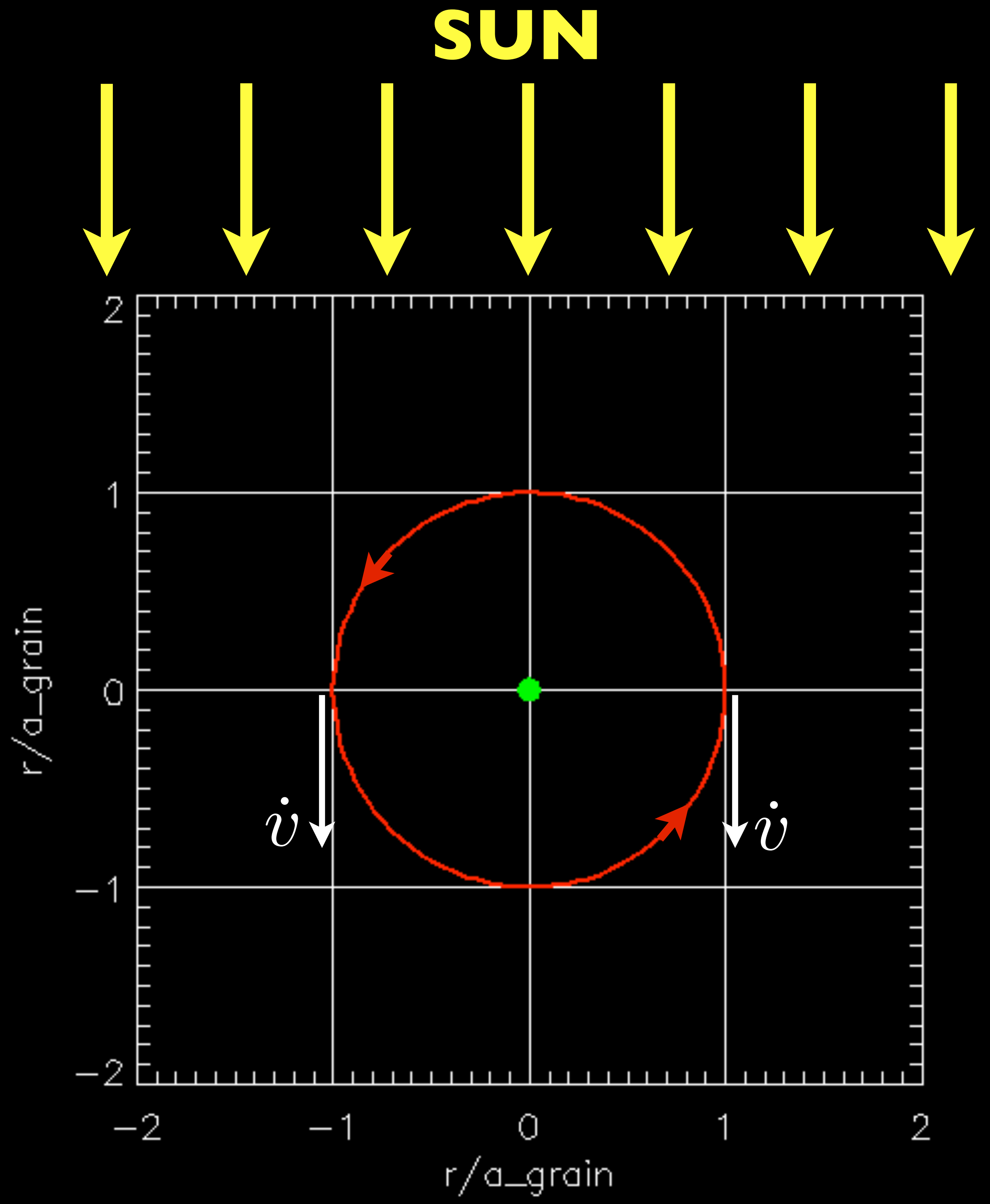



B
Cassini Division
A
lit side
low phase
unlit side
high
phase
angle
charming ringlet

consider dust grain on circular orbit



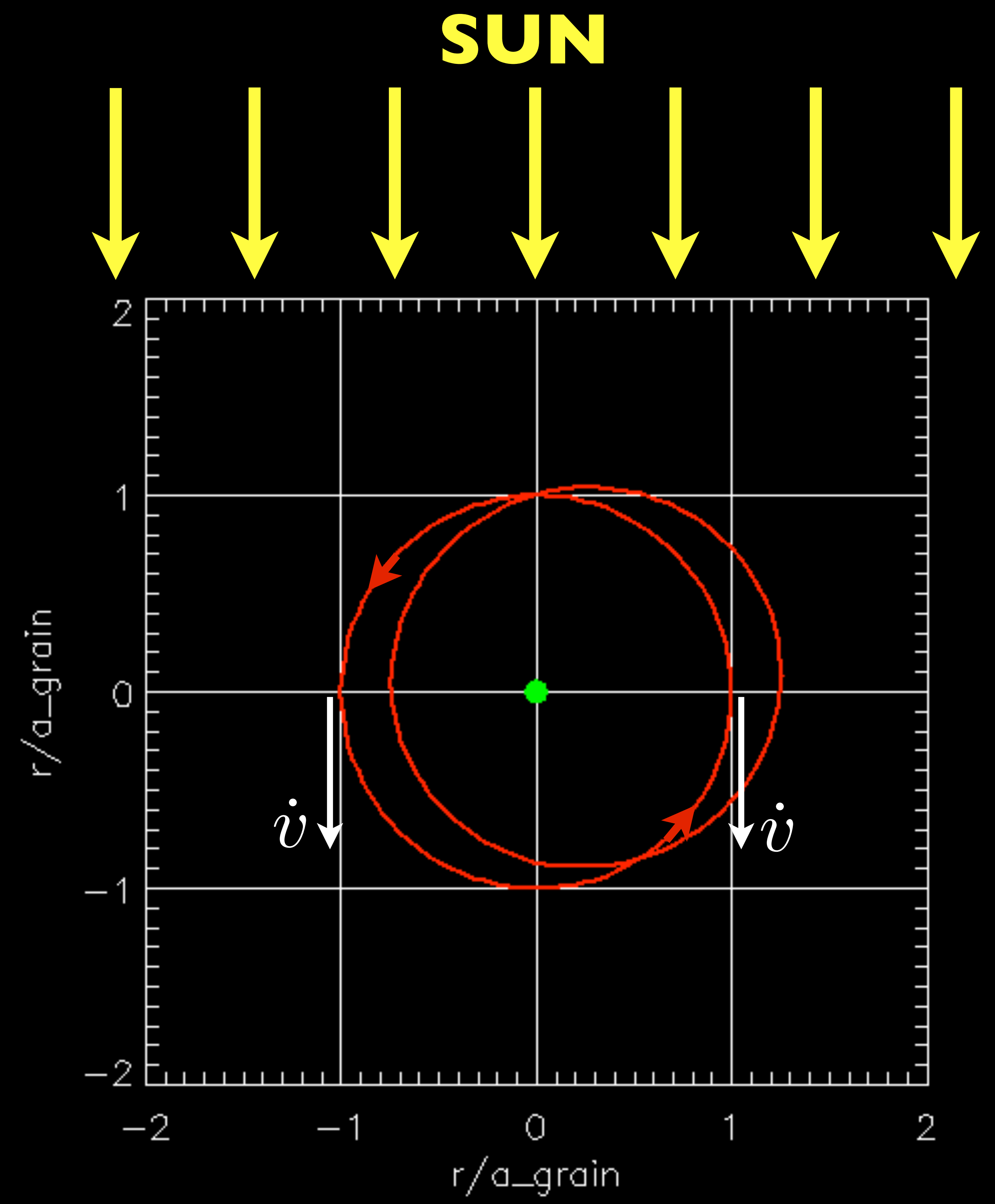
consider dust grain on circular orbit



(Horanyi et al, 1992, Hedman et al., 2010)

consider dust grain on circular orbit

-> radiation pressure
   induces eccentricity

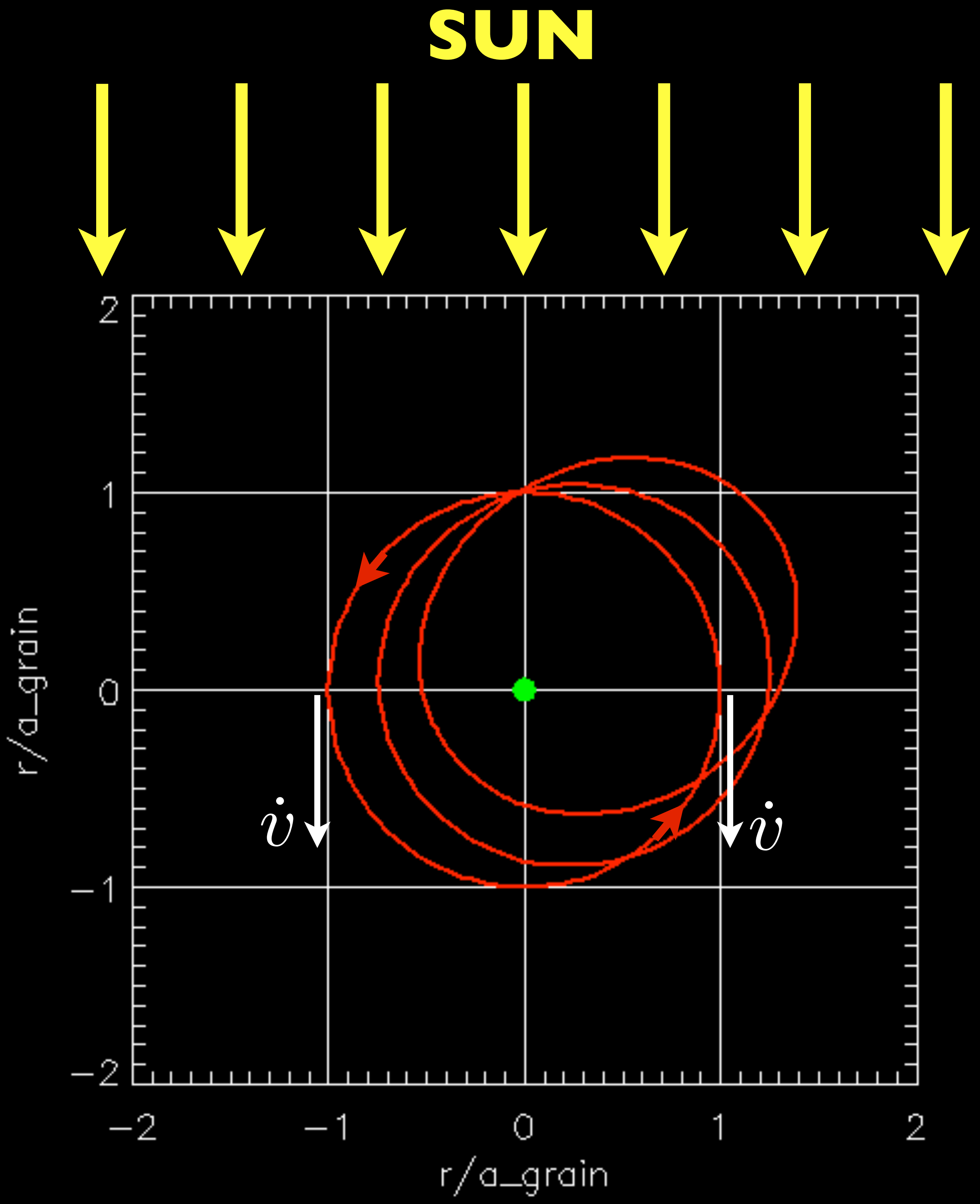


(Horanyi et al, 1992,
Hedman et al., 2010)

consider dust grain on circular orbit

-> radiation pressure induces eccentricity

-> planetary oblateness: advance of pericenter

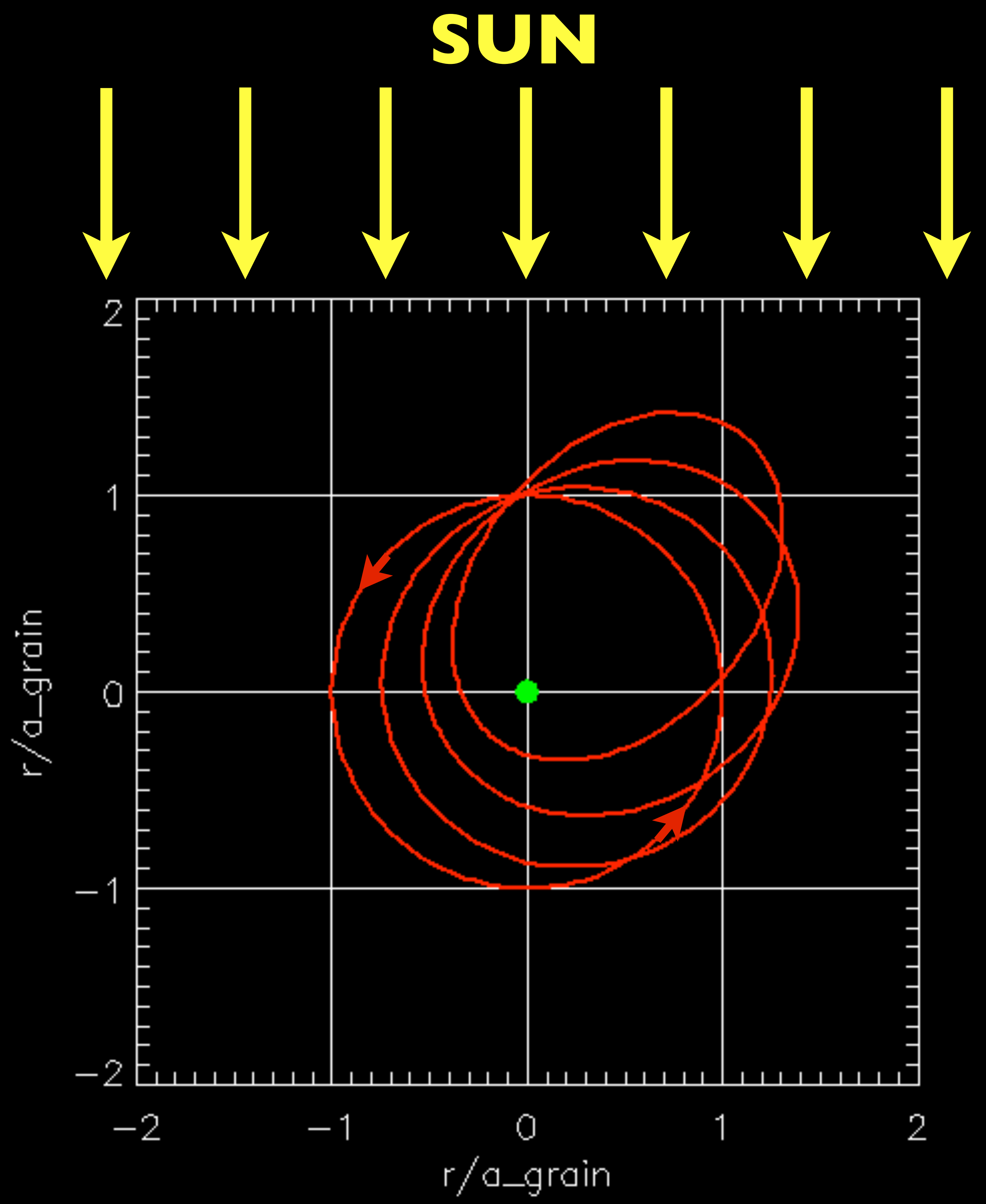


(Horanyi et al, 1992, Hedman et al., 2010)

consider dust grain on circular orbit

-> radiation pressure induces eccentricity

-> planetary oblateness: advance of pericenter

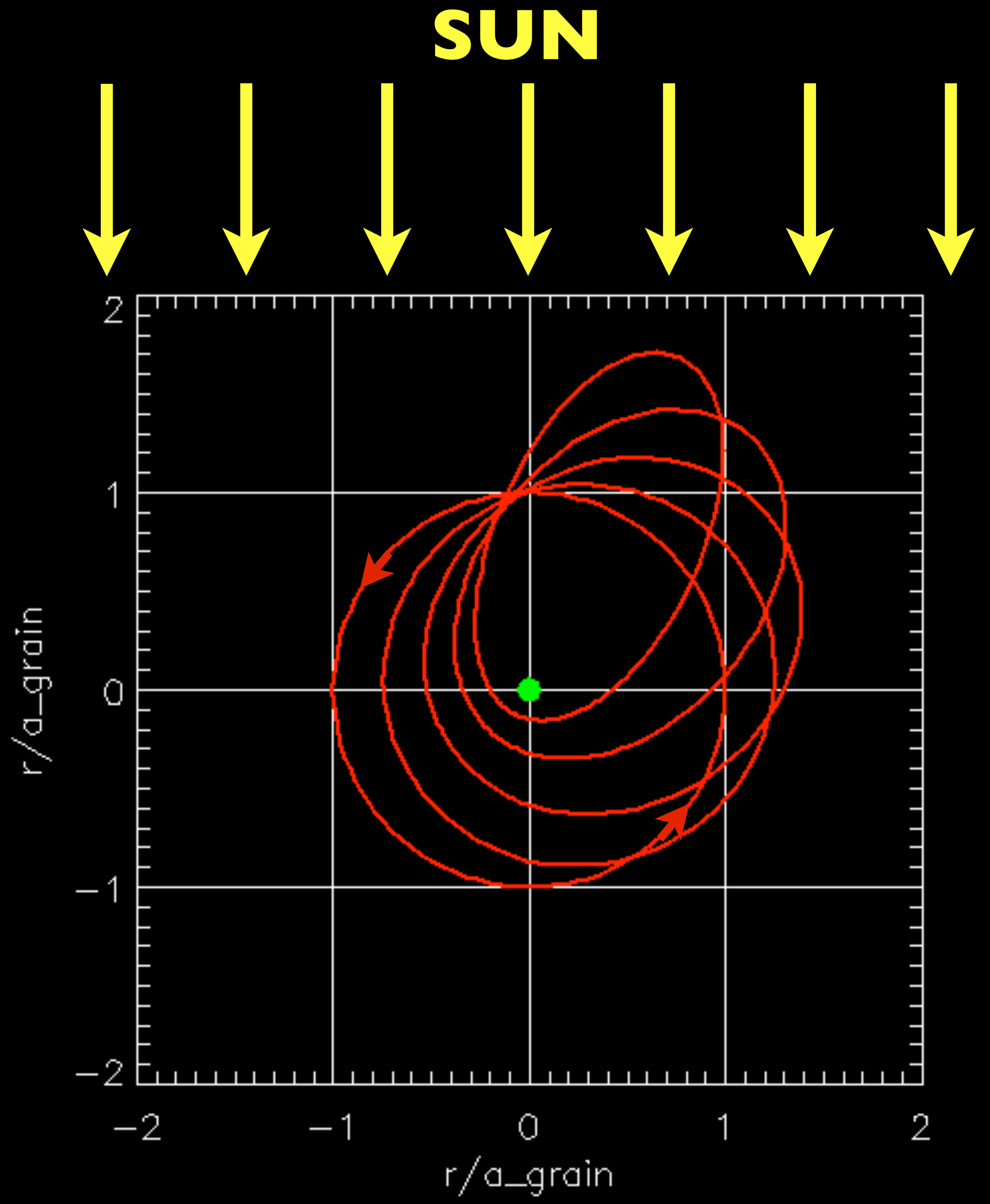


(Horanyi et al, 1992, Hedman et al., 2010)

consider dust grain on circular orbit

-> radiation pressure induces eccentricity

-> planetary oblateness: advance of pericenter

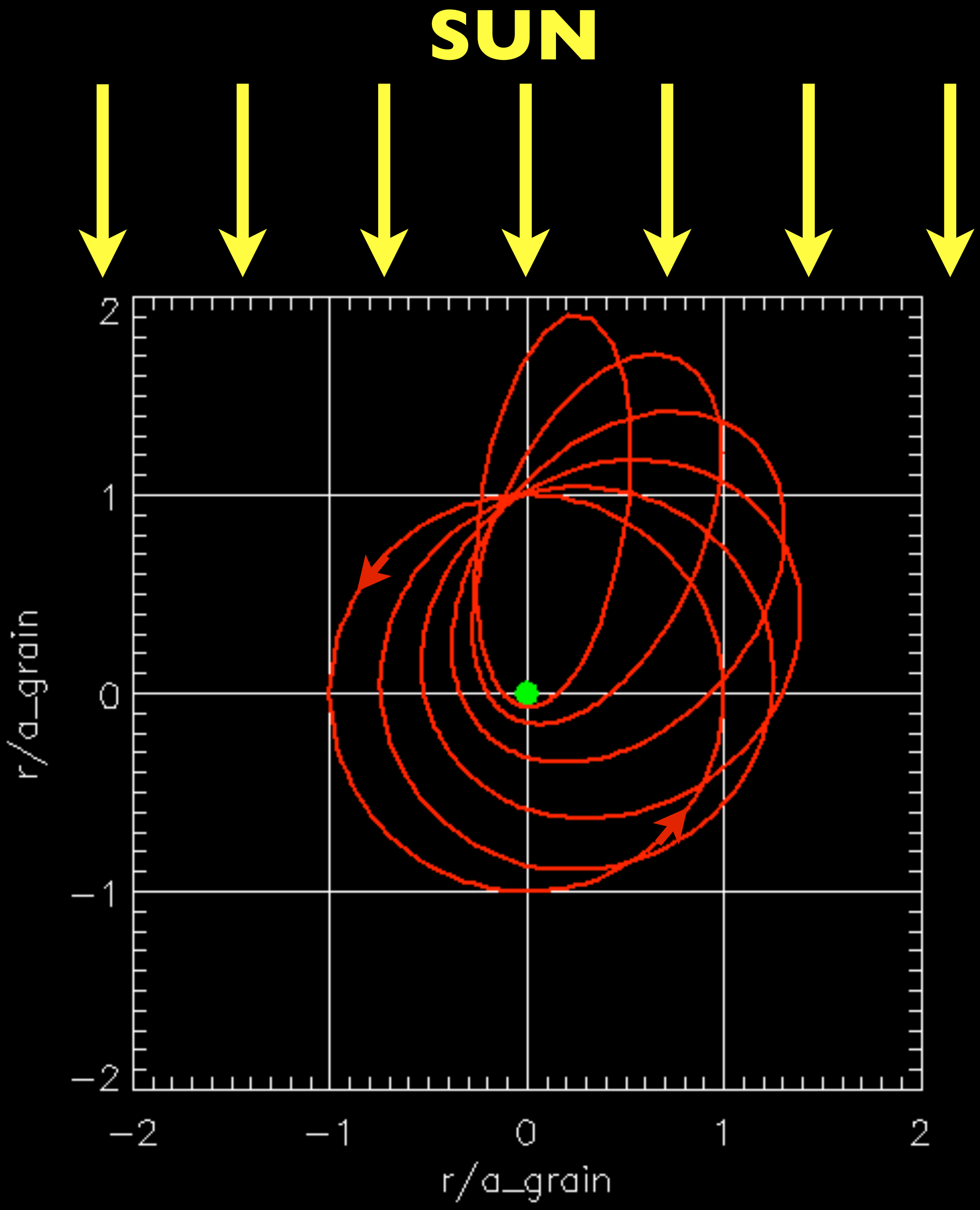


(Horanyi et al, 1992, Hedman et al., 2010)

consider dust grain on circular orbit

-> radiation pressure induces eccentricity

-> planetary oblateness: advance of pericenter



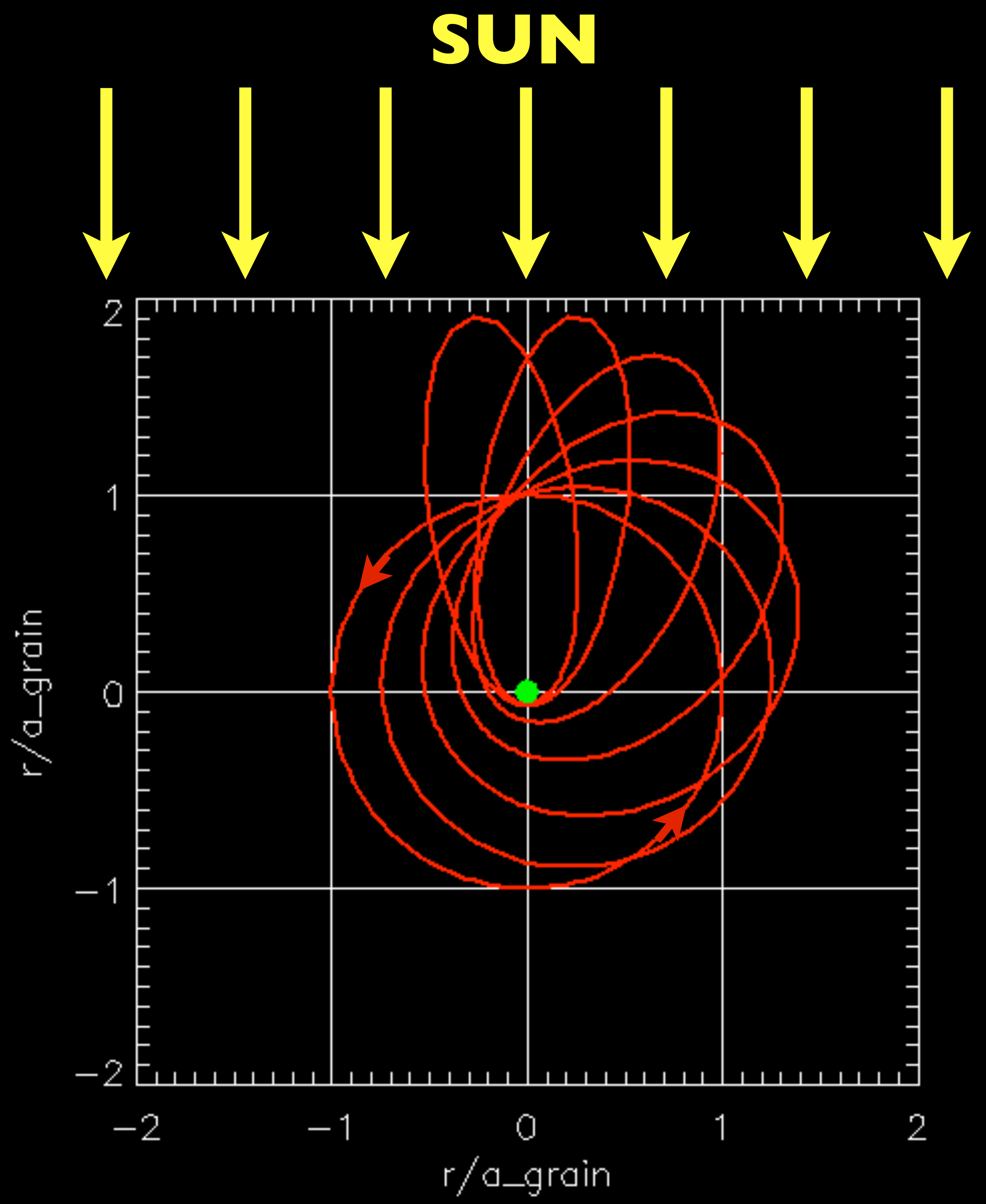
(Horanyi et al, 1992, Hedman et al., 2010)

consider dust grain on circular orbit

-> radiation pressure induces eccentricity

-> planetary oblateness: advance of pericenter

-> eccentricity begins to shrink after apocenter has passed the solar direction



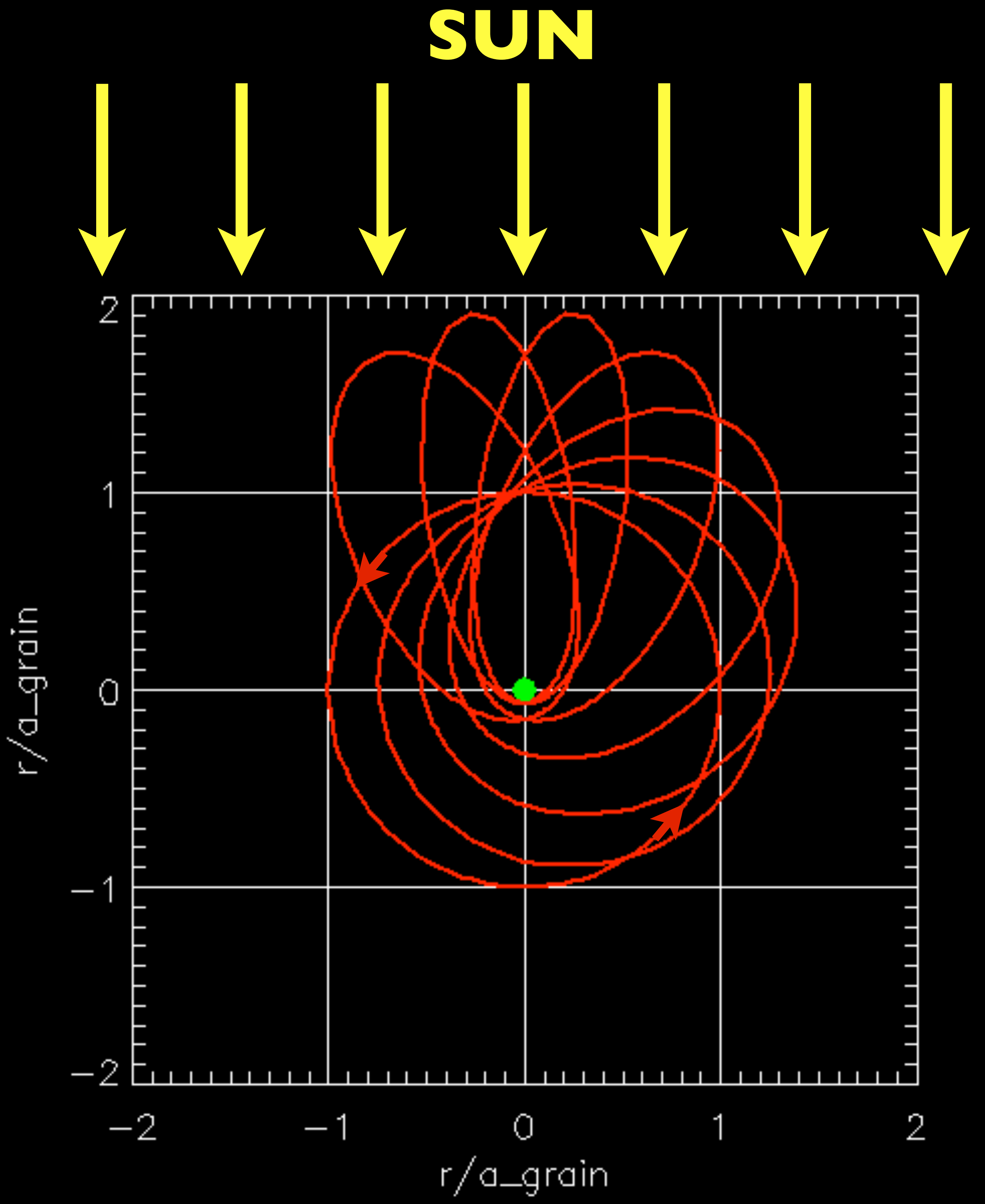
(Horanyi et al, 1992, Hedman et al., 2010)

consider dust grain on circular orbit

-> radiation pressure induces eccentricity

-> planetary oblateness: advance of pericenter

-> eccentricity begins to shrink after apocenter has passed the solar direction



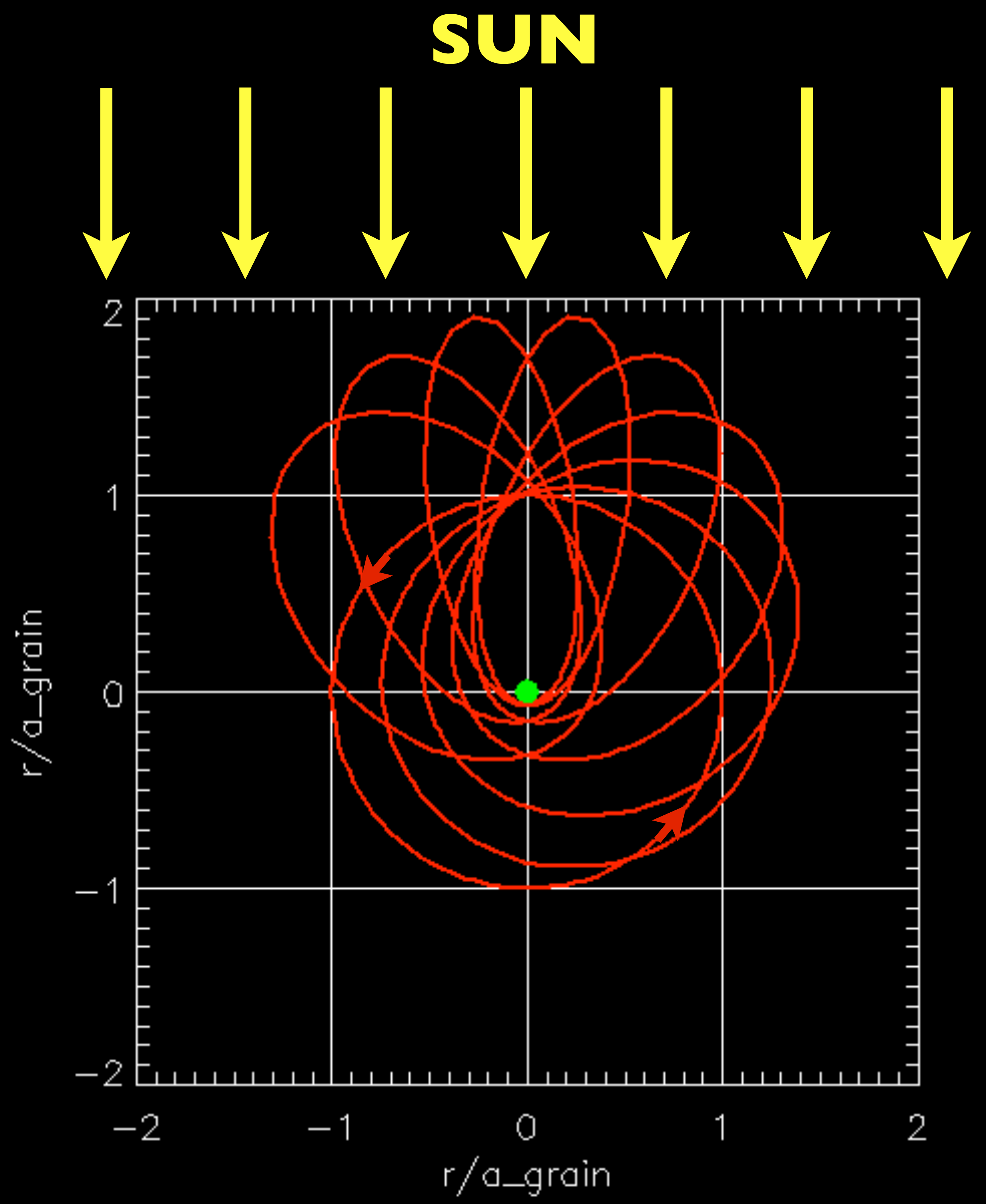
(Horanyi et al, 1992, Hedman et al., 2010)

consider dust grain on circular orbit

-> radiation pressure induces eccentricity

-> planetary oblateness: advance of pericenter

-> eccentricity begins to shrink after apocenter has passed the solar direction



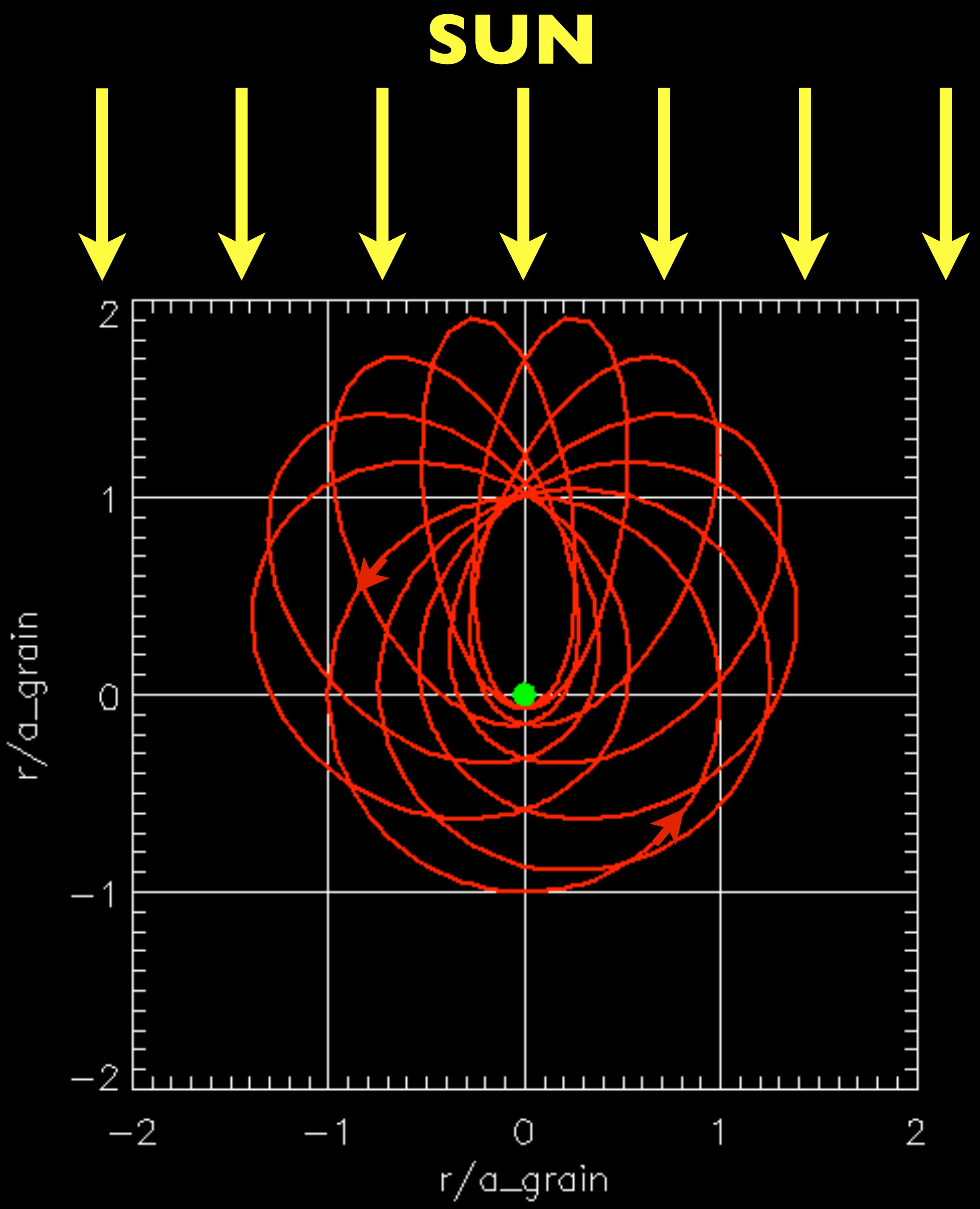
(Horanyi et al, 1992, Hedman et al., 2010)

consider dust grain on circular orbit

-> radiation pressure induces eccentricity

-> planetary oblateness: advance of pericenter

-> eccentricity begins to shrink after apocenter has passed the solar direction



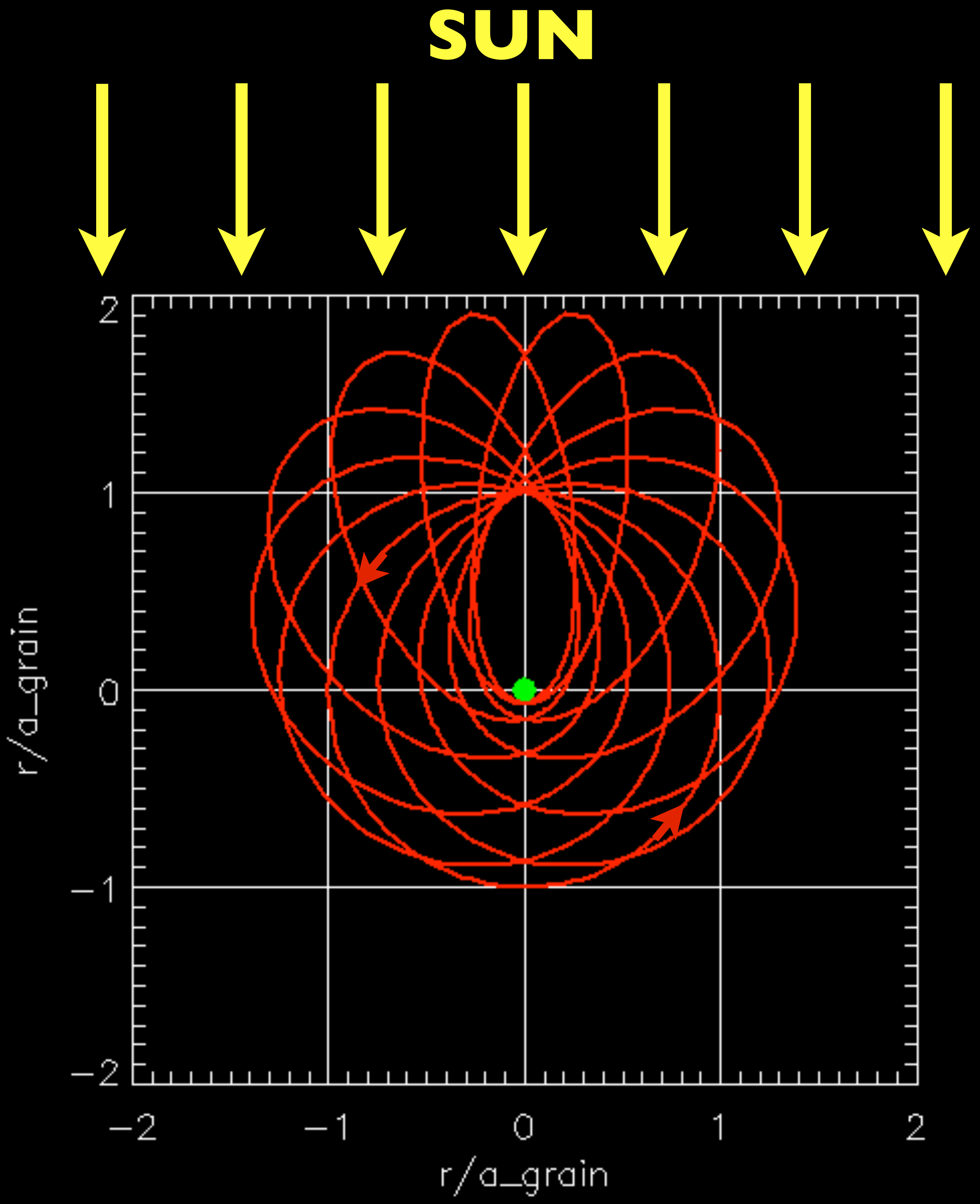
(Horanyi et al, 1992, Hedman et al., 2010)

consider dust grain on circular orbit

-> radiation pressure induces eccentricity

-> planetary oblateness: advance of pericenter

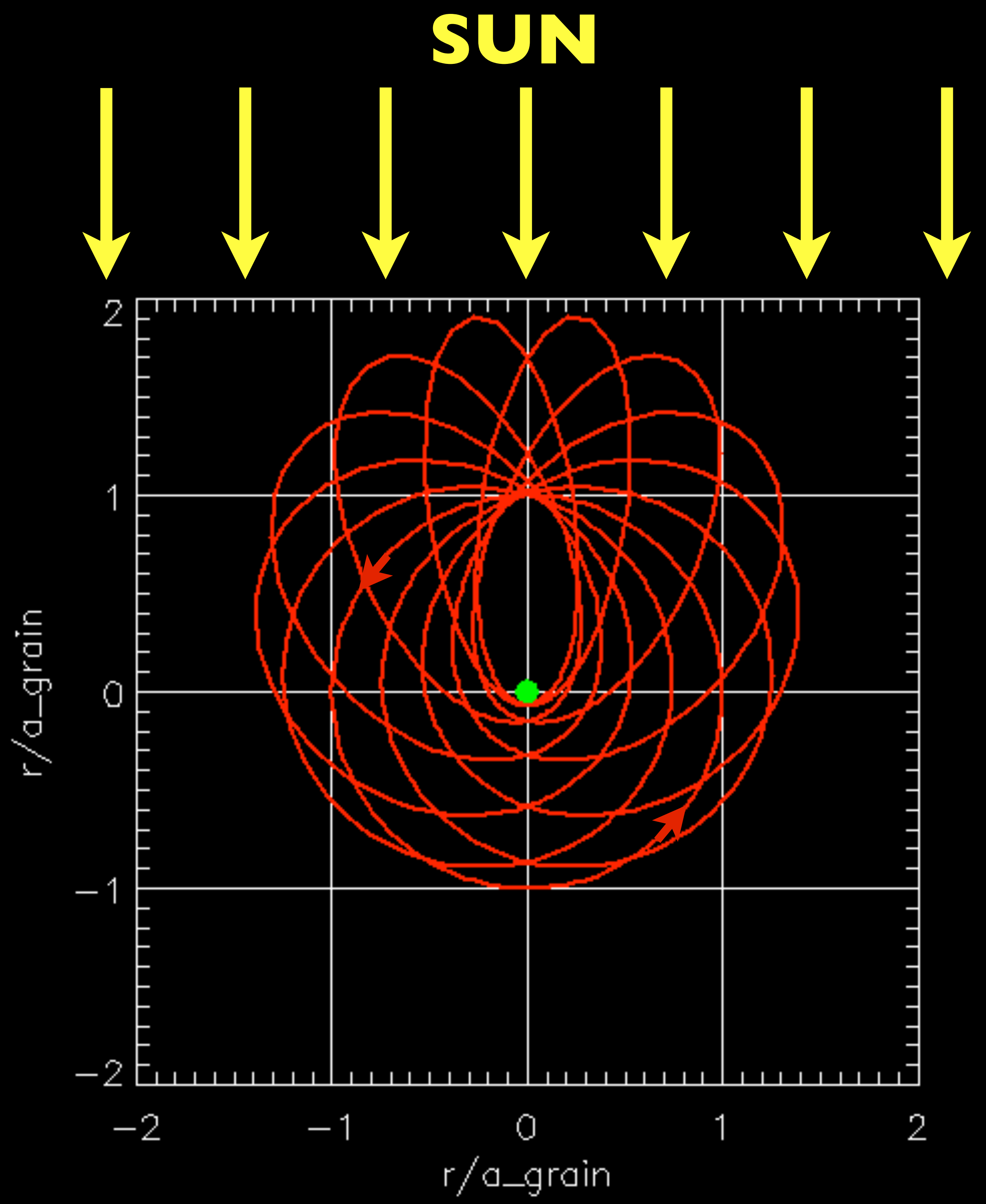
-> eccentricity begins to shrink after apocenter has passed the solar direction



(Horanyi et al, 1992, Hedman et al., 2010)

consider dust grain on circular orbit

- -> radiation pressure induces eccentricity
- -> planetary oblateness: advance of pericenter
- -> eccentricity begins to shrink after apocenter has passed the solar direction



(Horanyi et al, 1992, Hedman et al., 2010)

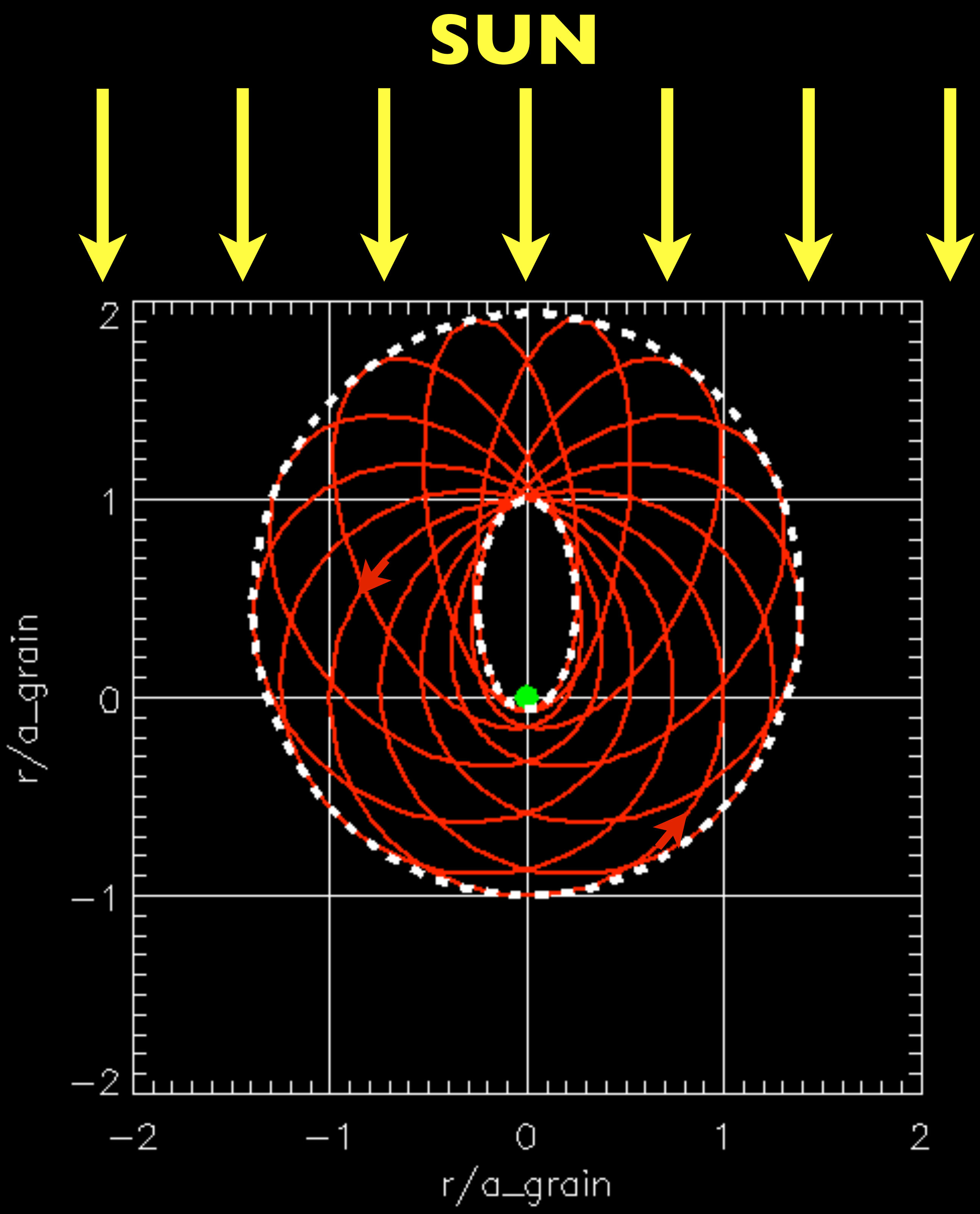
consider dust grain on circular orbit

-> radiation pressure induces eccentricity

-> planetary oblateness: advance of pericenter

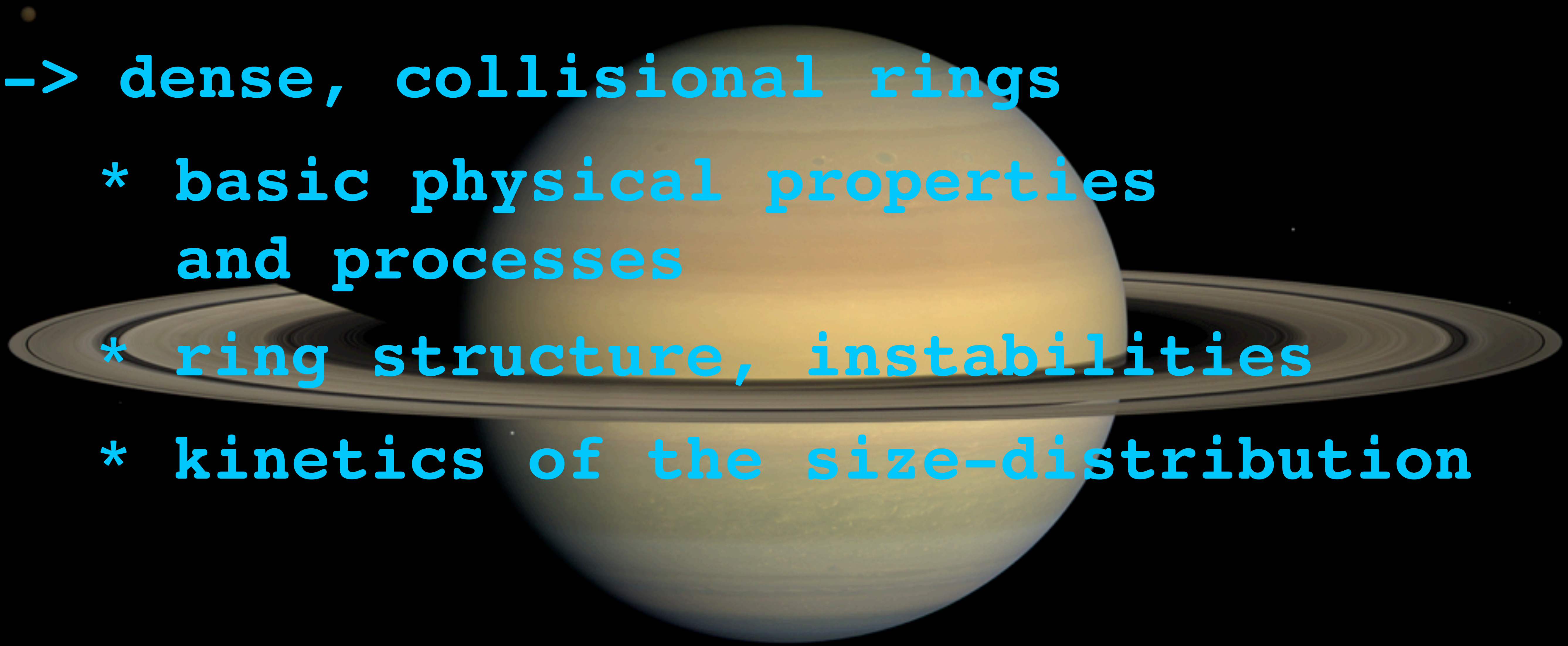
-> eccentricity begins to shrink after apocenter has passed the solar direction

-> fixed envelope points towards the sun "heliotropic" ring



(Horanyi et al, 1992, Hedman et al., 2010)

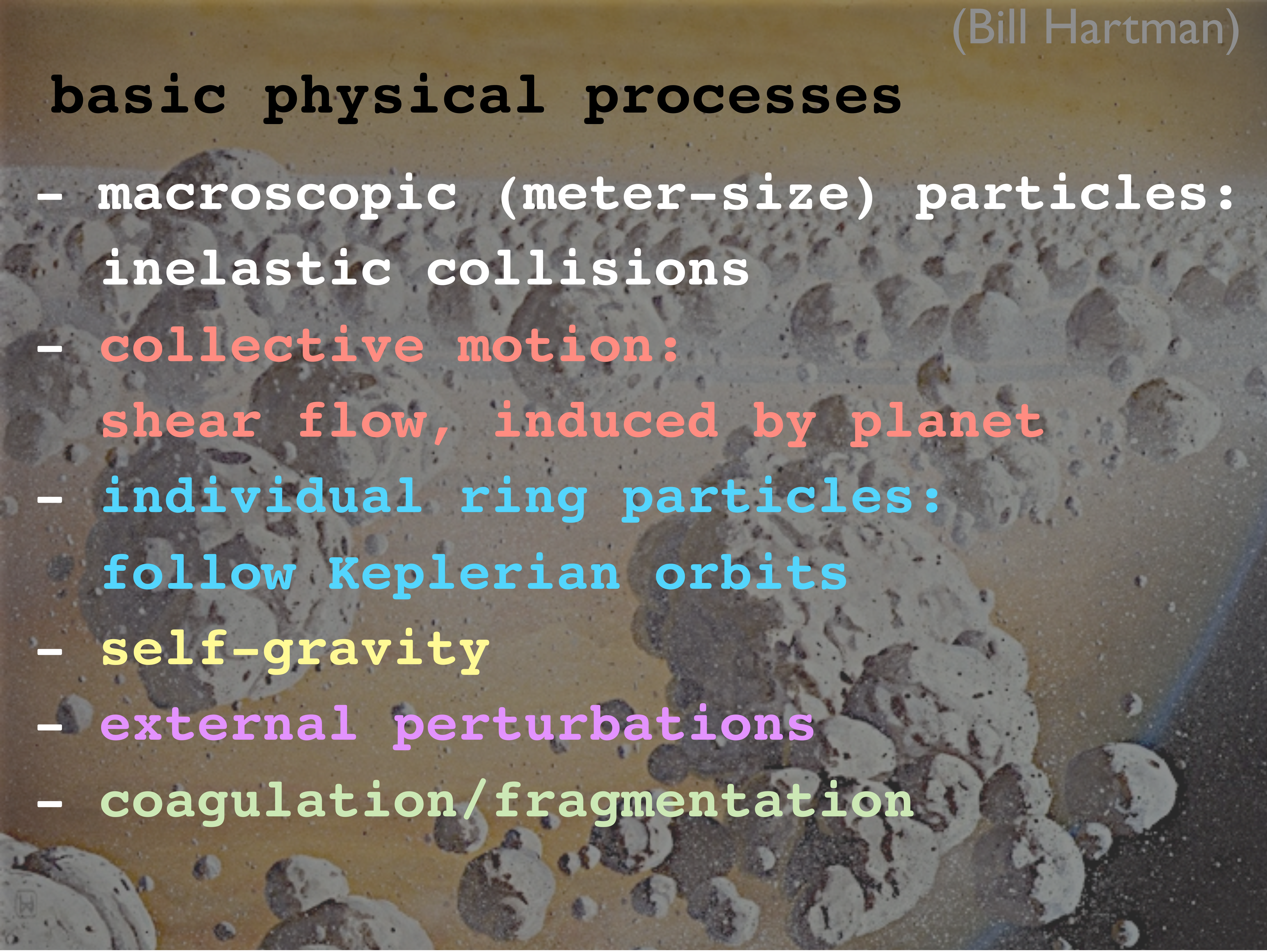
dense
collisional rings



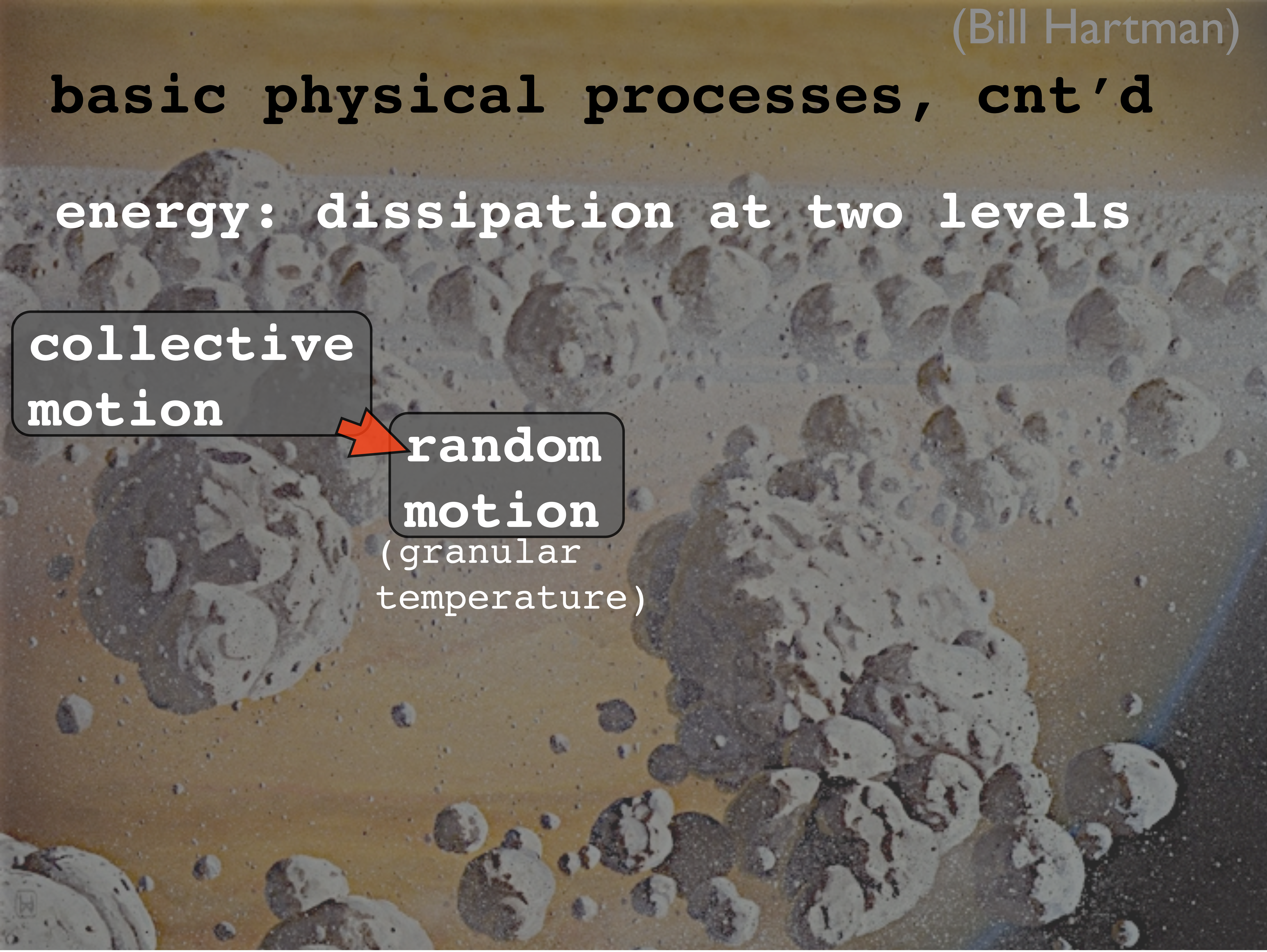
-> dense, collisional rings

- basic physical properties and processes
- ring structure, instabilities
- kinetics of the size-distribution

basic physical processes


(Bill Hartman)

- macroscopic (meter-size) particles: inelastic collisions
- collective motion: shear flow, induced by planet
- individual ring particles: follow Keplerian orbits
- self-gravity
- external perturbations
- coagulation/fragmentation



(Bill Hartman)

basic physical processes, cnt’d

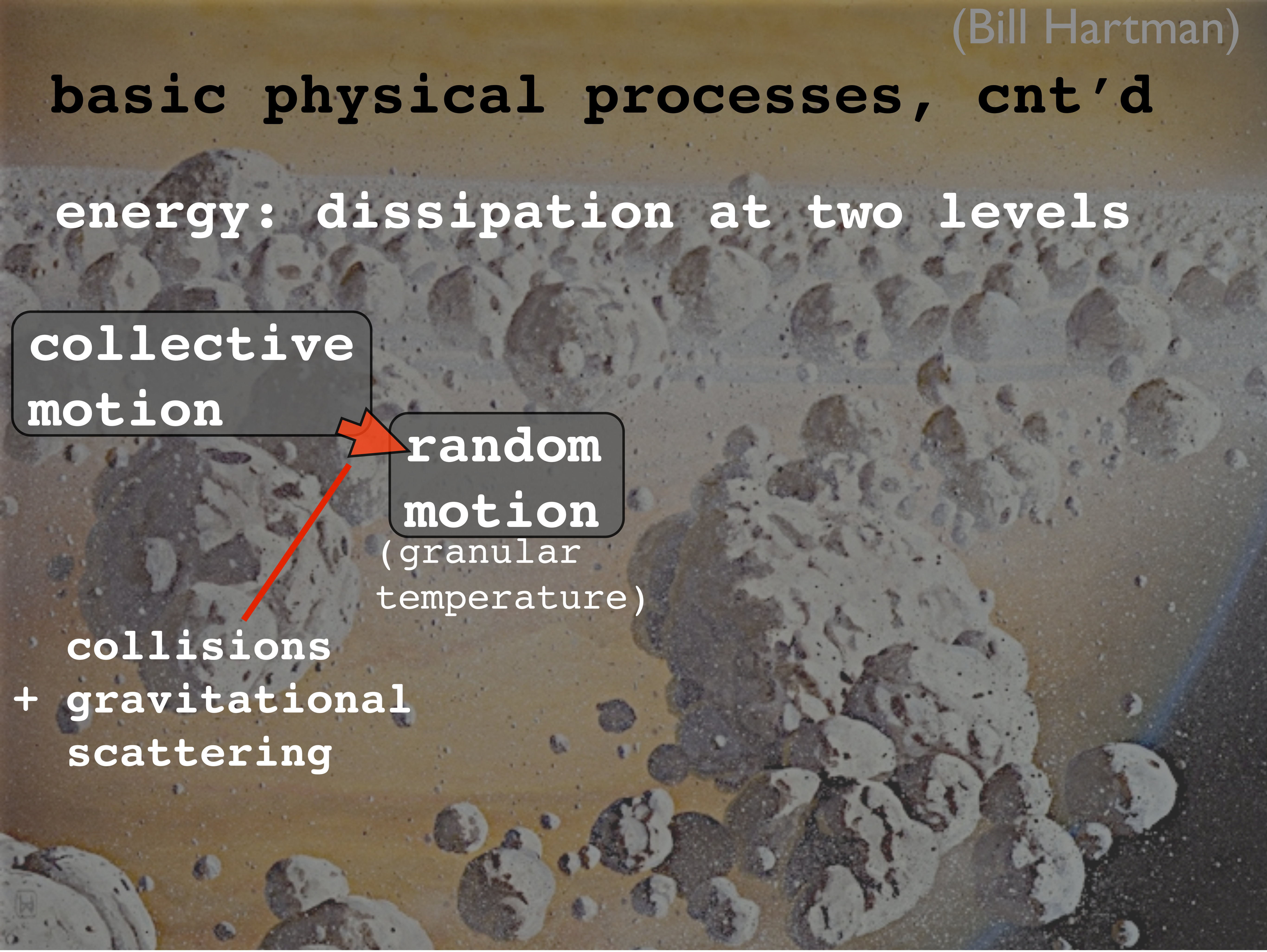
energy: dissipation at two levels

collective motion

random motion

(granular temperature)

basic physical processes, cnt'd


(Bill Hartman)

energy: dissipation at two levels

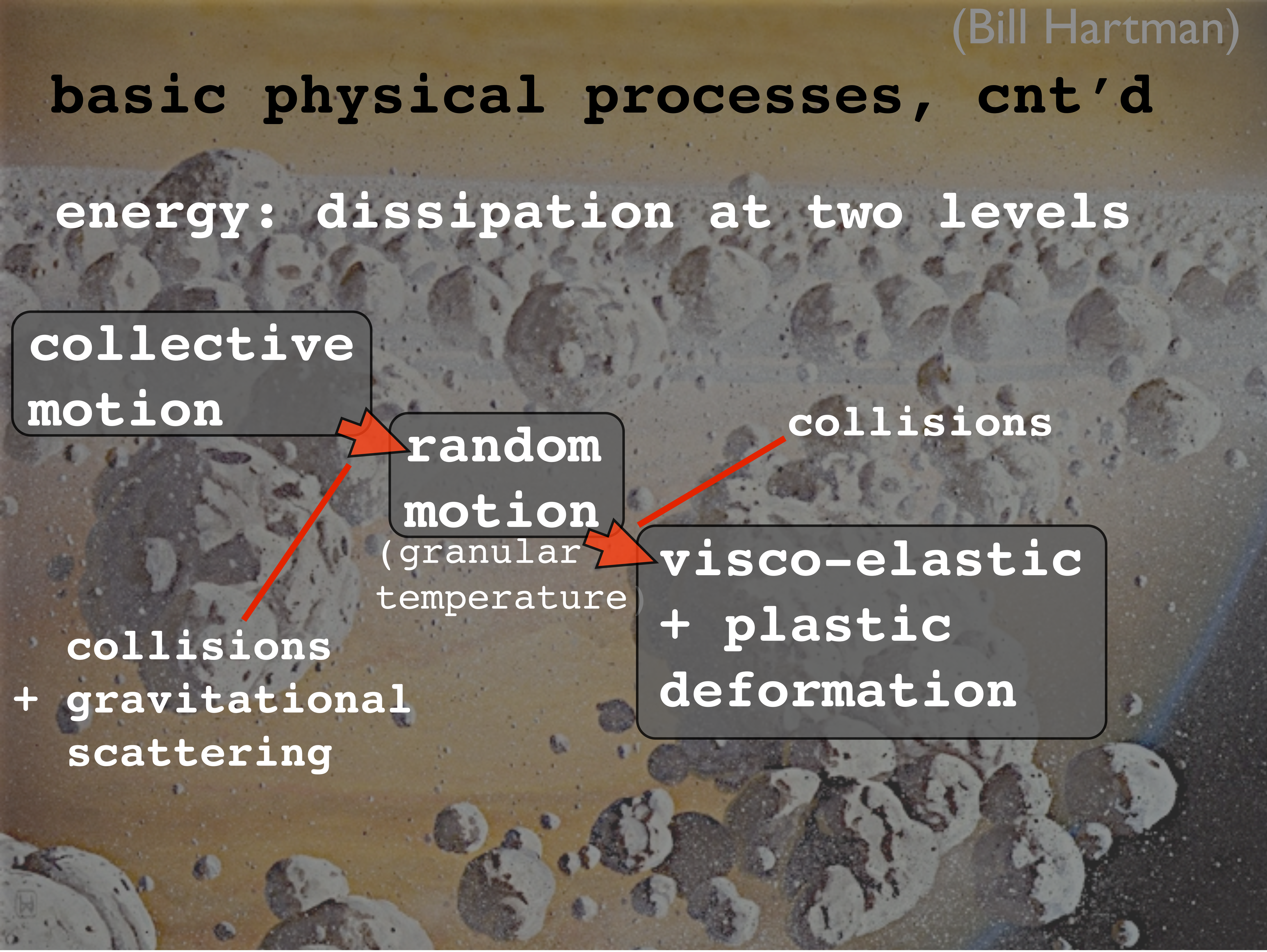
collective motion

random motion

(granular temperature)

collisions
+ gravitational scattering

basic physical processes, cnt'd



(Bill Hartman)

energy: dissipation at two levels

collective motion

random motion

(granular temperature)

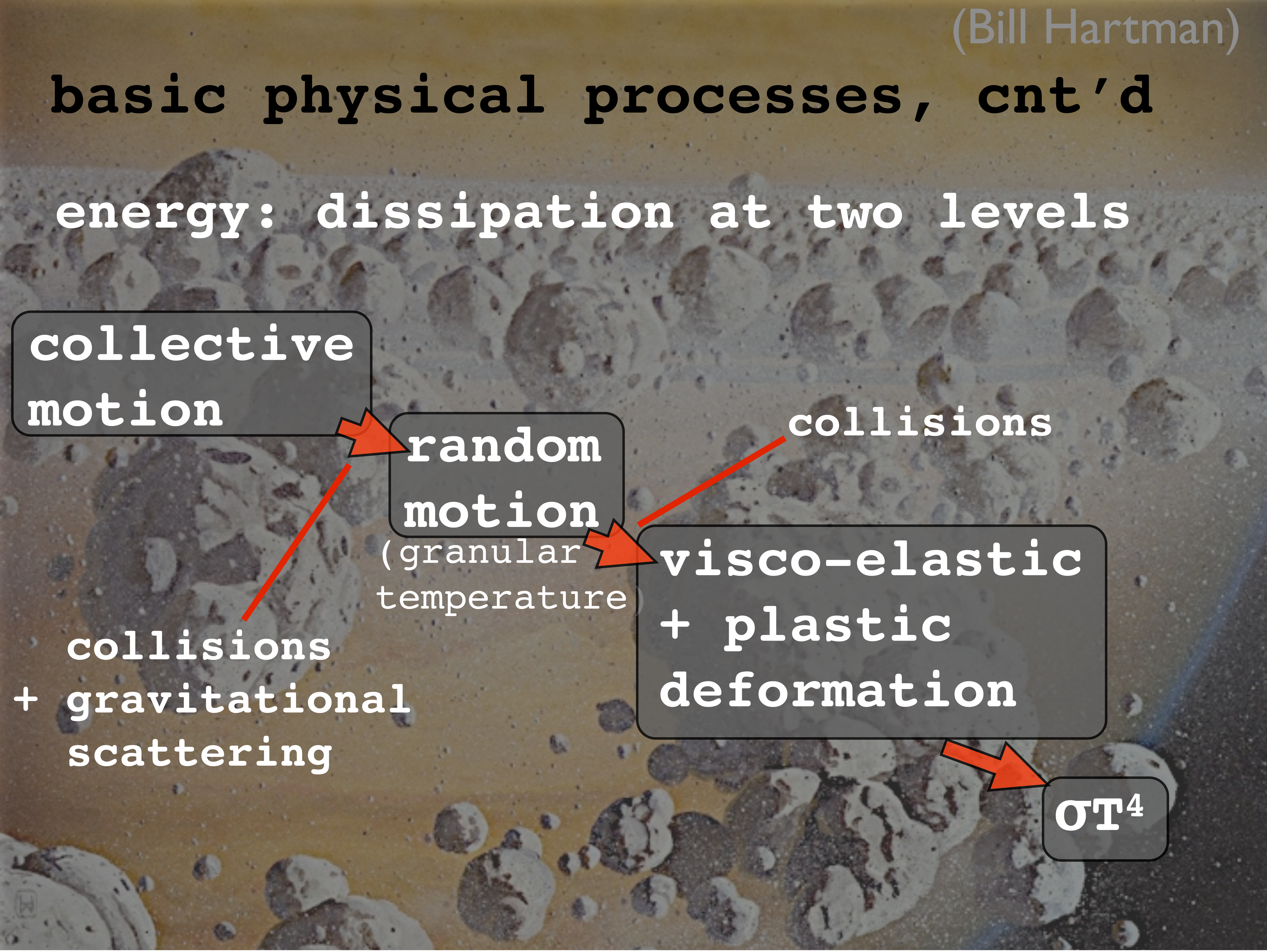
visco-elastic + plastic deformation

collisions

collisions + gravitational scattering

(Bill Hartman)

basic physical processes, cnt'd



energy: dissipation at two levels

collective motion

collisions + gravitational scattering

random motion (granular temperature)

collisions

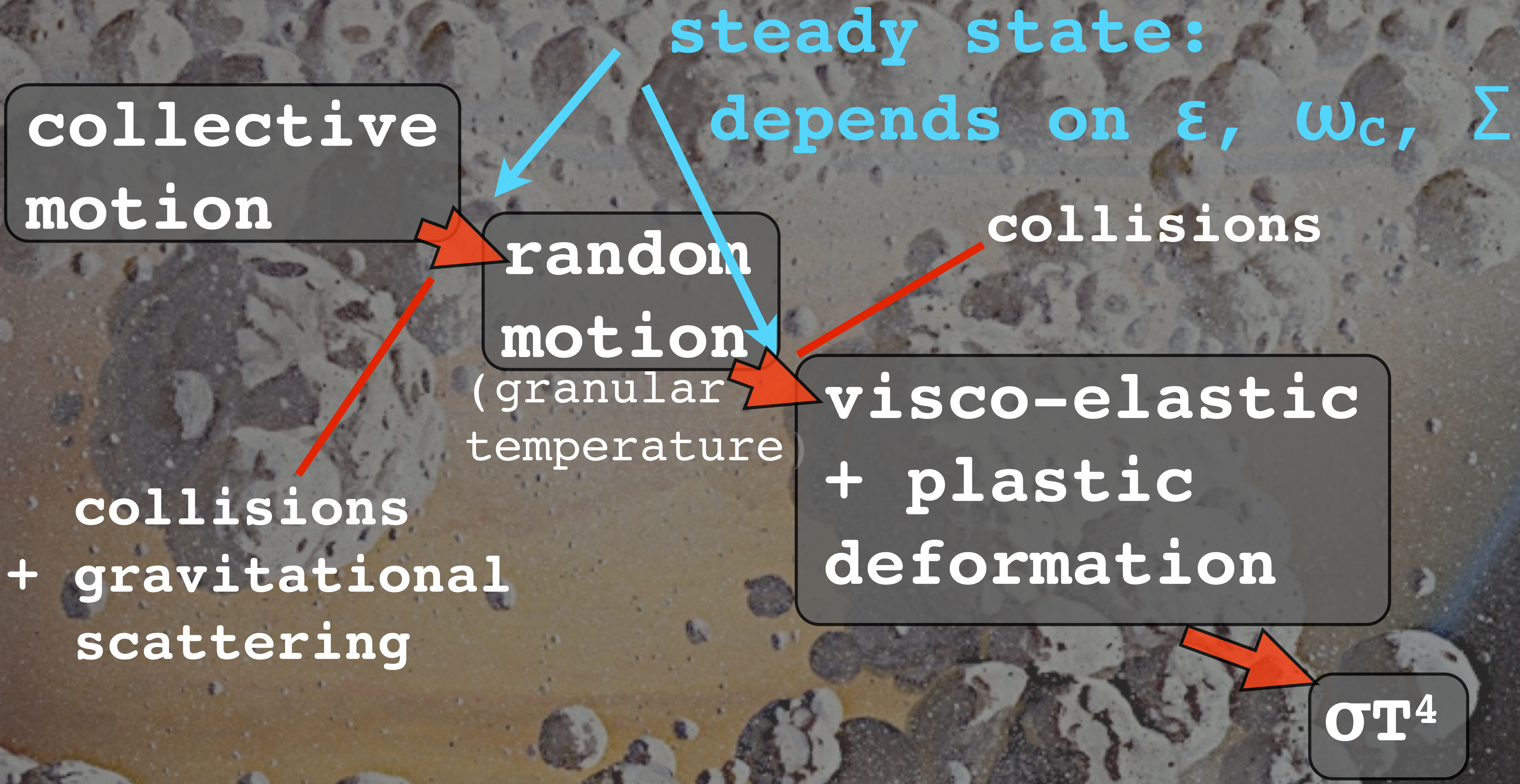
visco-elastic + plastic deformation

σT^4

(Bill Hartman)

basic physical processes, cnt'd

energy: dissipation at two levels



steady state:

depends on ε, ω_c, Σ

collective motion

random motion

(granular temperature)

collisions

visco-elastic + plastic deformation

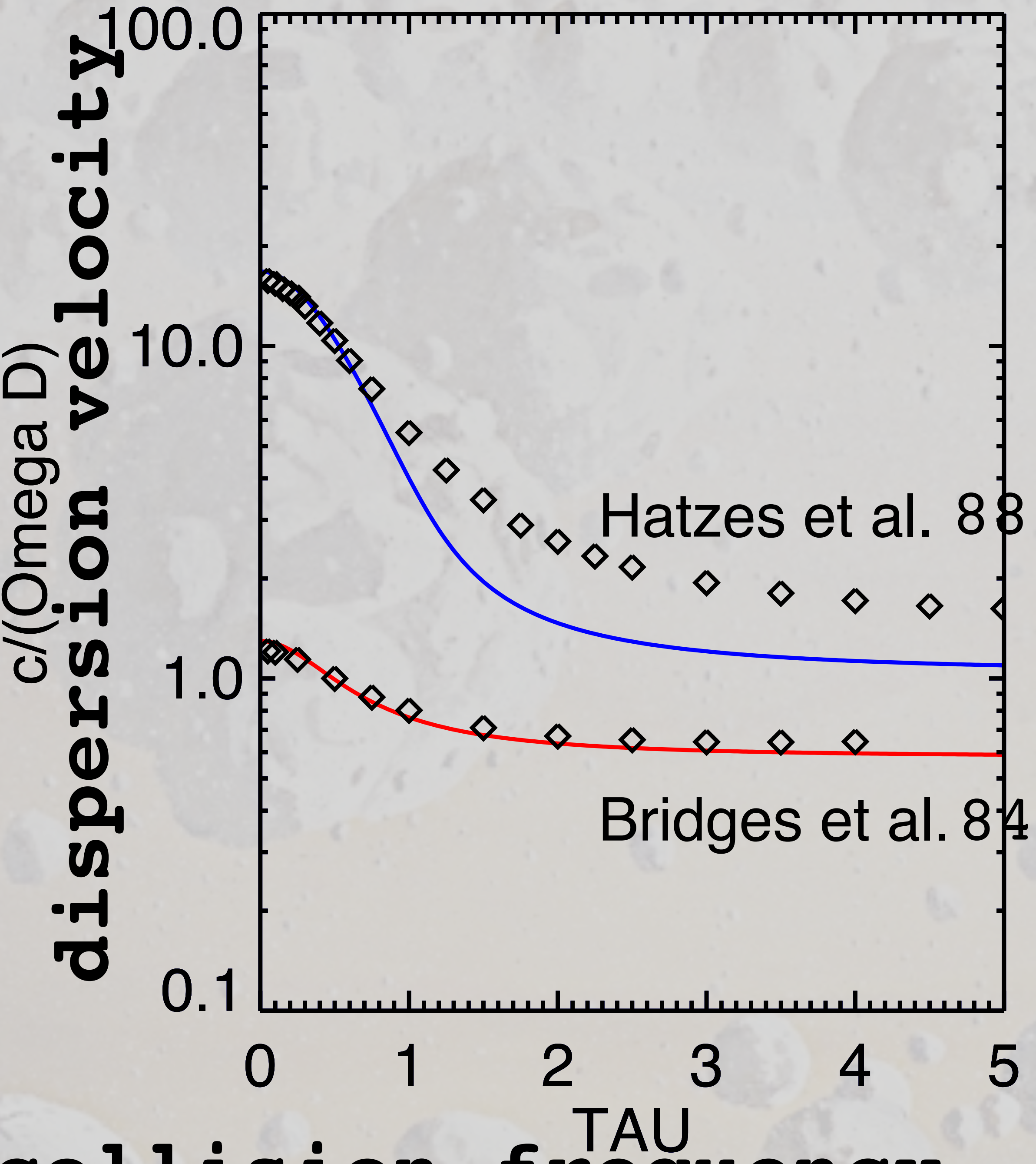
collisions + gravitational scattering

σT^4

(Bill Hartman)

basic physical processes, cnt'd

steady state



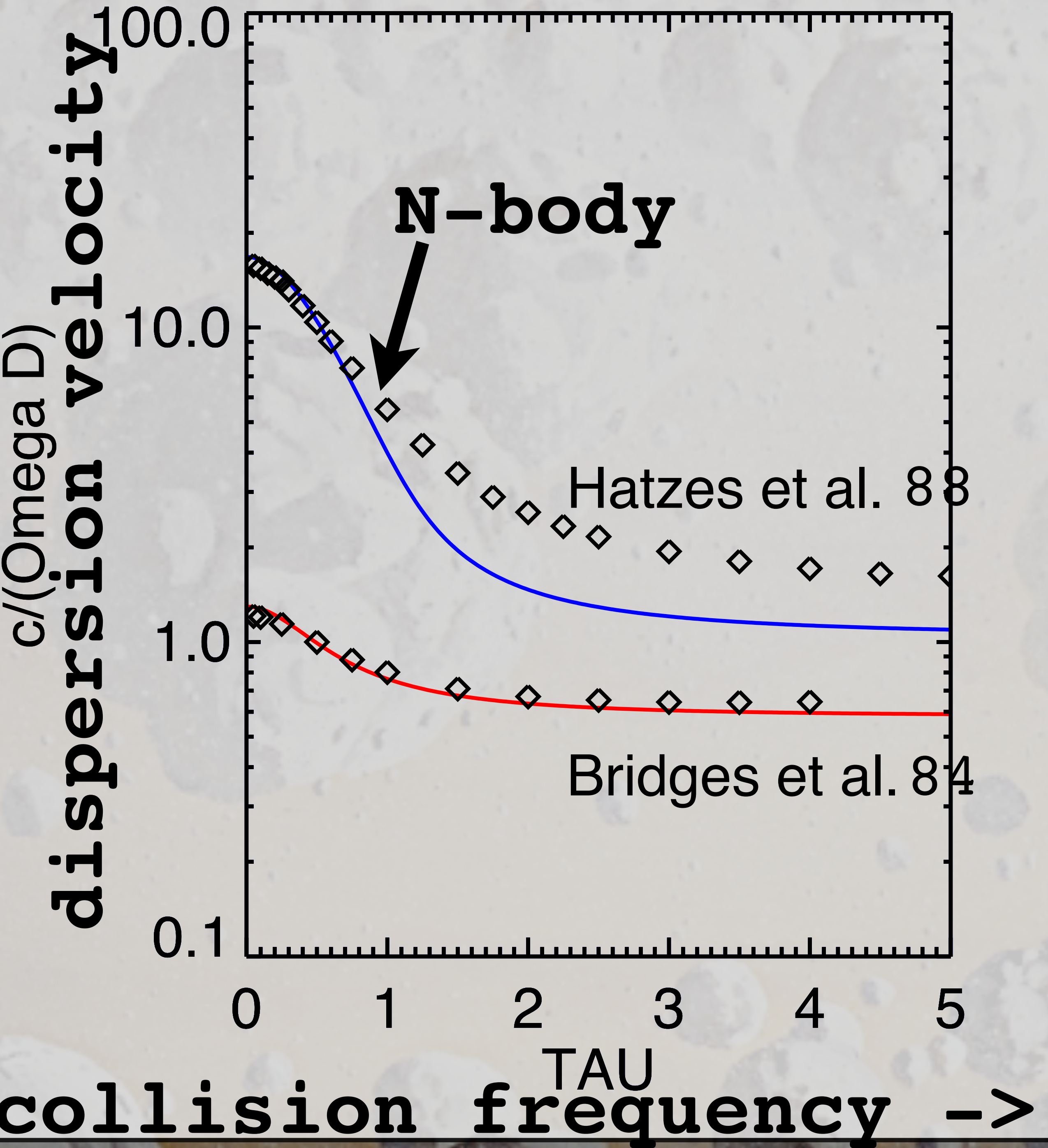
dispersion velocity

collision frequency ->

(Bill Hartman)

basic physical processes, cnt'd

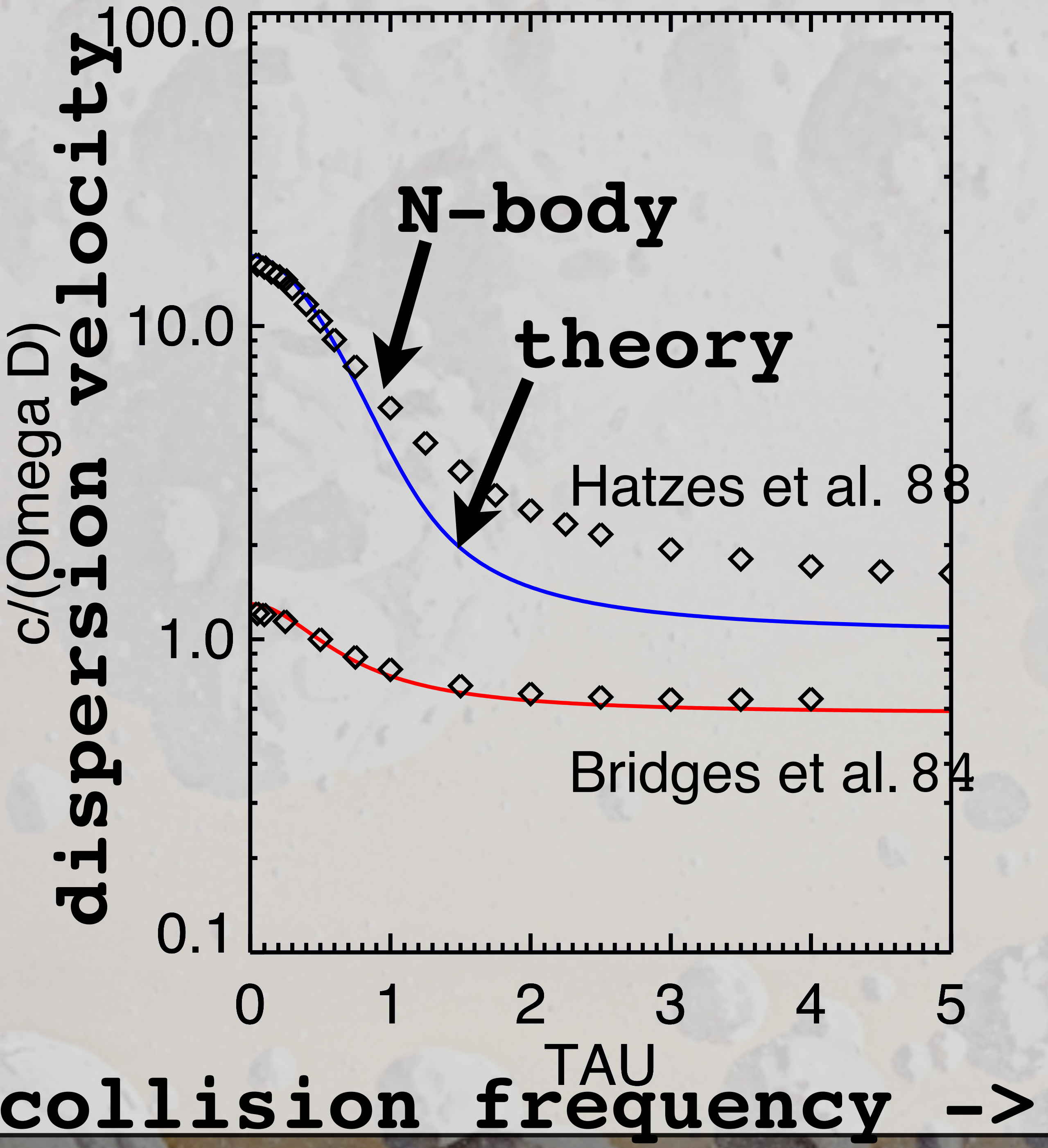
steady state



(Bill Hartman)

basic physical processes, cnt'd

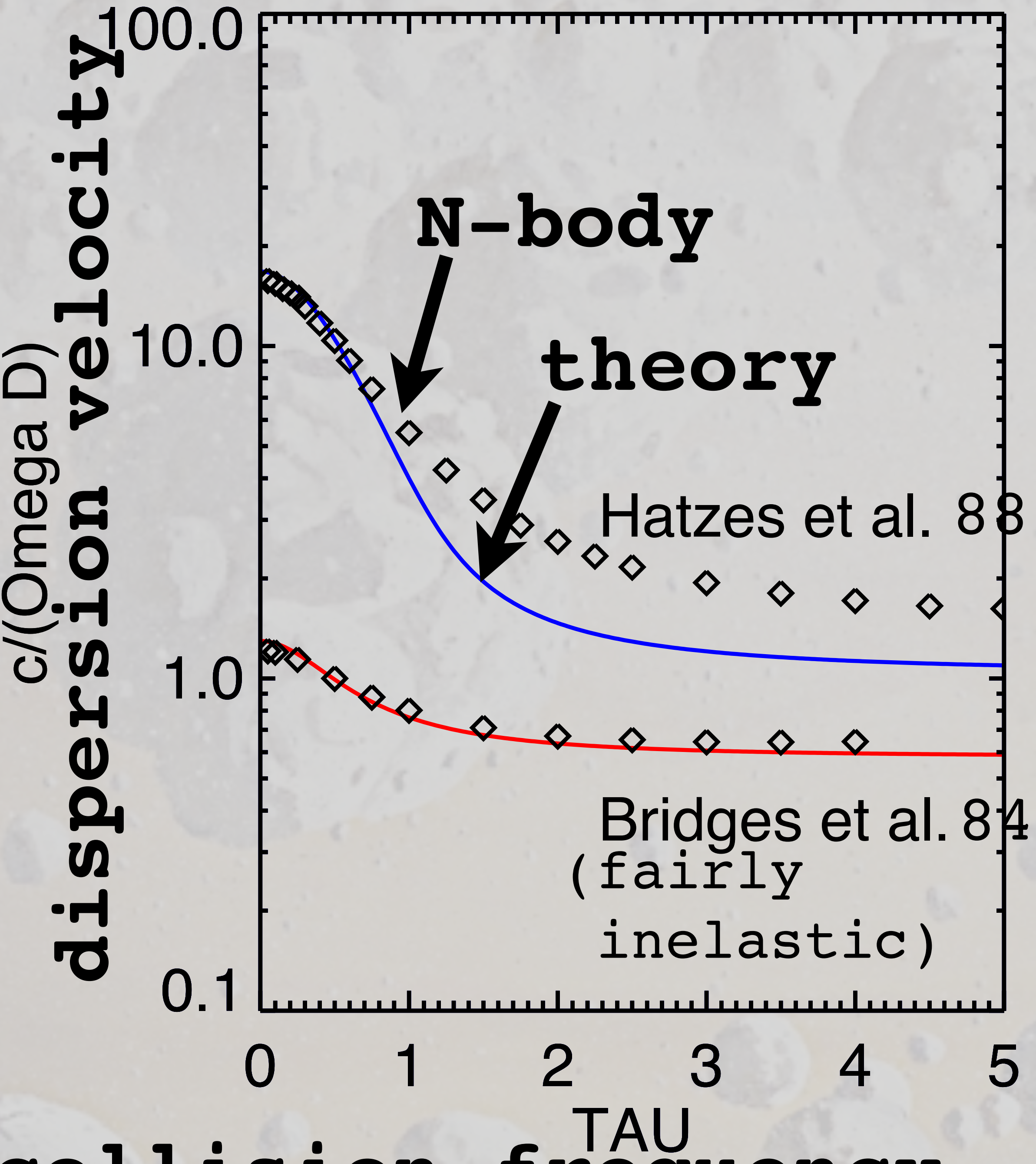
steady state



(Bill Hartman)

basic physical processes, cnt'd

steady state



N-body

theory

(fairly inelastic)

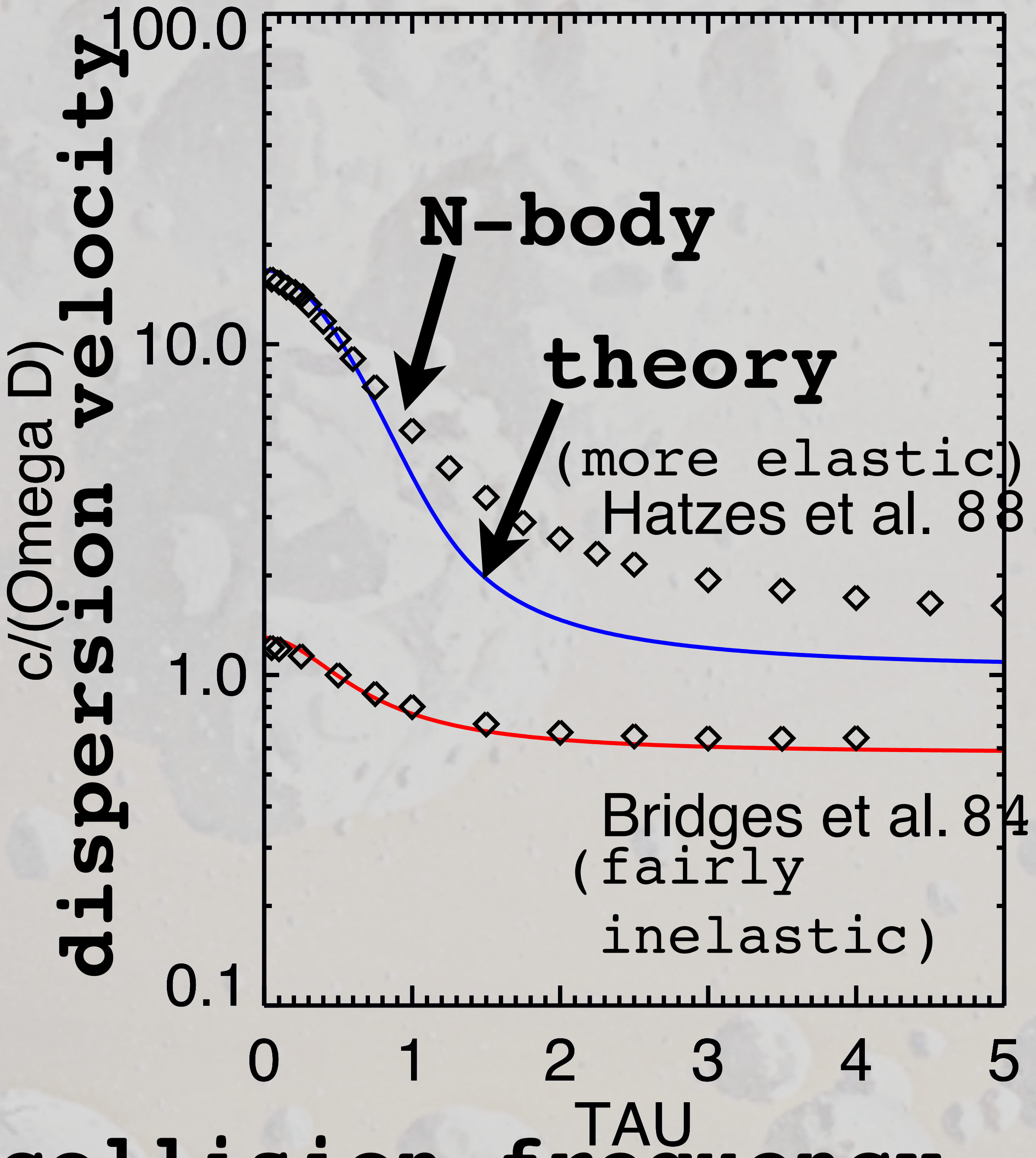
dispersion velocity

collision frequency ->

(Bill Hartman)

basic physical processes, cnt’d

steady state



dispersion velocity

N-body

theory

(more elastic)
Hatzes et al. 88

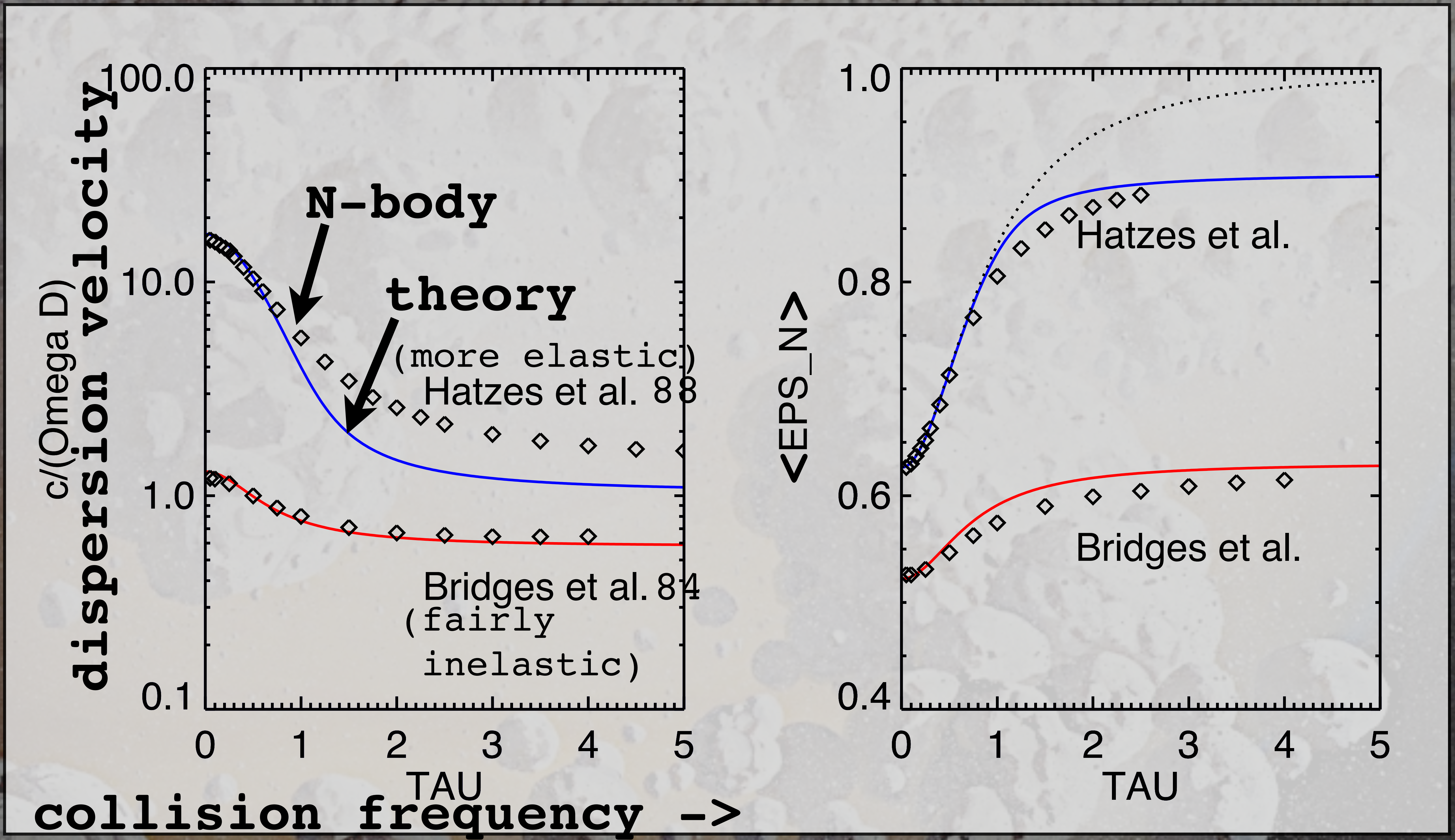
Bridges et al. 84
(fairly
inelastic)

collision frequency ->

(Bill Hartman)

basic physical processes, cnt’d

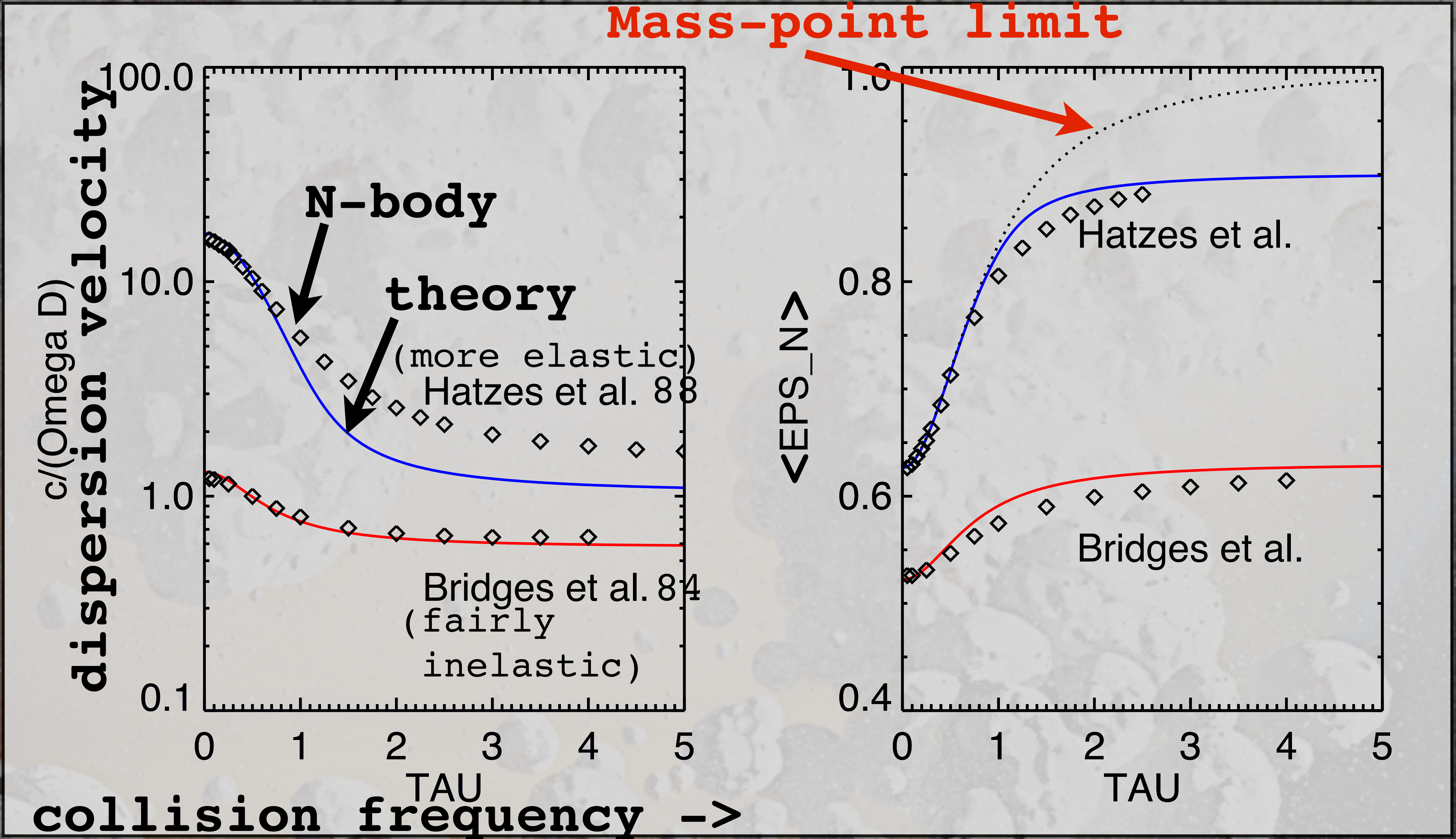
steady state



(Bill Hartman)

basic physical processes, cnt'd

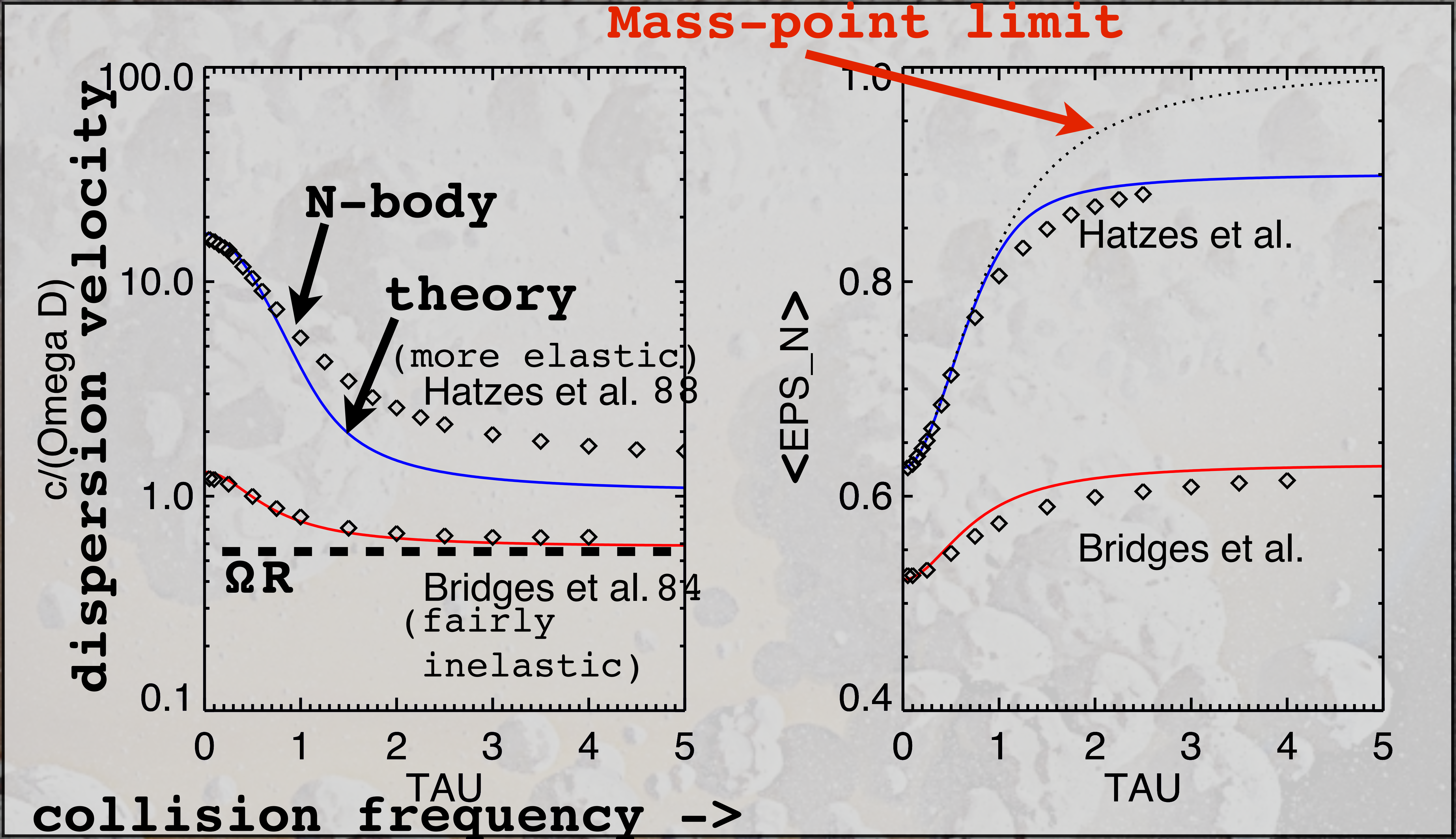
steady state

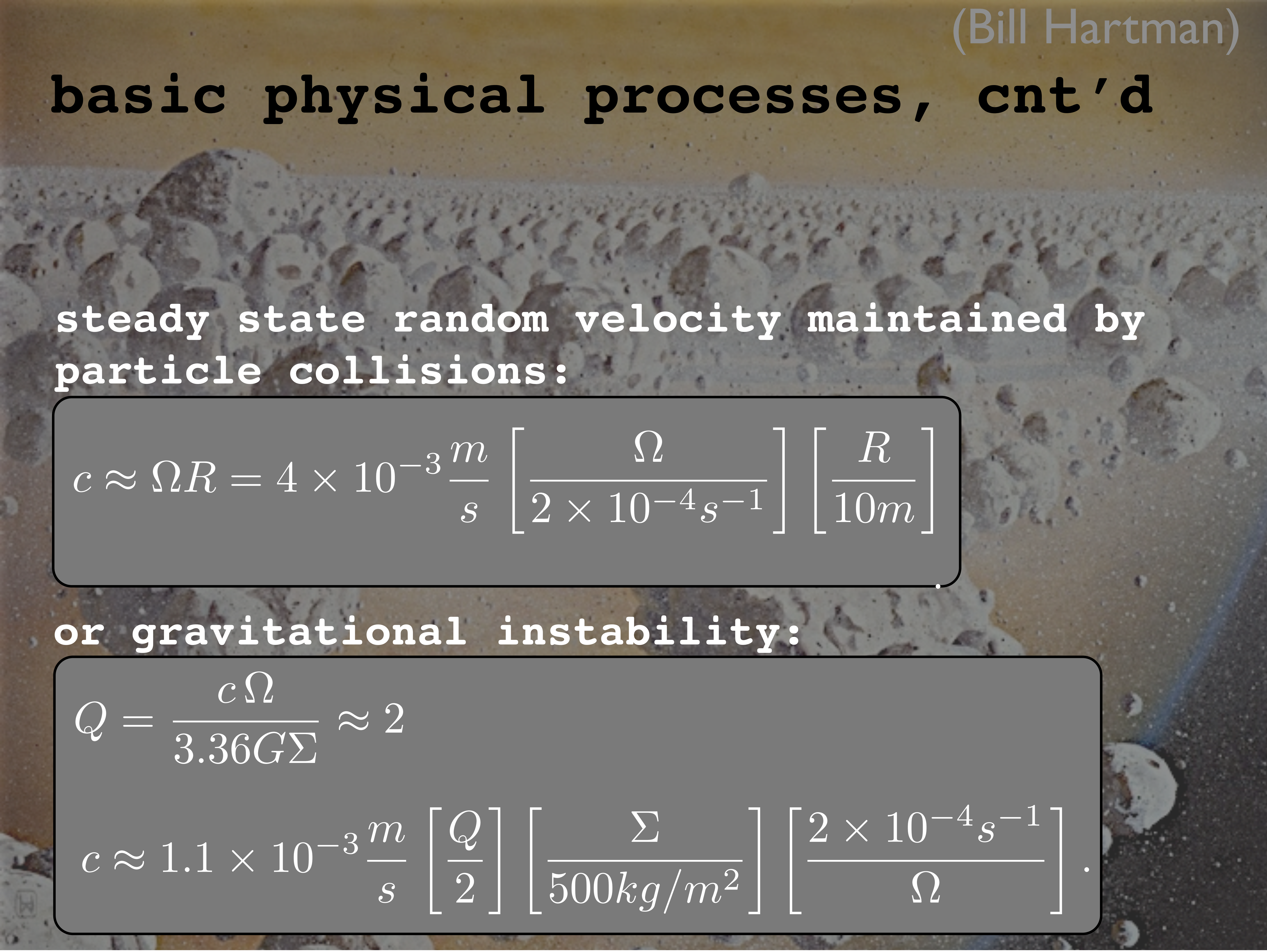


(Bill Hartman)

basic physical processes, cnt'd

steady state





(Bill Hartman)

basic physical processes, cnt'd

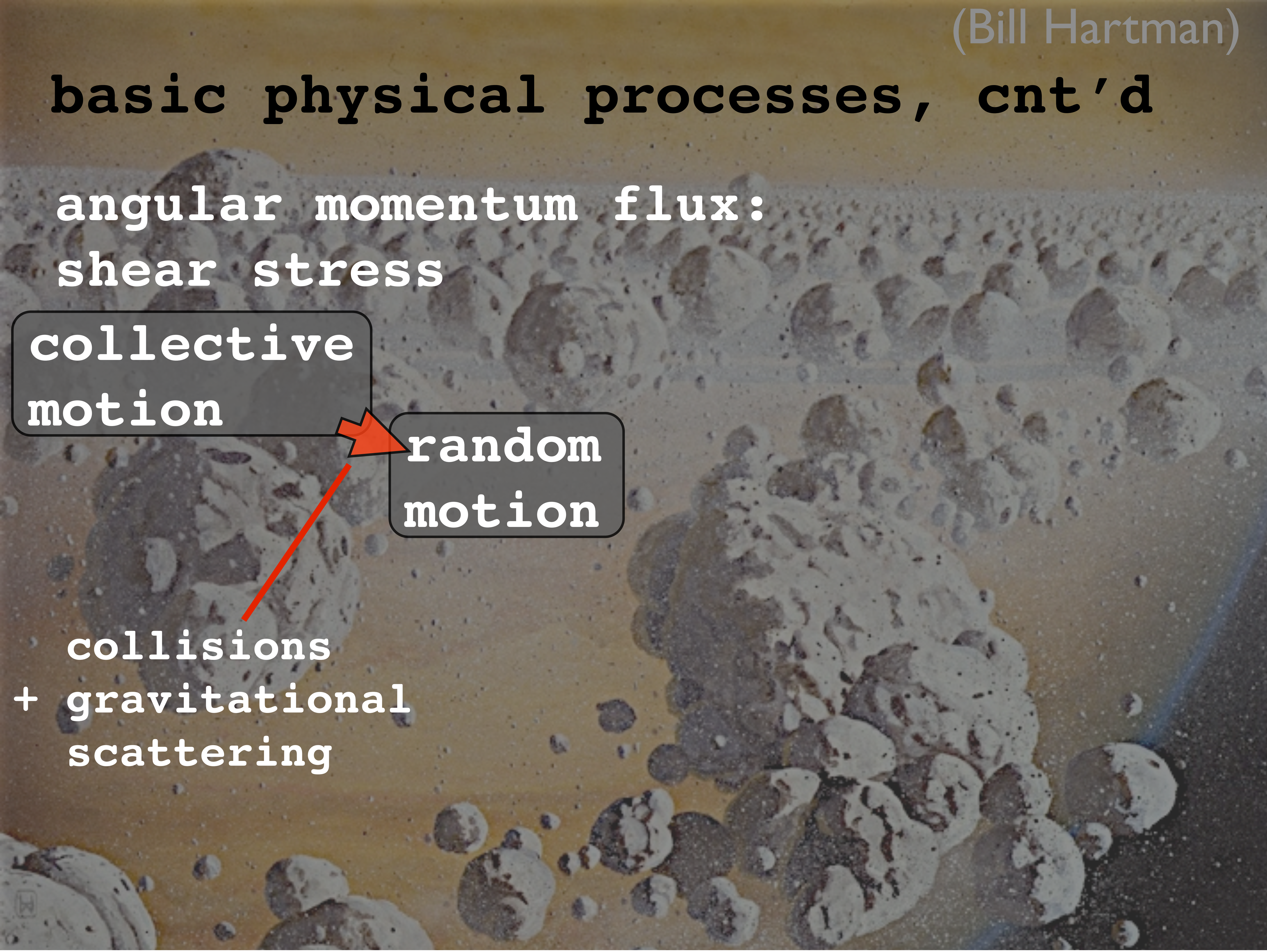
steady state random velocity maintained by particle collisions:

$$c \approx \Omega R = 4\times 10^{-3}\frac{m}{s}\left[\frac{\Omega}{2\times 10^{-4}s^{-1}}\right]\left[\frac{R}{10m}\right].$$

or gravitational instability:

$$Q = \frac{c\Omega}{3.36G\Sigma} \approx 2$$

$$c \approx 1.1\times 10^{-3}\frac{m}{s}\left[\frac{Q}{2}\right]\left[\frac{\Sigma}{500kg/m^2}\right]\left[\frac{2\times 10^{-4}s^{-1}}{\Omega}\right].$$



(Bill Hartman)

basic physical processes, cnt'd

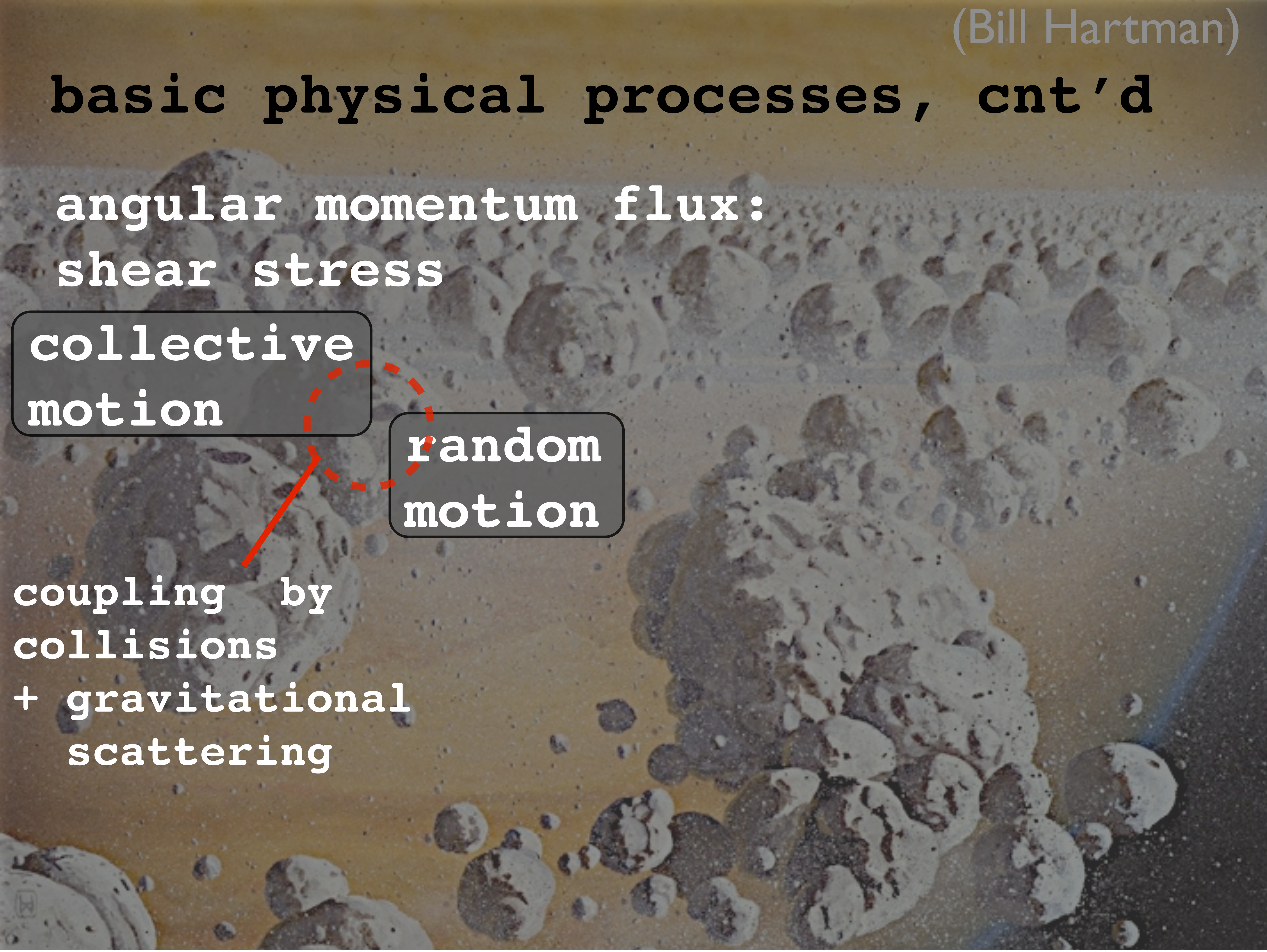
angular momentum flux:
shear stress

collective motion

random motion

collisions
+ gravitational scattering

basic physical processes, cnt'd



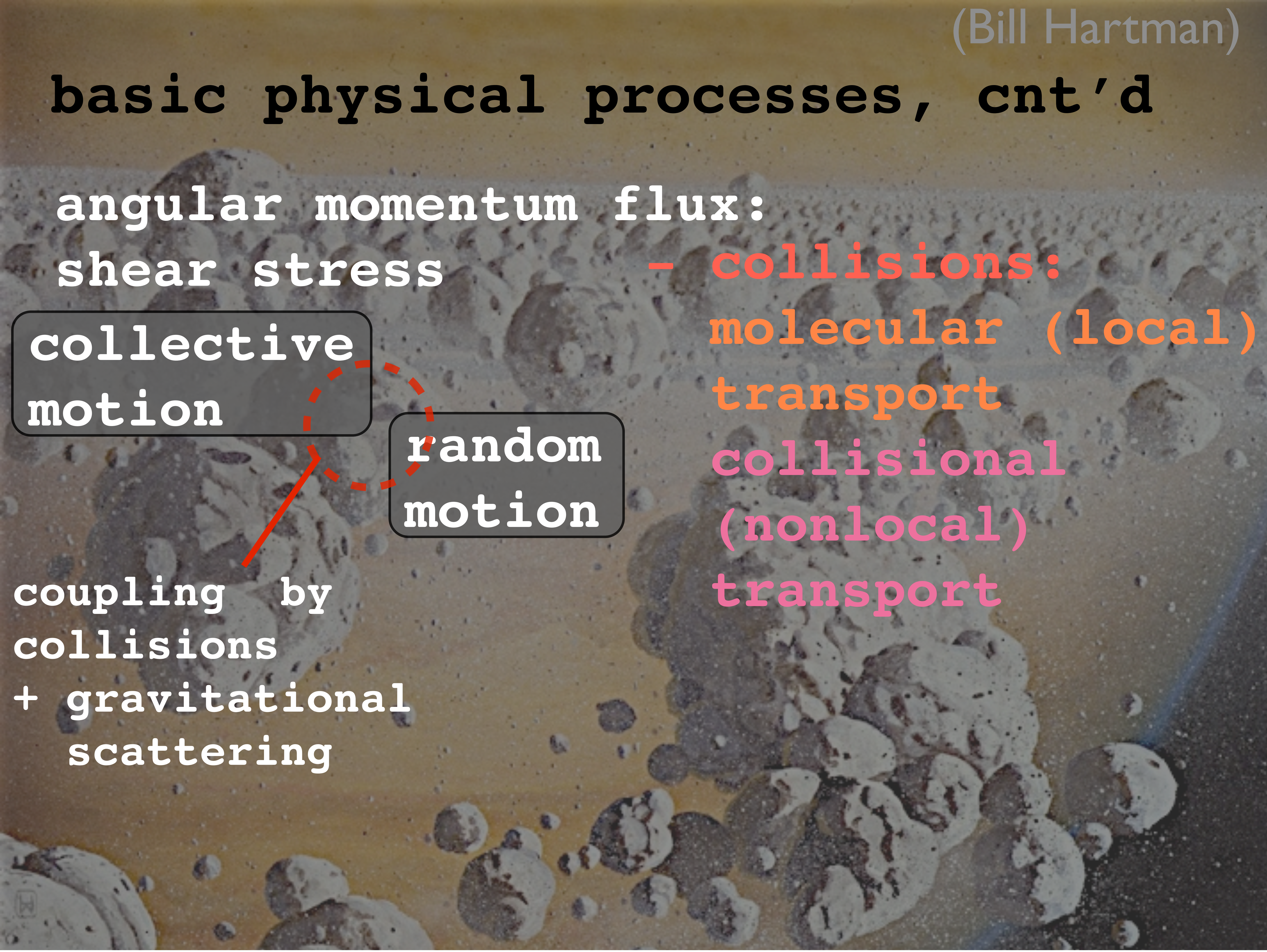
(Bill Hartman)

angular momentum flux:
shear stress

collective motion

random motion

coupling by collisions
+ gravitational scattering



(Bill Hartman)

basic physical processes, cnt'd

angular momentum flux:

shear stress

- collisions:

molecular (local) transport

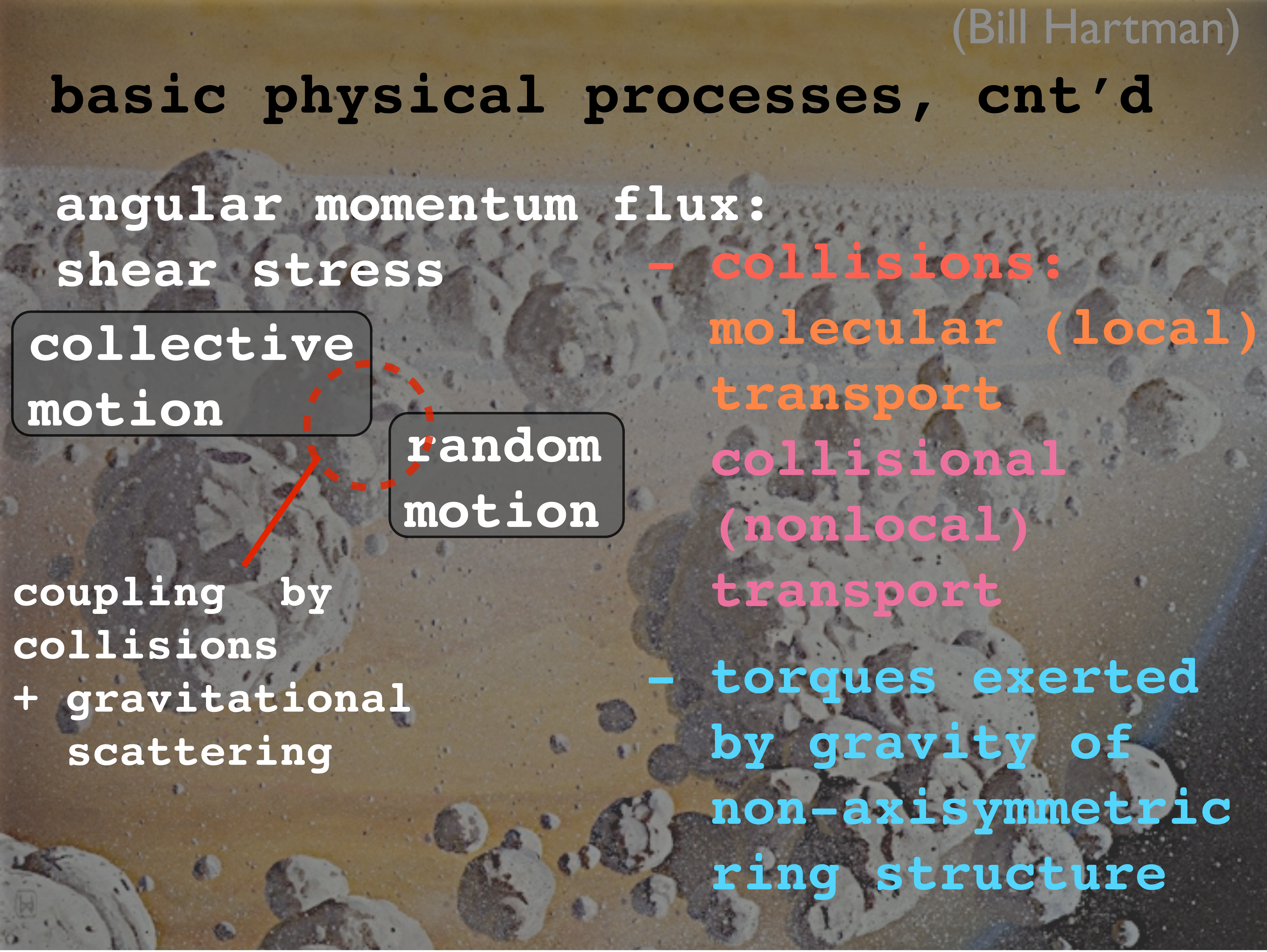
collisional (nonlocal) transport

collective motion

random motion

coupling by collisions

+ gravitational scattering


(Bill Hartman)

basic physical processes, cnt'd

angular momentum flux:

shear stress

collective motion

random motion

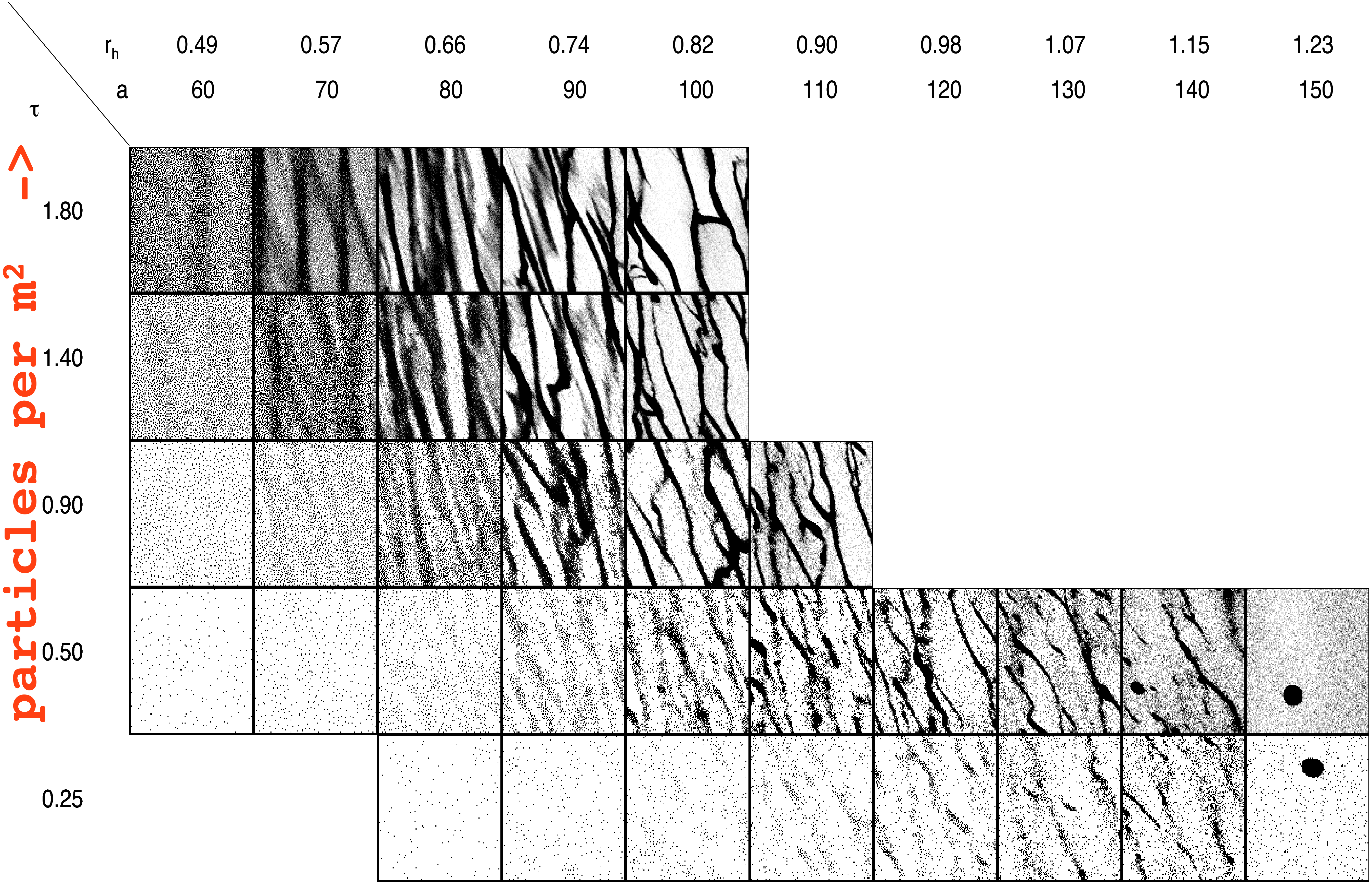
coupling by collisions
+ gravitational scattering

- collisions: molecular (local) transport
  collisional (nonlocal) transport
- torques exerted by gravity of non-axisymmetric ring structure

particle bulk density ->
distance from Saturn ->

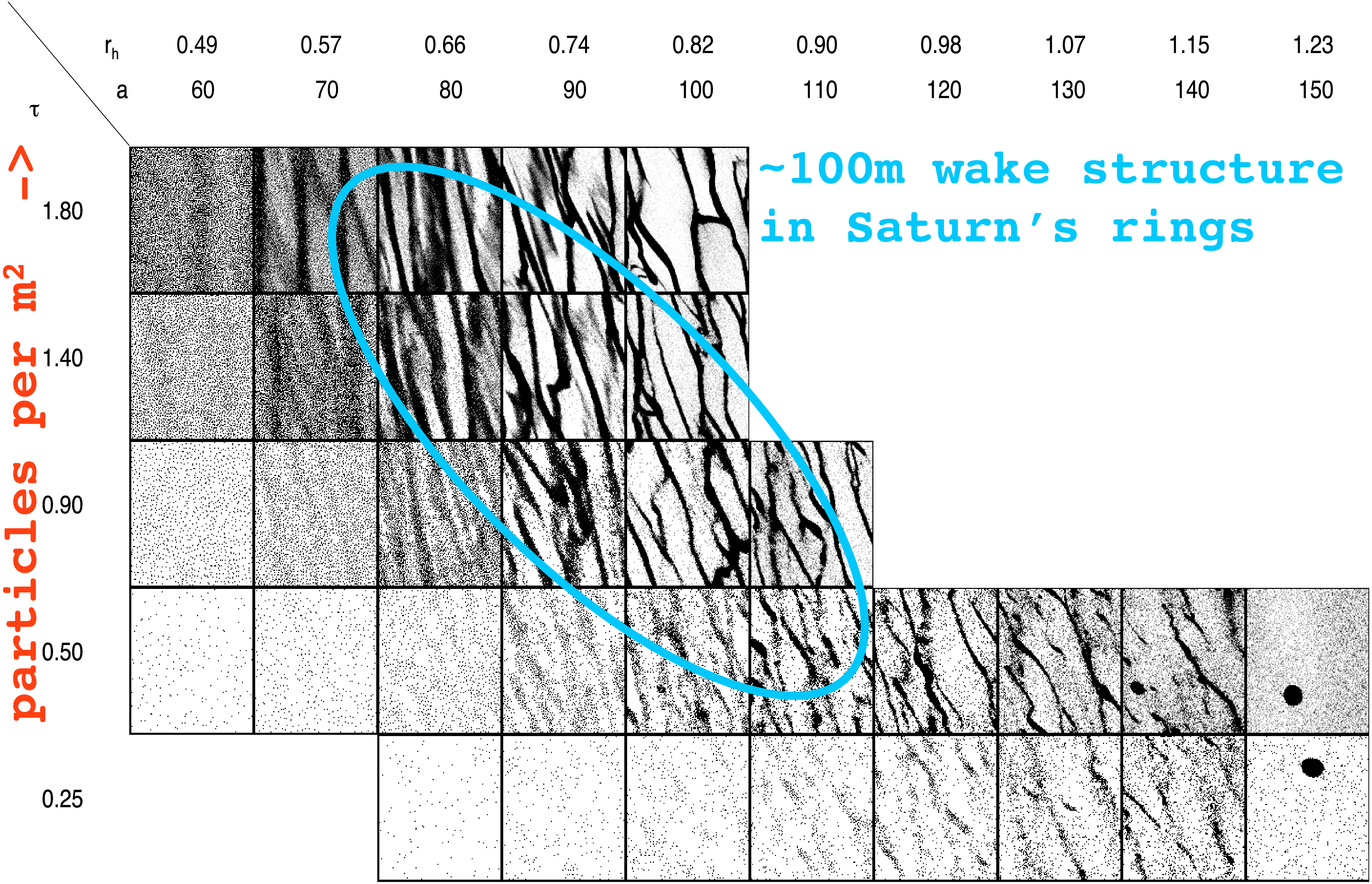
(Salo, BAAS, 2008
Schmidt et al, 2009)

particles per m^2 ->



particle bulk density ->
distance from Saturn ->

(Salo, BAAS, 2008
Schmidt et al, 2009)



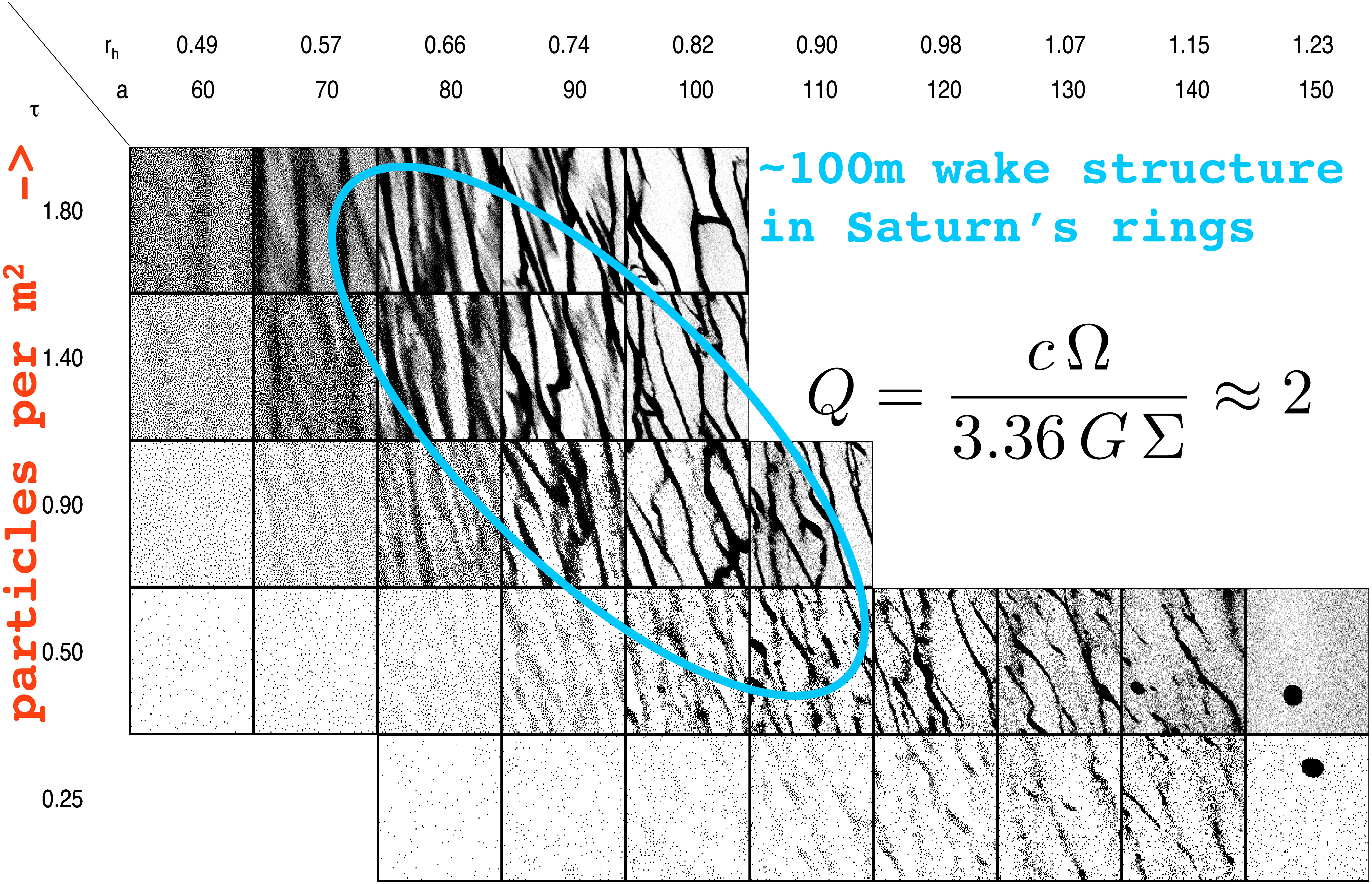
~100m wake structure in Saturn's rings

particle bulk density ->

distance from Saturn ->

(Salo, BAAS, 2008
Schmidt et al, 2009)

particles per m^2 ->

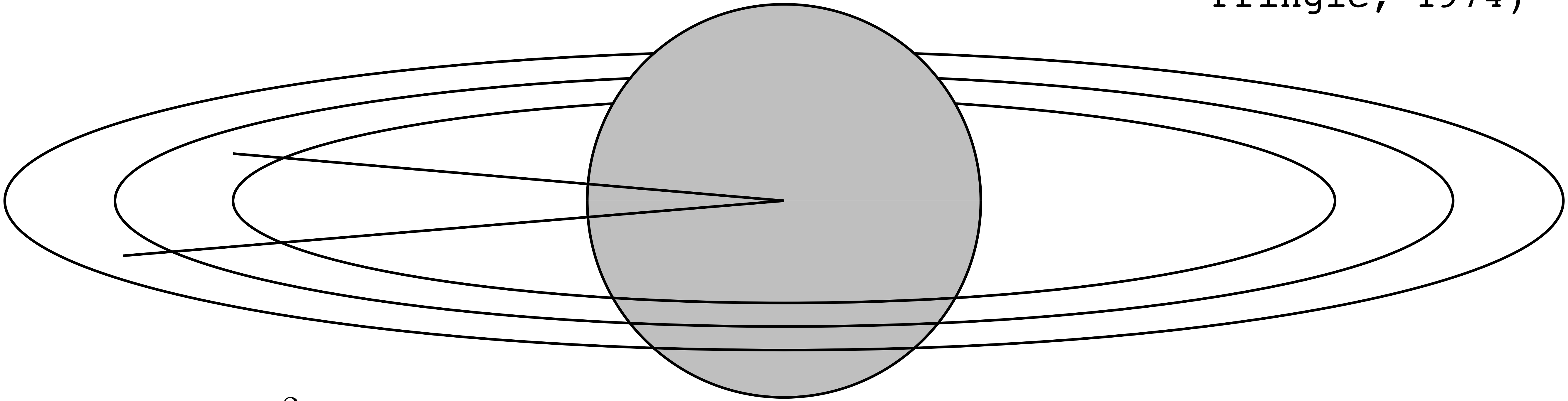


~100m wake structure in Saturn's rings

$$Q = \frac{c\,\Omega}{3.36\,G\,\Sigma} \approx 2$$

Global budget of energy and angular momentum

(Lynden-Bell and Pringle, 1974)

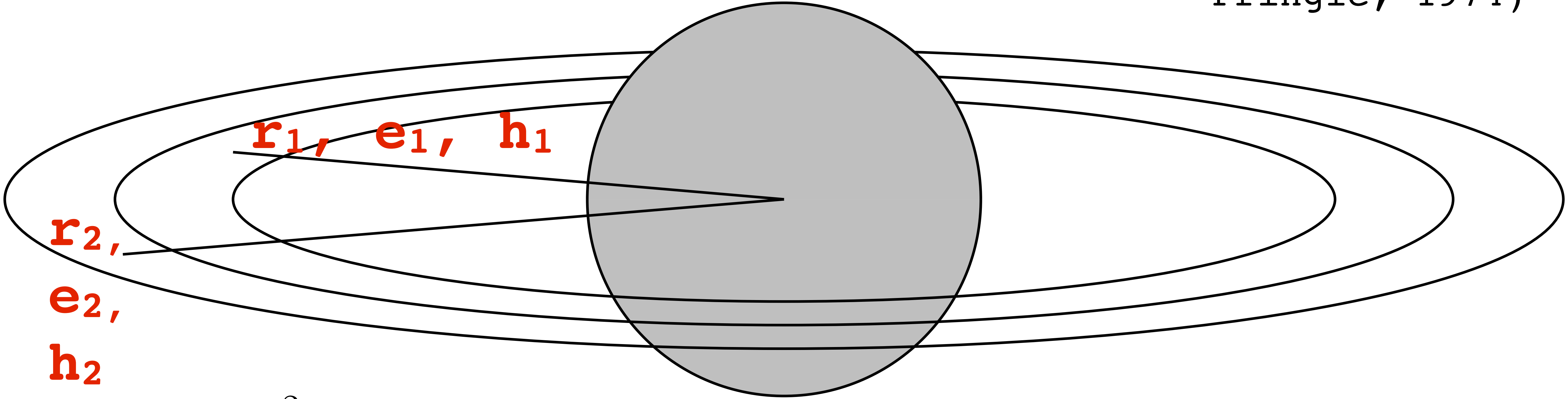


$$e = \frac{h^2}{2\,r^2} + \Phi(r) \quad \text{energy per unit mass}$$

$$h = \Omega\,r^2 \quad \text{angular momentum per unit mass}$$

Global budget of energy and angular momentum

(Lynden-Bell and Pringle, 1974)



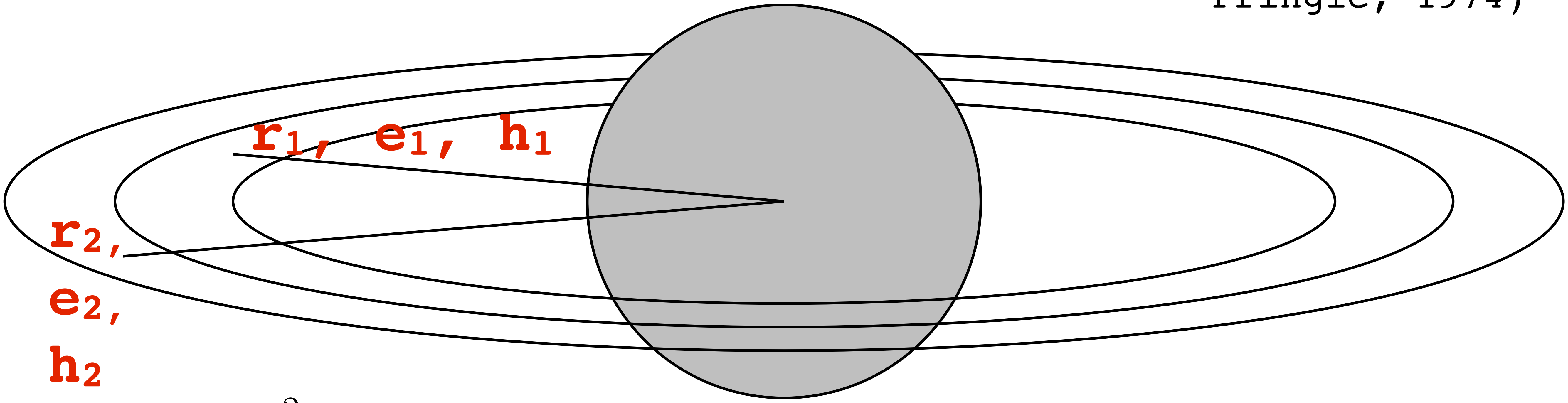
$$e = \frac{h^2}{2\,r^2} + \Phi(r)$$ **energy per unit mass**

$$h = \Omega\, r^2$$ **angular momentum per unit mass**

allow two neighboring segments to exchange mass and angular momentum

Global budget of energy and angular momentum

(Lynden-Bell and Pringle, 1974)



$$e = \frac{h^2}{2\,r^2} + \Phi(r)$$ **energy per unit mass**

$$h = \Omega\, r^2$$ **angular momentum per unit mass**

allow two neighboring segments to exchange mass and angular momentum

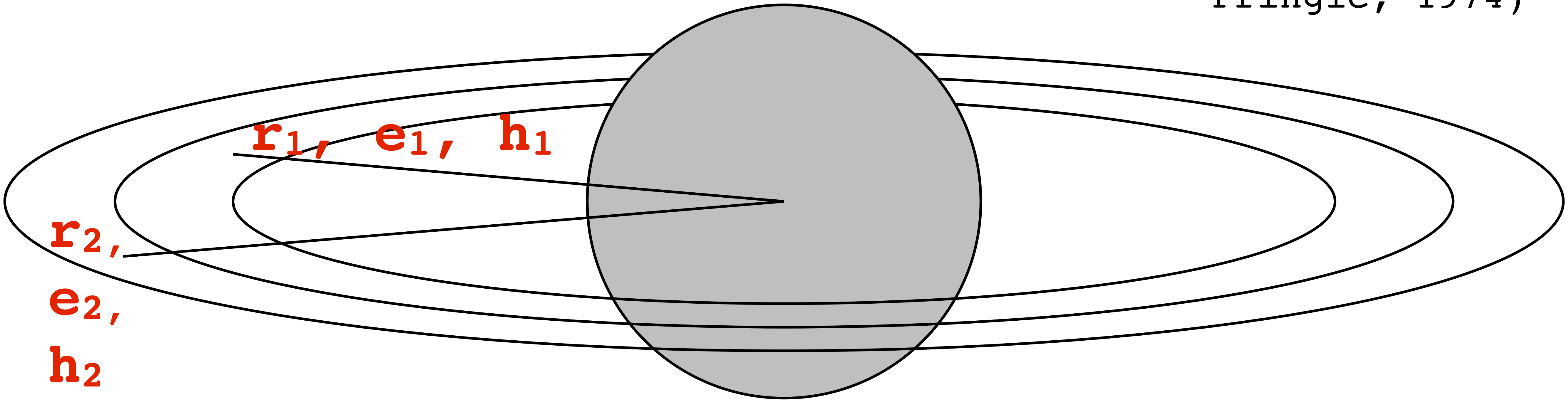
$$\delta E = \delta(m_1 e_1) + \delta(m_2 e_2)$$ **should be negative**

$$\delta H = \delta(m_1 h_1) + \delta(m_2 h_2) \equiv \delta H_1 + \delta H_2 = 0$$ **conserved**

$$\delta M = \delta m_1 + \delta m_2 = 0$$ **conserved**

Global budget of energy and angular momentum

(Lynden-Bell and Pringle, 1974)

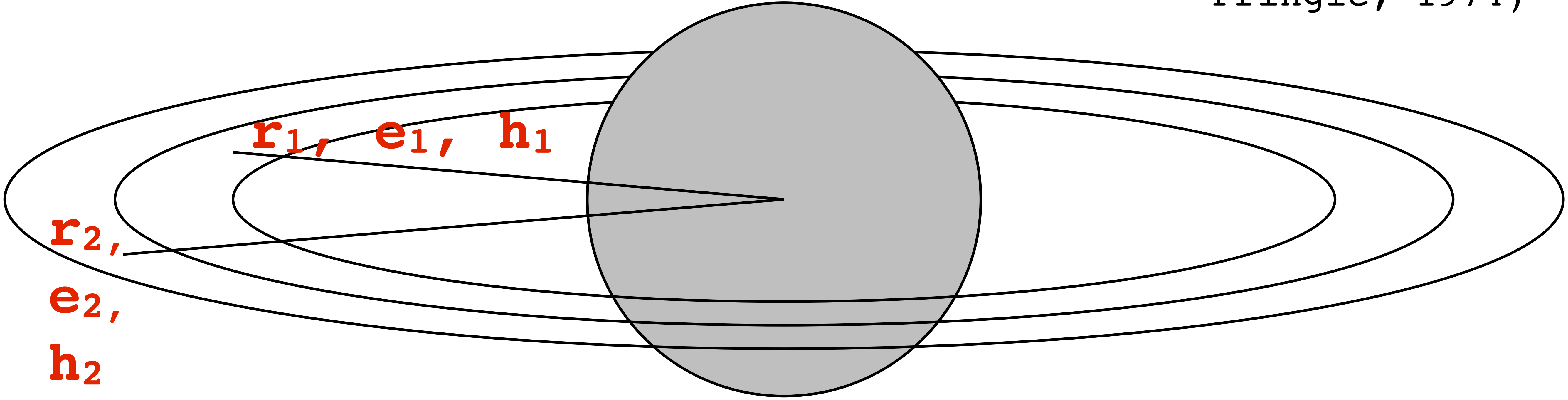


total change in energy:

$$\begin{aligned}\delta E &= \delta(m_1 e_1) + \delta(m_2 e_2) \\ &= \delta m_1 e_1 + \delta m_2 e_2 + \Omega_1 m_1 \delta h_1 + \Omega_2 m_2 \delta h_2 \\ &= \delta m_1 \left[(e_1 - \Omega_1 h_1) - (e_2 - \Omega_2 h_2)\right] + \delta H_1 (\Omega_1 - \Omega_2)\end{aligned}$$

Global budget of energy and angular momentum

(Lynden-Bell and Pringle, 1974)

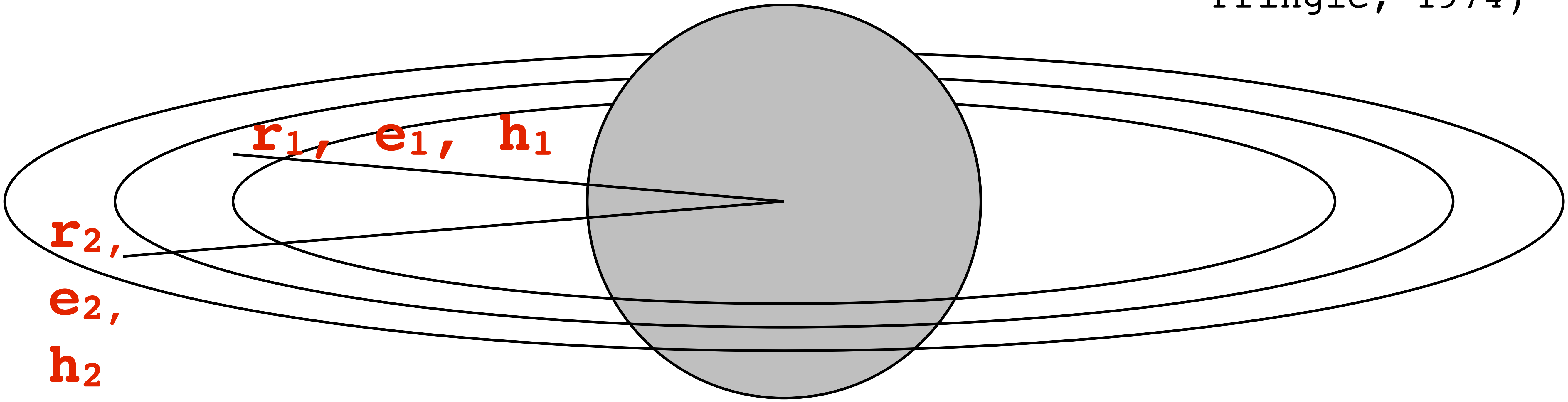


total change in energy:

$$
\begin{aligned}
\delta E &= \delta(m_1 e_1) + \delta(m_2 e_2) \\
&= \delta m_1 e_1 + \delta m_2 e_2 + \Omega_1 m_1 \delta h_1 + \Omega_2 m_2 \delta h_2 \\
&= \delta m_1 \underbrace{\left[(e_1 - \Omega_1 h_1) - (e_2 - \Omega_2 h_2)\right]}_{\text{positive}} + \delta H_1 \underbrace{(\Omega_1 - \Omega_2)}_{\text{negative}}
\end{aligned}
$$

Global budget of energy and angular momentum

(Lynden-Bell and Pringle, 1974)

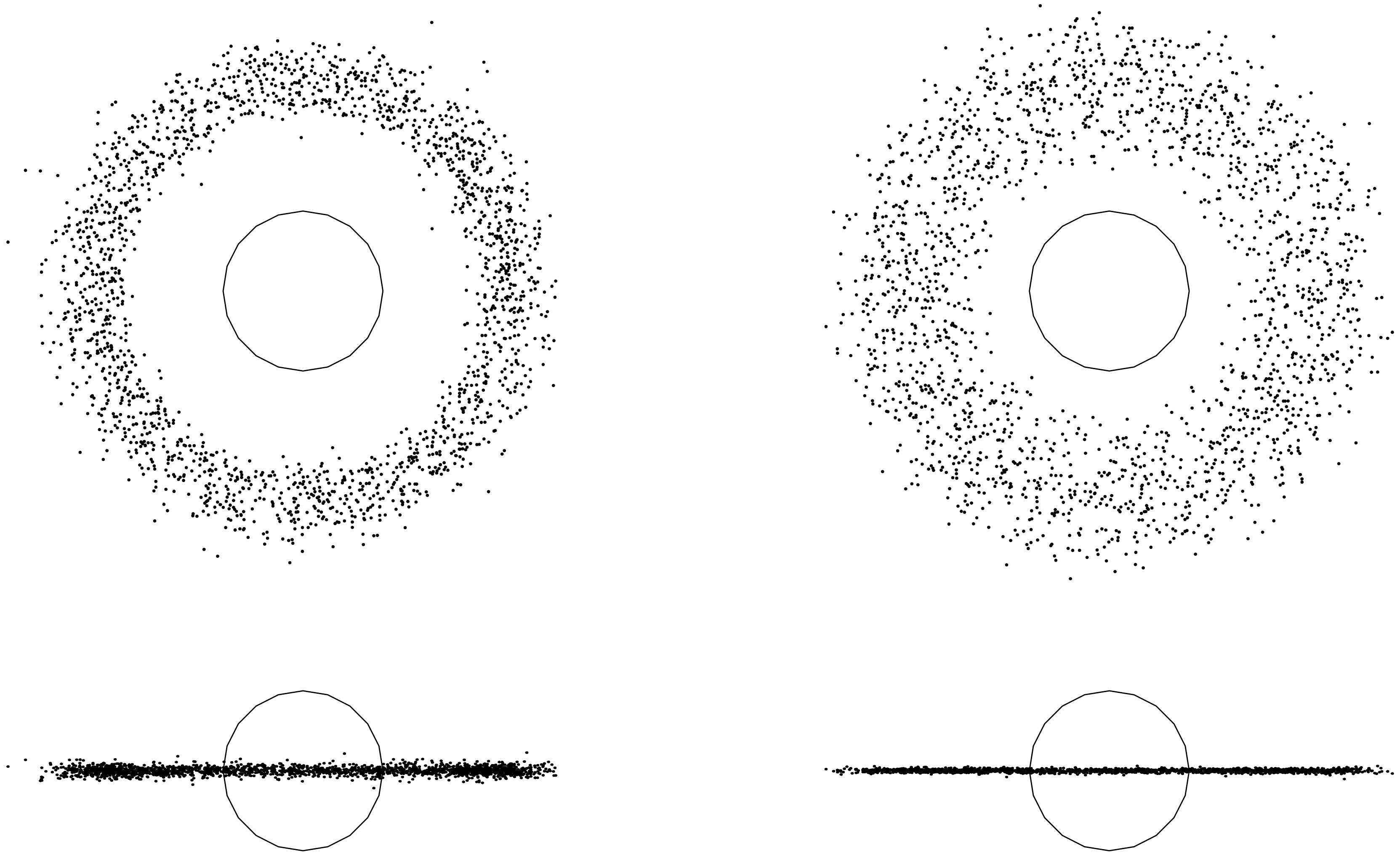


total change in energy:

$$\begin{aligned}\delta E &= \delta(m_1 e_1) + \delta(m_2 e_2) \\ &= \delta m_1 e_1 + \delta m_2 e_2 + \Omega_1 m_1 \delta h_1 + \Omega_2 m_2 \delta h_2 \\ &= \delta m_1 \underbrace{\left[(e_1 - \Omega_1 h_1) - (e_2 - \Omega_2 h_2)\right]}_{\text{positive}} + \delta H_1 \underbrace{(\Omega_1 - \Omega_2)}_{\text{negative}}\end{aligned}$$

=> energy is lowered if mass flows inward and/or angular momentum flows outward

INITIAL DISTRIBUTION AFTER 150 REVOLUTIONS



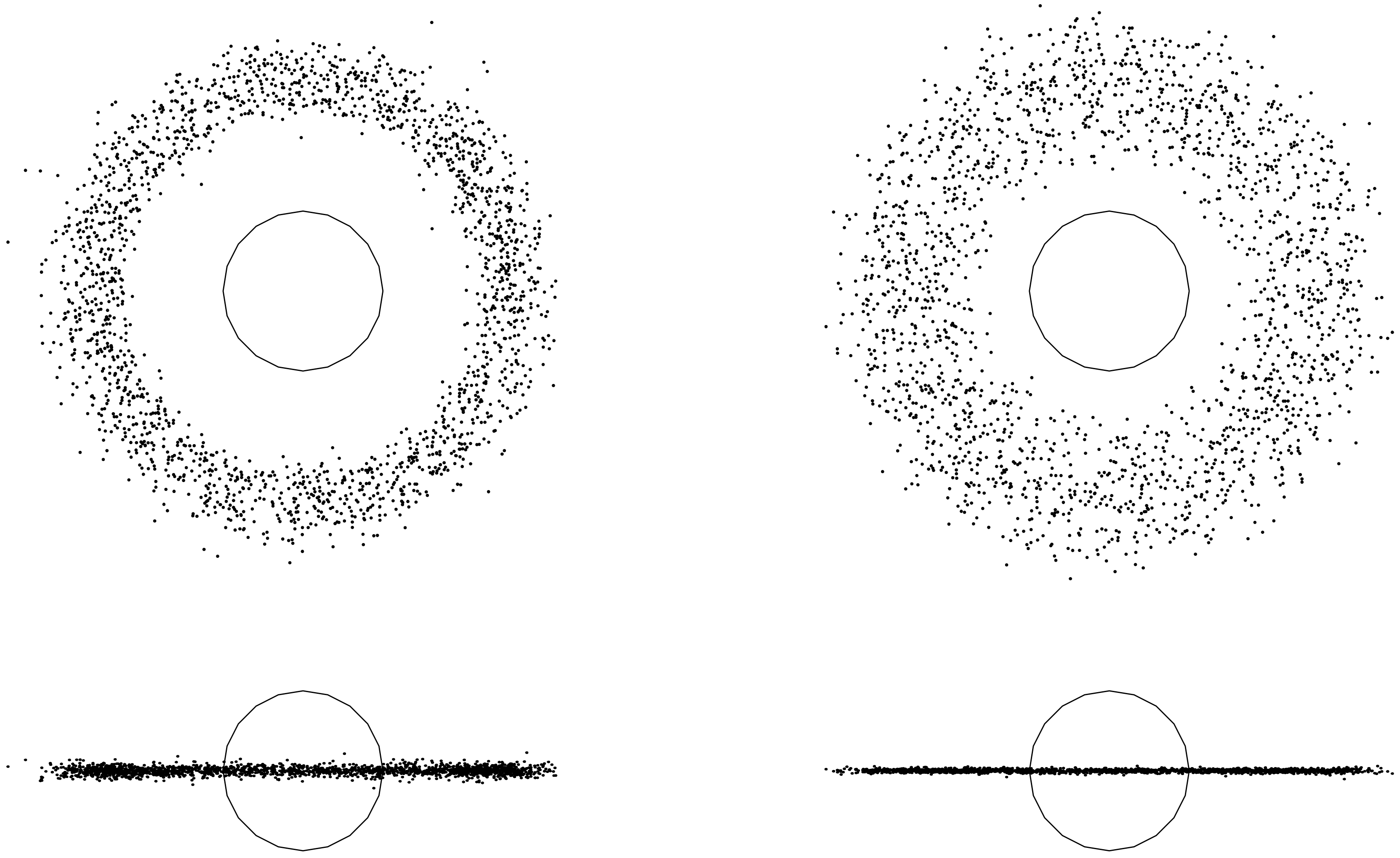
top
view

side
view

=> the disk flattens and spreads

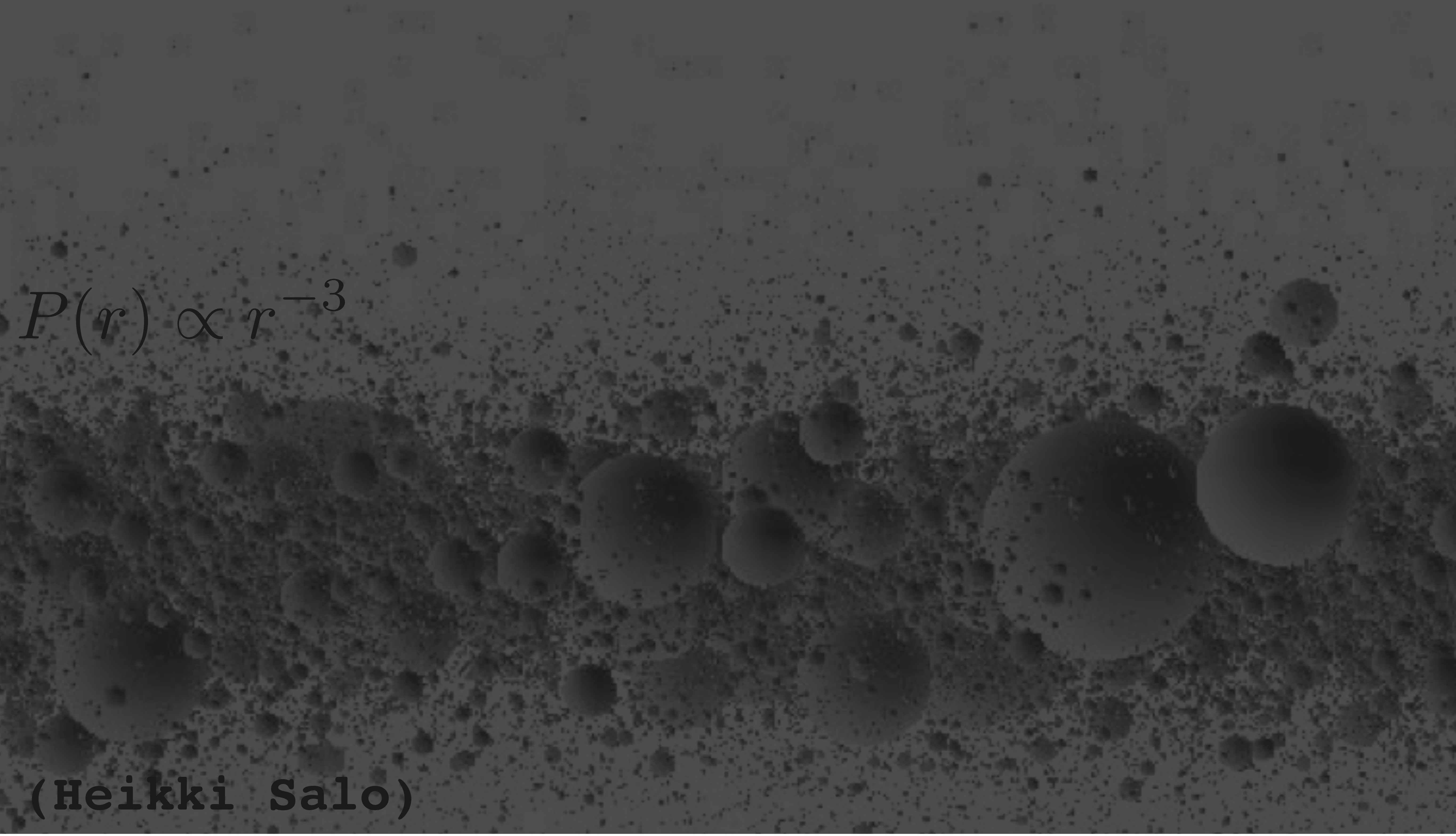
INITIAL DISTRIBUTION

AFTER 150 REVOLUTIONS



top view

side view

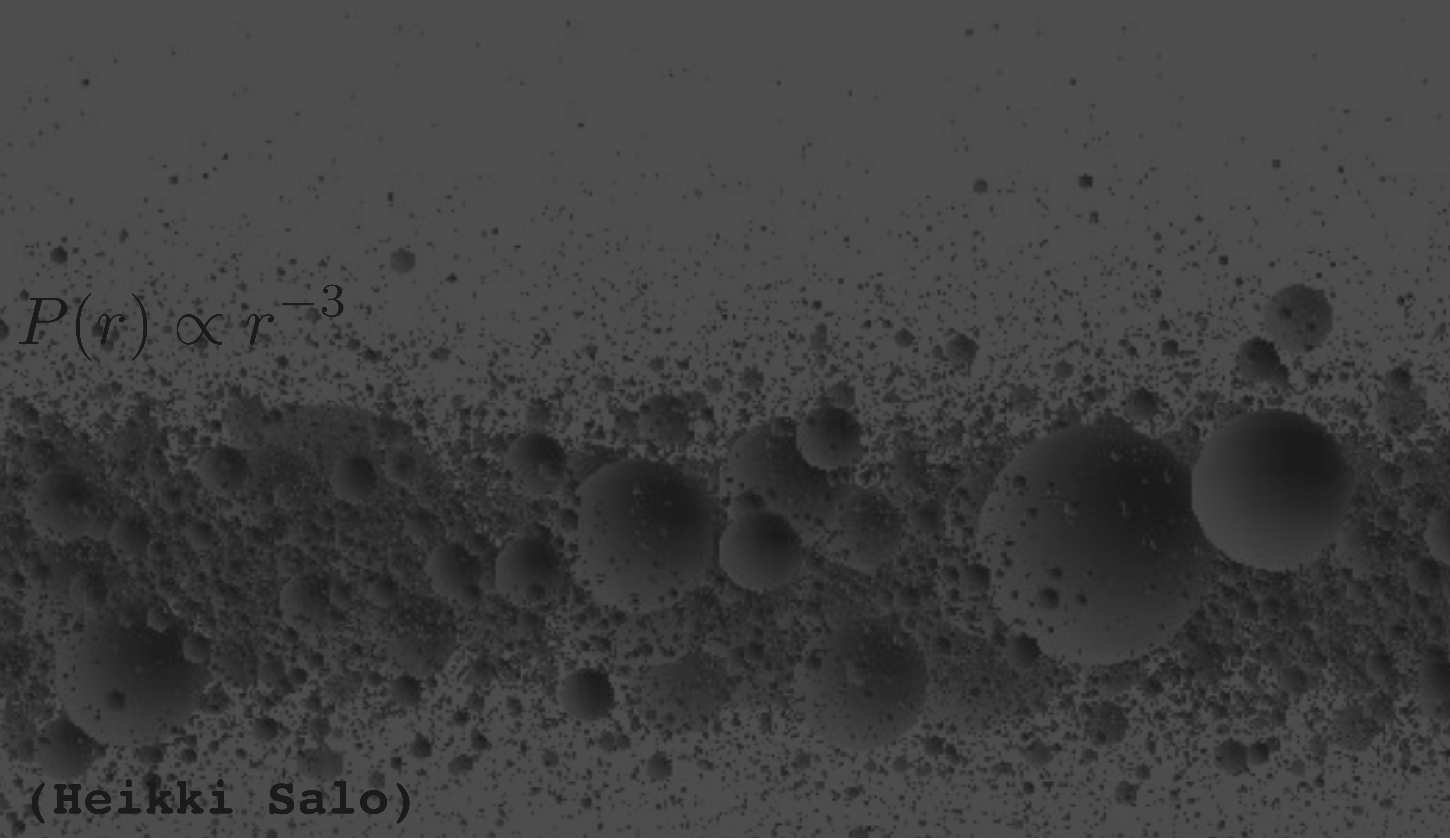


$$P(r) \propto r^{-3}$$

(Heikki Salo)

scale hight: H ~ c/Ω

(pressure vs vertical
Saturn gravity)



$$P(r) \propto r^{-3}$$

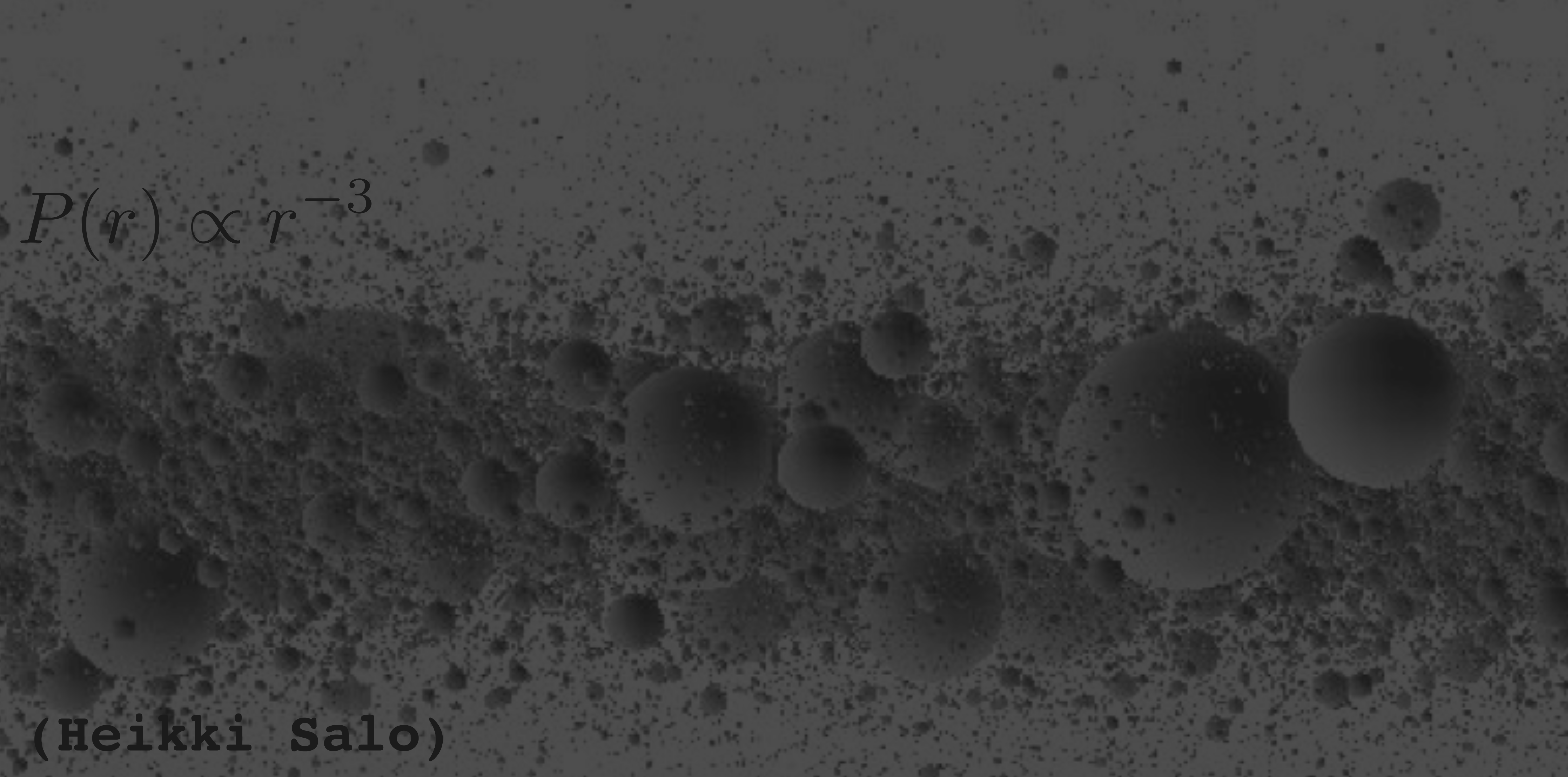
(Heikki Salo)

scale hight: H ~ c/Ω

(pressure vs vertical Saturn gravity)

surface number density

number density: $$n \propto \frac{n_2}{H} e^{-\frac{z^2}{2H^2}}$$



$$P(r) \propto r^{-3}$$

(Heikki Salo)

scale hight: H ~ c/Ω

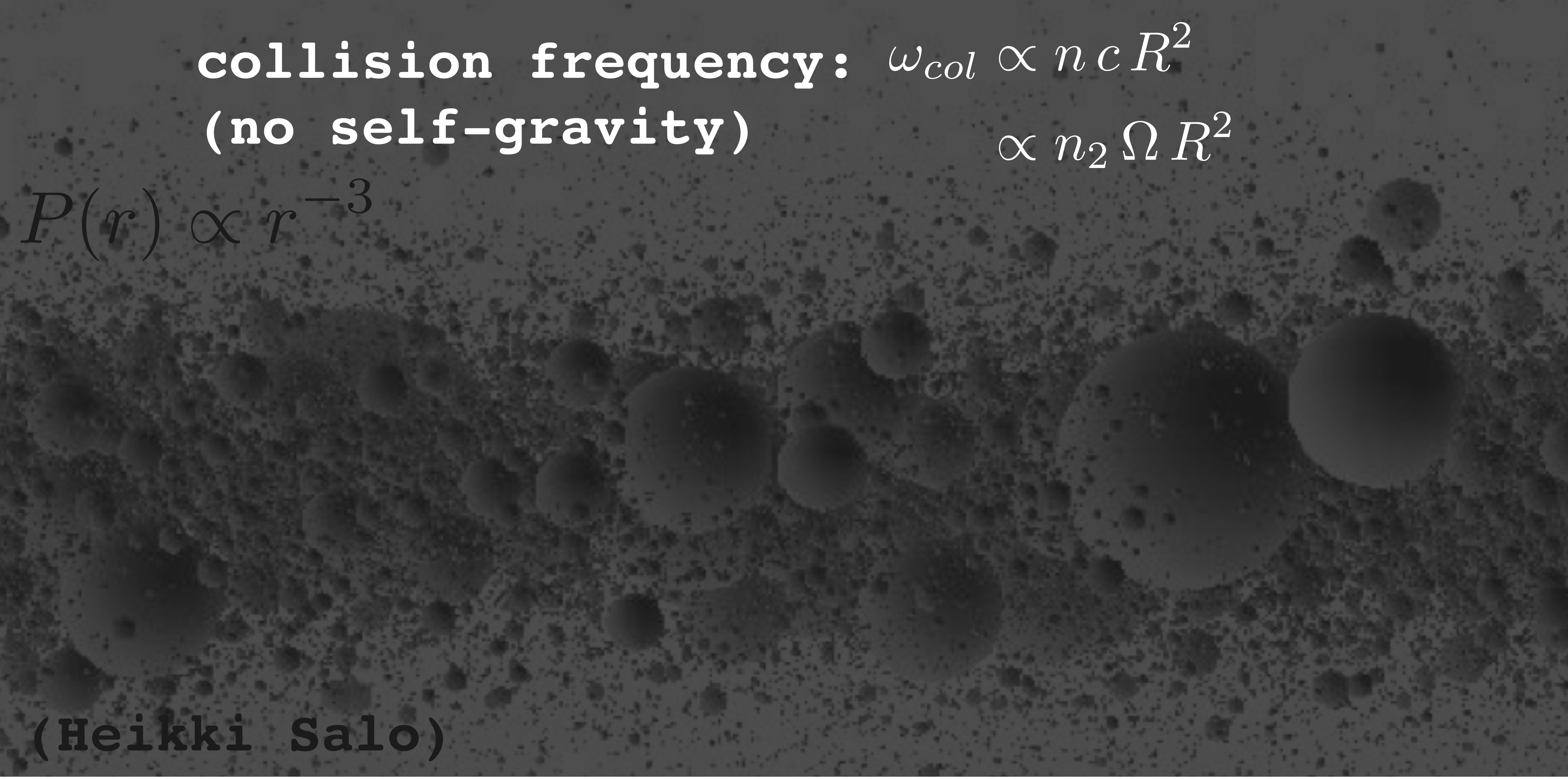
(pressure vs vertical Saturn gravity)

surface number density

number density: $n \propto \frac{n_2}{H} e^{-\frac{z^2}{2H^2}}$

collision frequency: $\omega_{col} \propto n\, c\, R^2$

(no self-gravity) $\propto n_2\, \Omega\, R^2$



$P(r) \propto r^{-3}$

(Heikki Salo)

scale hight: H ~ c/Ω

(pressure vs vertical Saturn gravity)

surface number density

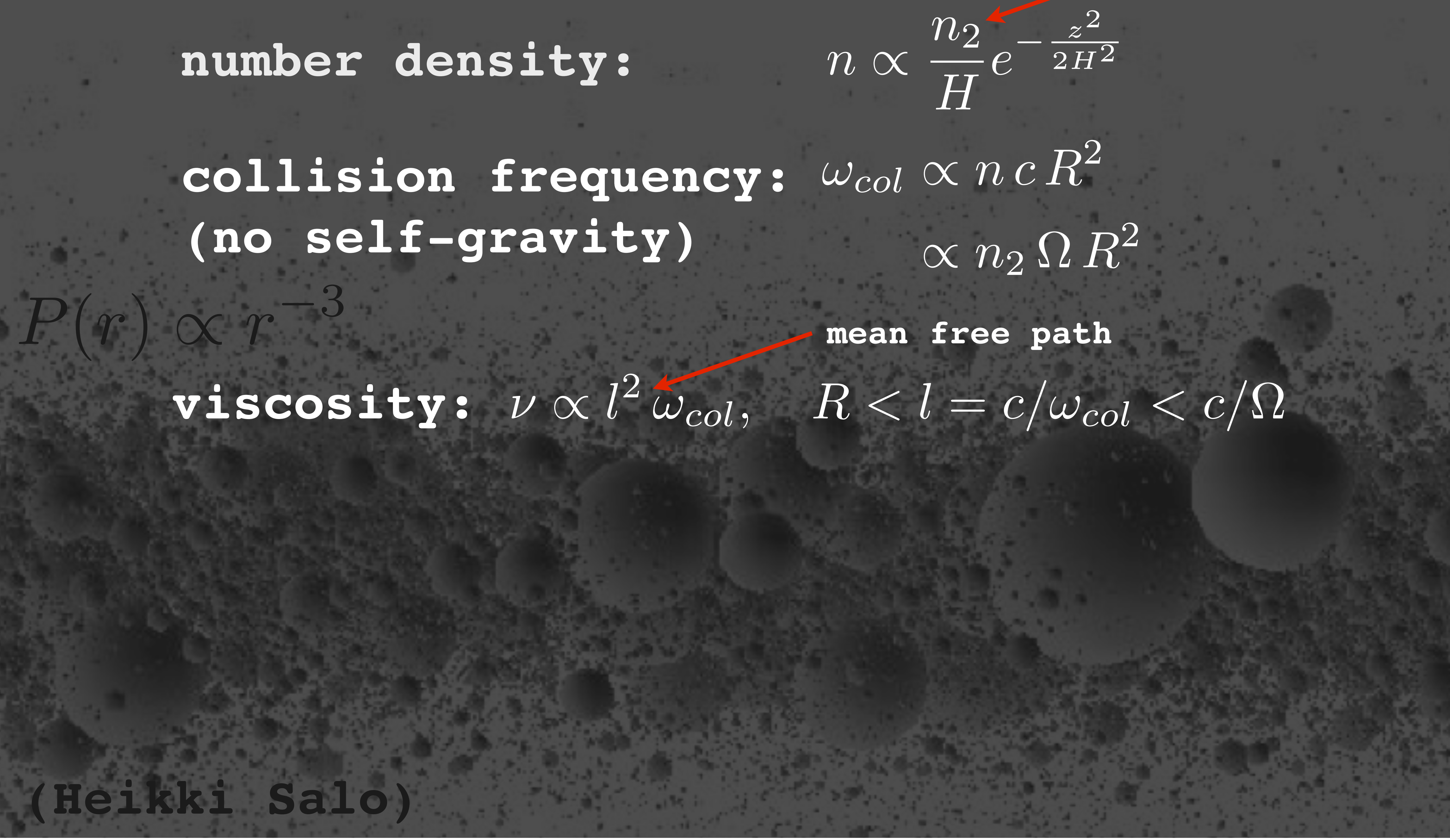
number density: $$n \propto \frac{n_2}{H} e^{-\frac{z^2}{2H^2}}$$

collision frequency: (no self-gravity) $$\omega_{col} \propto n\, c\, R^2$$
$$\propto n_2\, \Omega\, R^2$$

$$P(r) \propto r^{-3}$$

mean free path

viscosity: $\nu \propto l^2\, \omega_{col},\quad R < l = c/\omega_{col} < c/\Omega$



(Heikki Salo)

scale hight: H ~ c/Ω

(pressure vs vertical Saturn gravity)

surface number density

number density: $n \propto \frac{n_2}{H} e^{-\frac{z^2}{2H^2}}$

collision frequency: $\omega_{col} \propto n\, c\, R^2$

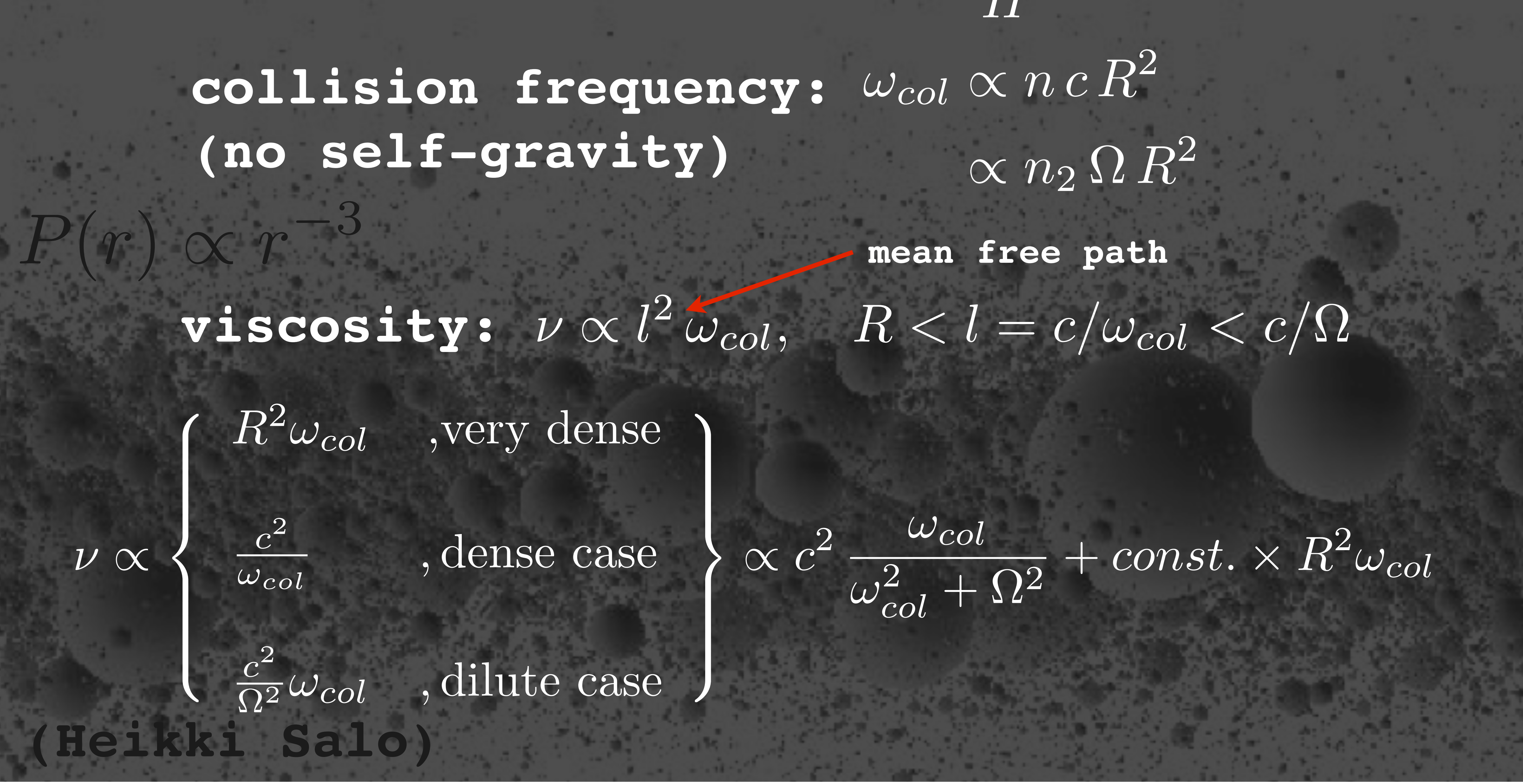
(no self-gravity) $\propto n_2\, \Omega\, R^2$

$P(r) \propto r^{-3}$

mean free path

viscosity: $\nu \propto l^2\, \omega_{col}, \quad R < l = c/\omega_{col} < c/\Omega$

$$\nu \propto \left\{ \begin{array}{ll} R^2\omega_{col} & \text{,very dense} \\ \frac{c^2}{\omega_{col}} & \text{, dense case} \\ \frac{c^2}{\Omega^2}\omega_{col} & \text{, dilute case} \end{array} \right\} \propto c^2 \frac{\omega_{col}}{\omega_{col}^2 + \Omega^2} + const. \times R^2\omega_{col}$$



(Heikki Salo)

scale hight: H ~ c/Ω

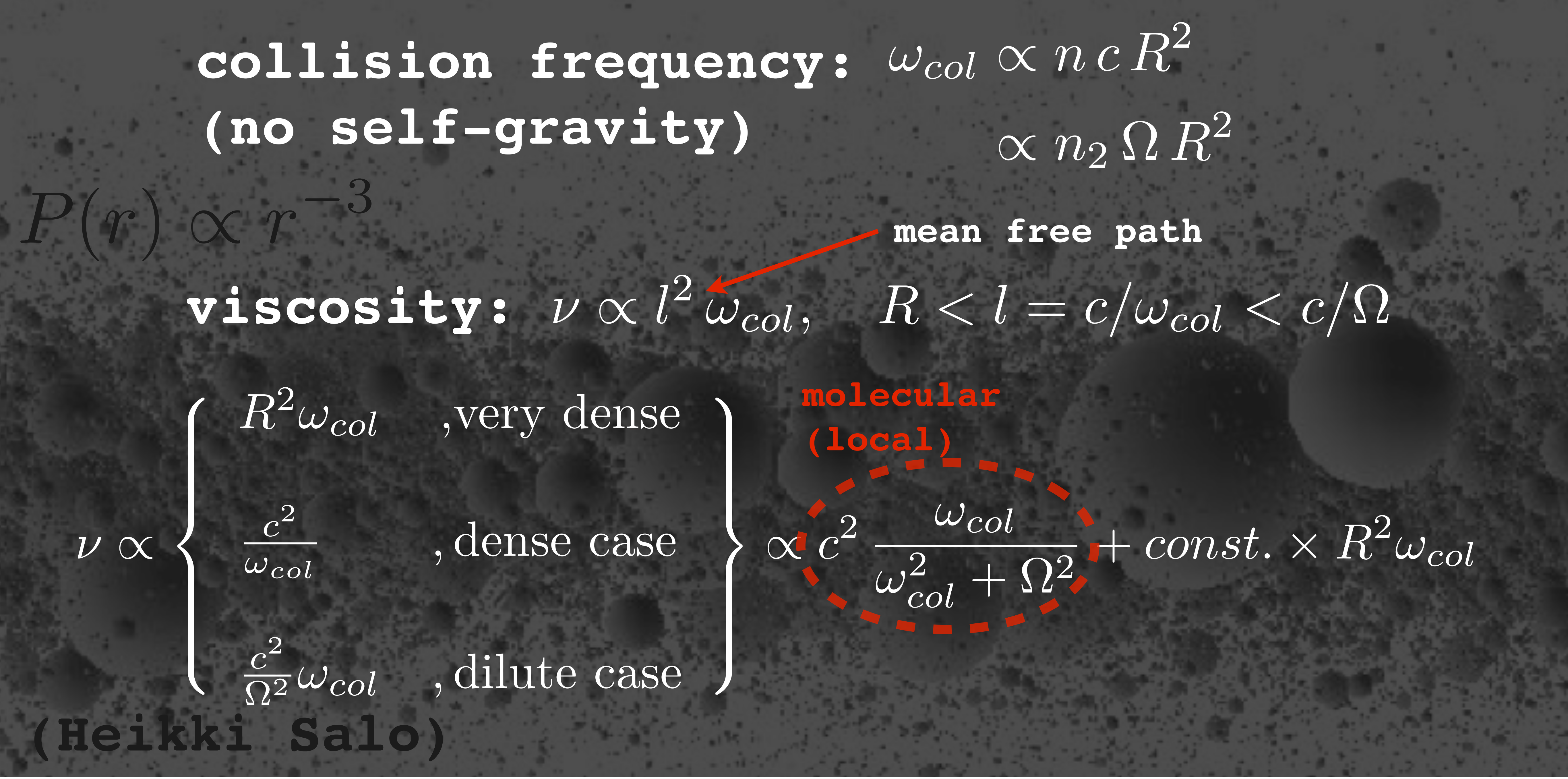
(pressure vs vertical Saturn gravity)

surface number density

number density: $$n \propto \frac{n_2}{H} e^{-\frac{z^2}{2H^2}}$$

collision frequency: $$\omega_{col} \propto n\, c\, R^2$$

(no self-gravity) $$\propto n_2\, \Omega\, R^2$$



$$P(r) \propto r^{-3}$$

mean free path

viscosity: $\nu \propto l^2\, \omega_{col}, \quad R < l = c/\omega_{col} < c/\Omega$

molecular (local)

$$\nu \propto \left\{ \begin{array}{ll} R^2 \omega_{col} & \text{,very dense} \\ \frac{c^2}{\omega_{col}} & \text{, dense case} \\ \frac{c^2}{\Omega^2}\omega_{col} & \text{, dilute case} \end{array} \right\} \propto c^2 \frac{\omega_{col}}{\omega_{col}^2 + \Omega^2} + const. \times R^2 \omega_{col}$$

(Heikki Salo)

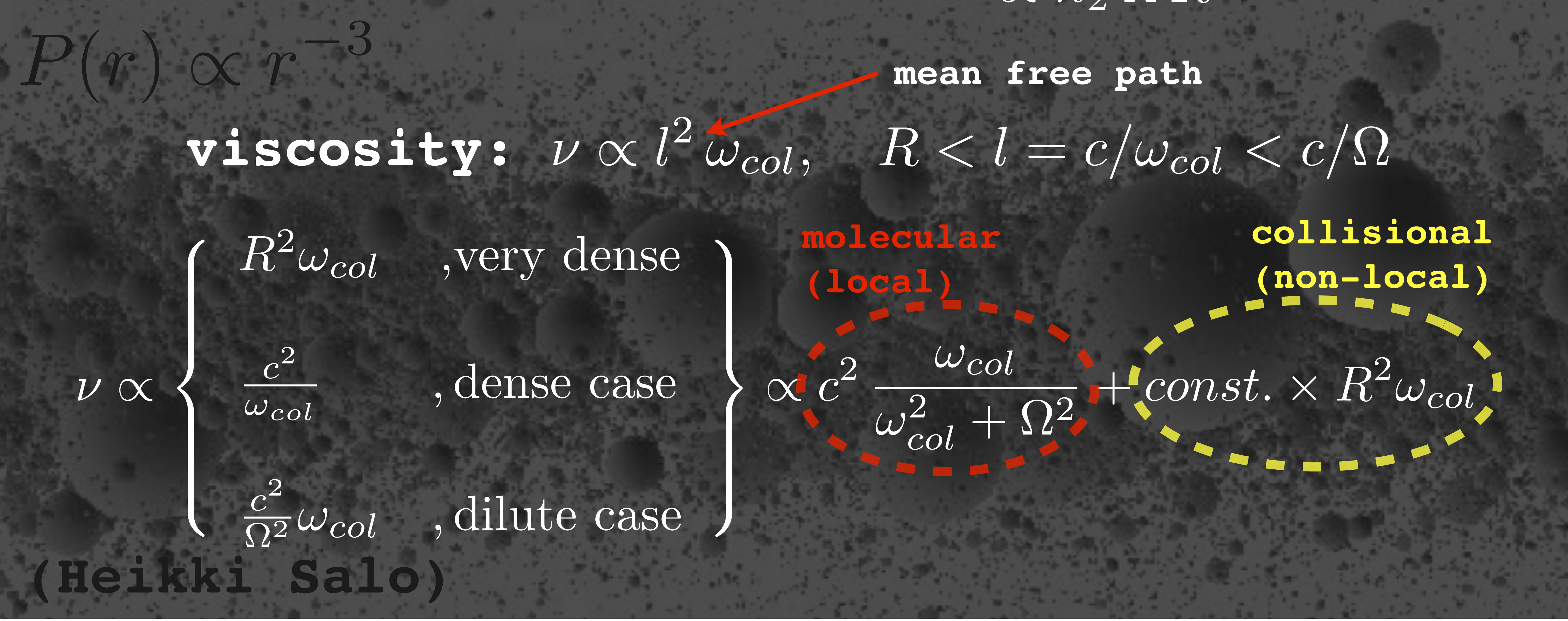
scale hight: H ~ c/Ω

(pressure vs vertical
Saturn gravity)

surface number density

number density: $$n \propto \frac{n_2}{H} e^{-\frac{z^2}{2H^2}}$$

collision frequency: $$\omega_{col} \propto n\, c\, R^2$$
(no self-gravity) $$\propto n_2\, \Omega\, R^2$$



$P(r) \propto r^{-3}$

mean free path

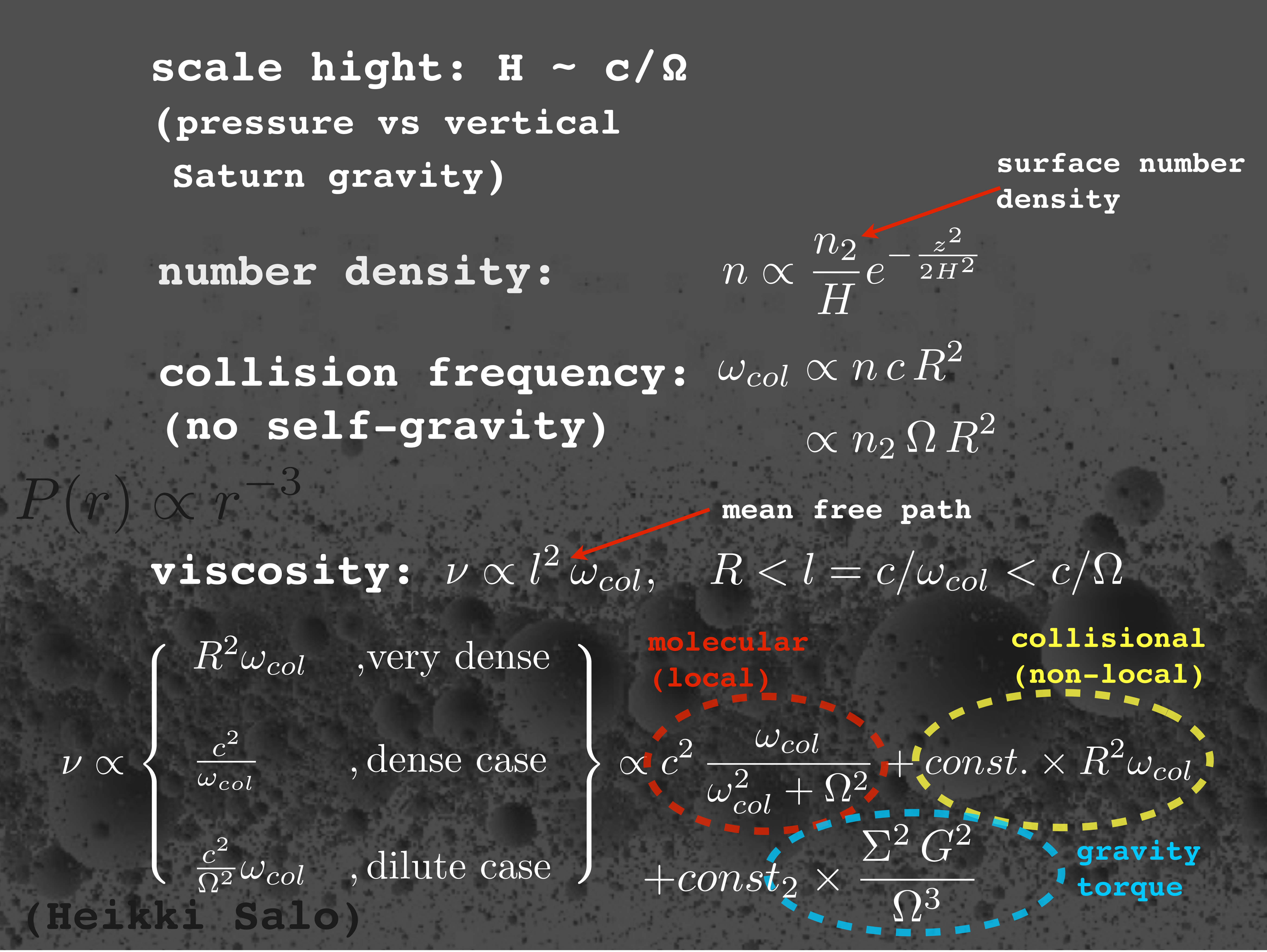
viscosity: $\nu \propto l^2\, \omega_{col}, \quad R < l = c/\omega_{col} < c/\Omega$

molecular (local)

collisional (non-local)

$$\nu \propto \left\{ \begin{array}{ll} R^2\omega_{col} & \text{,very dense} \\ \frac{c^2}{\omega_{col}} & \text{, dense case} \\ \frac{c^2}{\Omega^2}\omega_{col} & \text{, dilute case} \end{array} \right\} \propto c^2 \frac{\omega_{col}}{\omega_{col}^2 + \Omega^2} + const. \times R^2 \omega_{col}$$

(Heikki Salo)



scale hight: H ~ c/Ω

(pressure vs vertical
Saturn gravity)

surface number density

number density: $n \propto \frac{n_2}{H} e^{-\frac{z^2}{2H^2}}$

collision frequency: $\omega_{col} \propto n\, c\, R^2$

(no self-gravity) $\propto n_2\, \Omega\, R^2$

$P(r) \propto r^{-3}$

mean free path

viscosity: $\nu \propto l^2\, \omega_{col}, \quad R < l = c/\omega_{col} < c/\Omega$

molecular (local)

collisional (non-local)

$$\nu \propto \left\{ \begin{array}{ll} R^2 \omega_{col} & \text{,very dense} \\ \frac{c^2}{\omega_{col}} & \text{, dense case} \\ \frac{c^2}{\Omega^2}\omega_{col} & \text{, dilute case} \end{array} \right\} \propto c^2 \frac{\omega_{col}}{\omega_{col}^2 + \Omega^2} + const. \times R^2 \omega_{col}$$
$$+ const_2 \times \frac{\Sigma^2\, G^2}{\Omega^3}$$

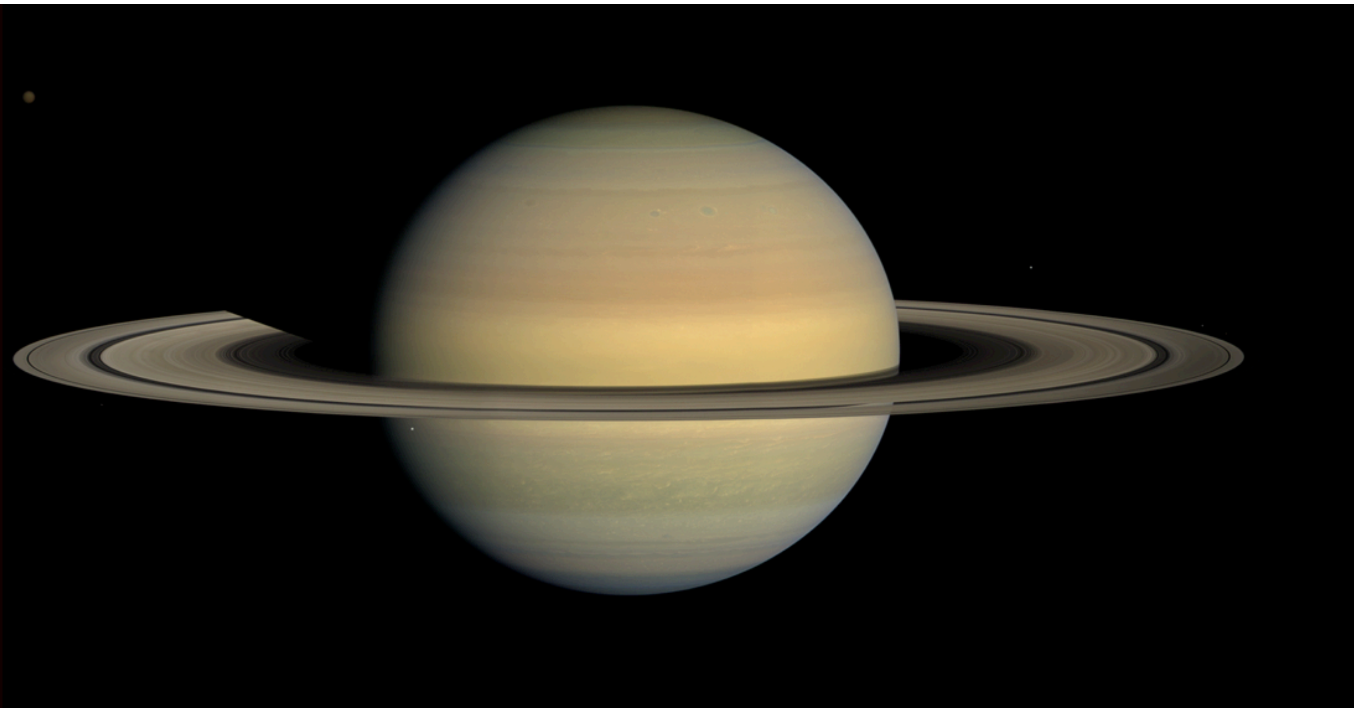
gravity torque

(Heikki Salo)

ring structure

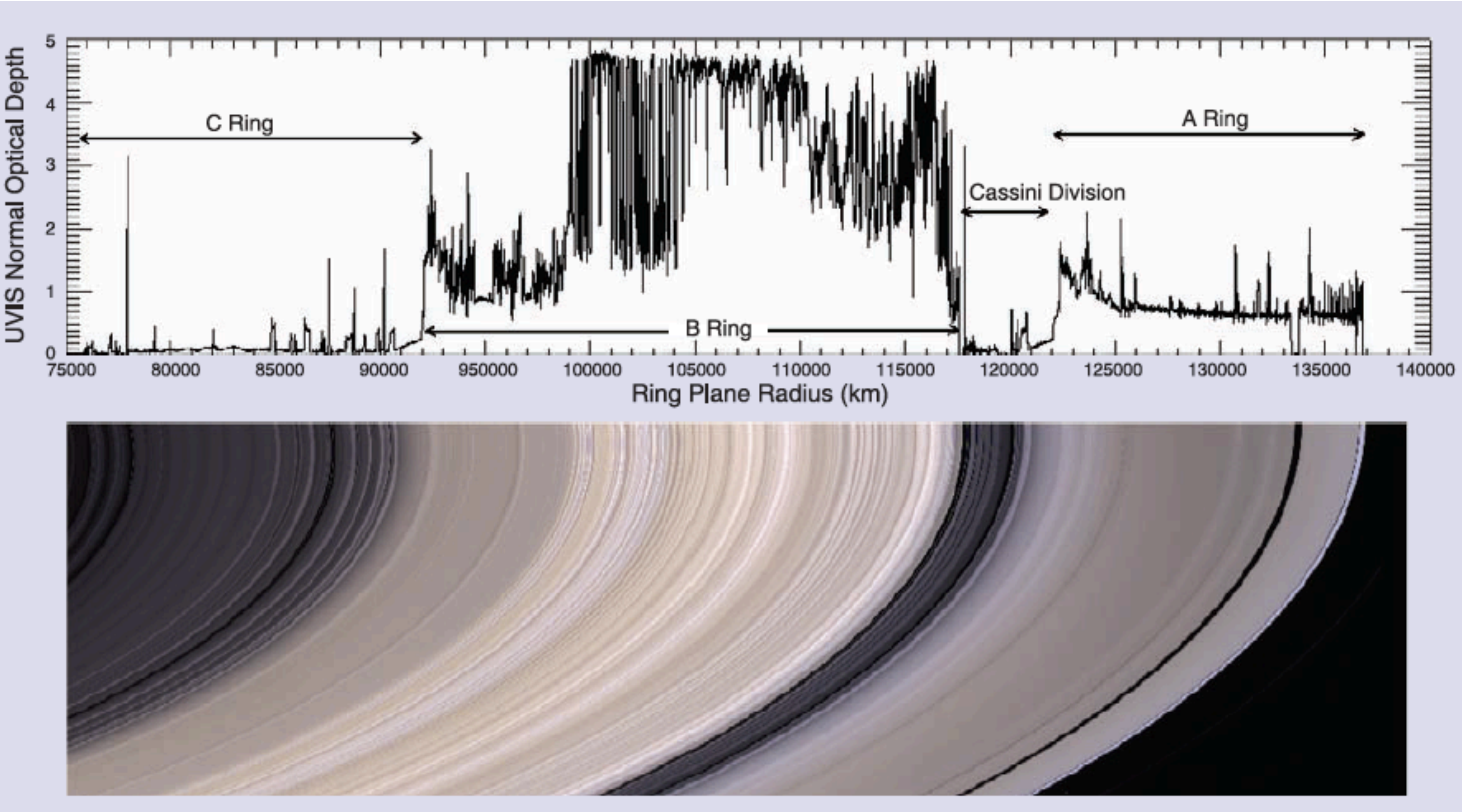
structure on all scales

first structure seen in the rings:
The Cassini Division



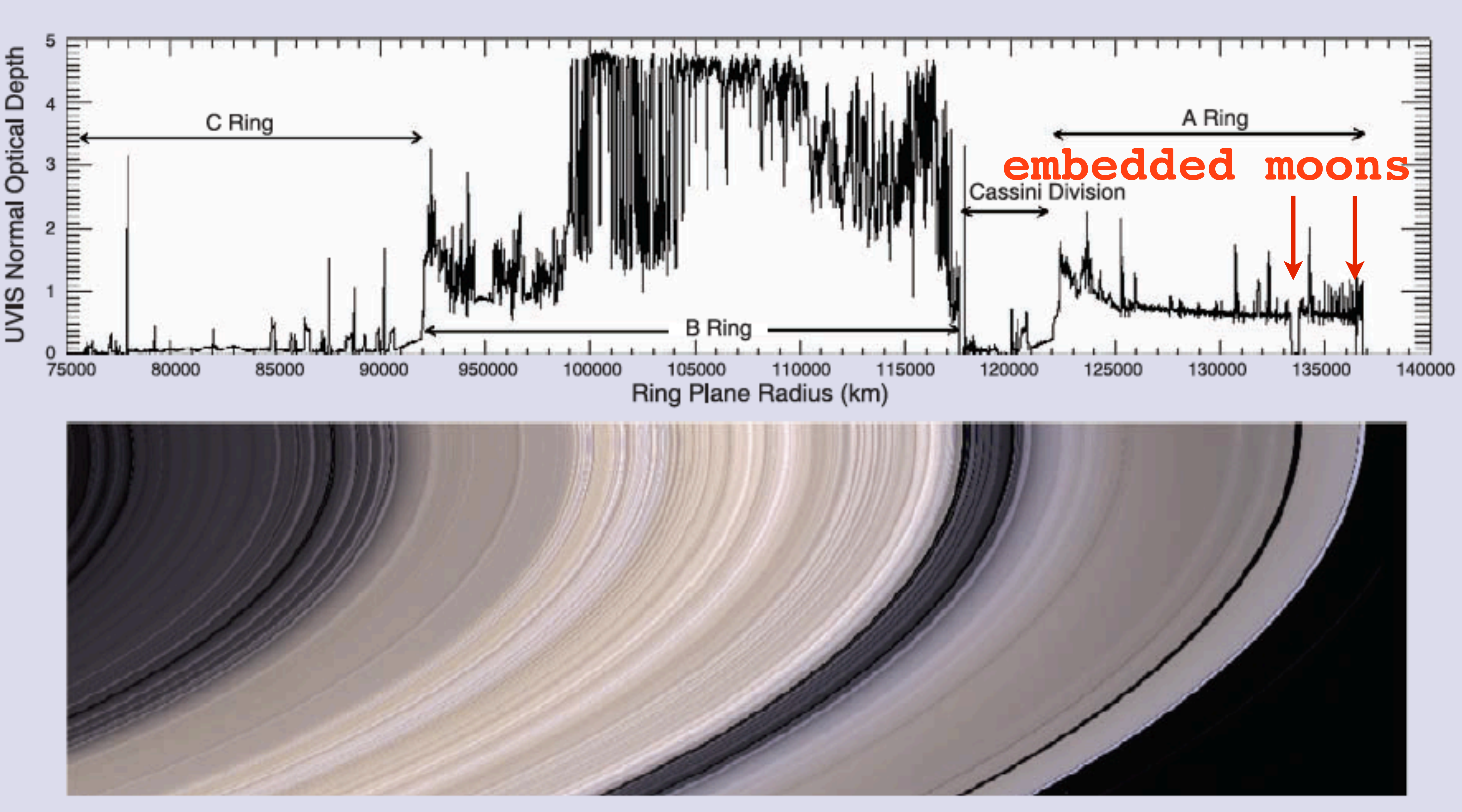
Giovanni Domenico Cassini

structure on all scales

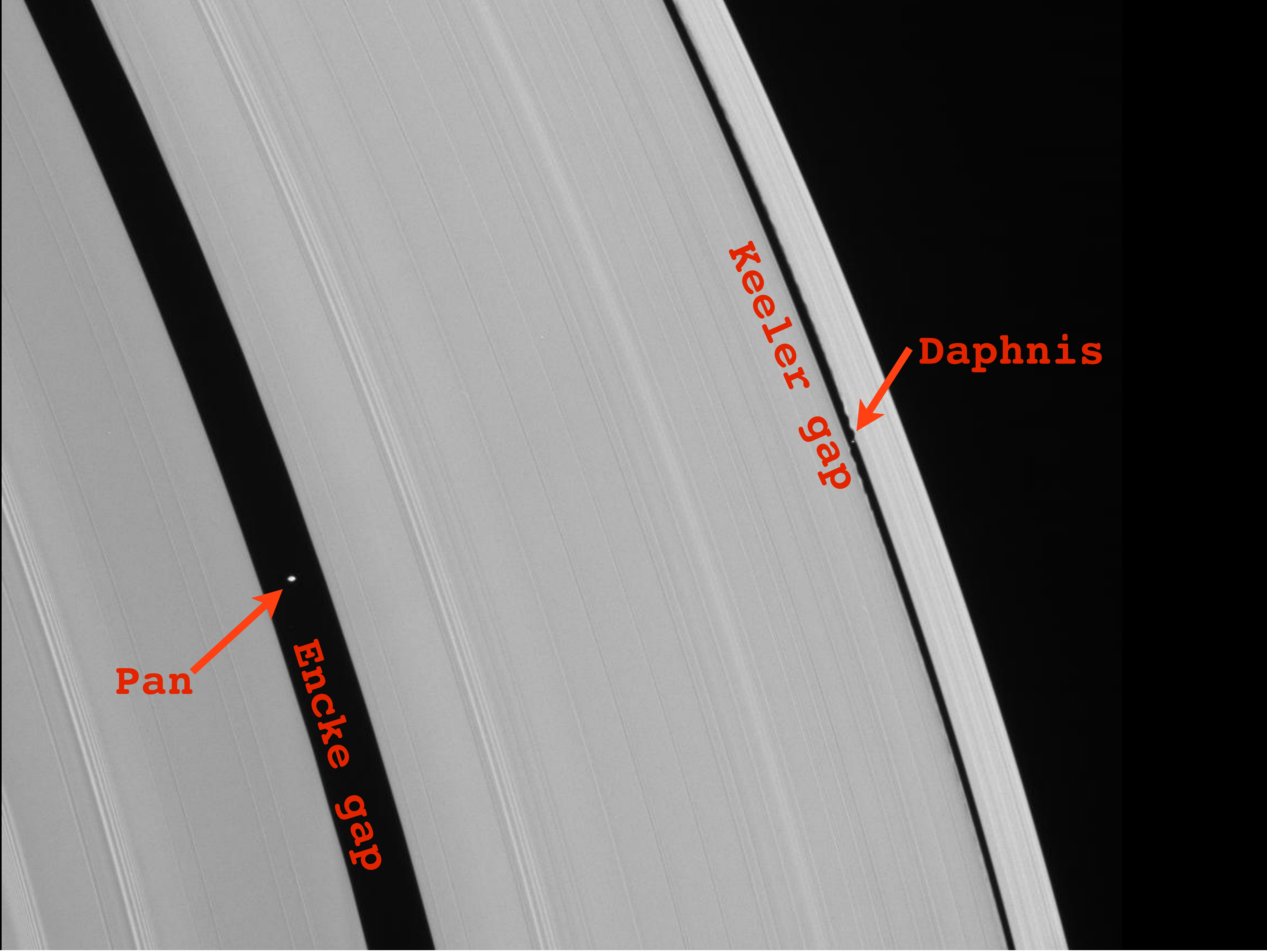


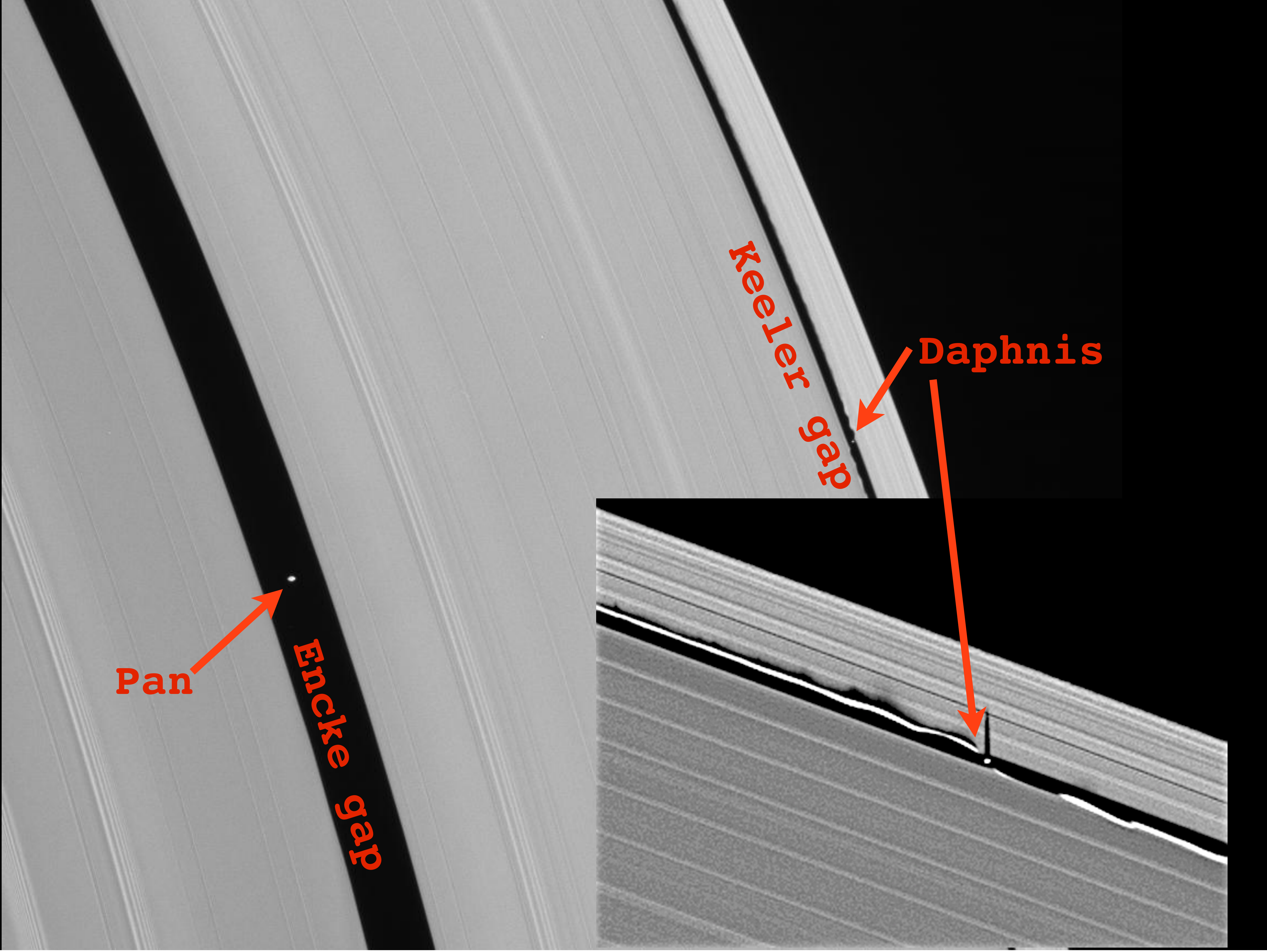
(from Cuzzi et al., Science, 2010)

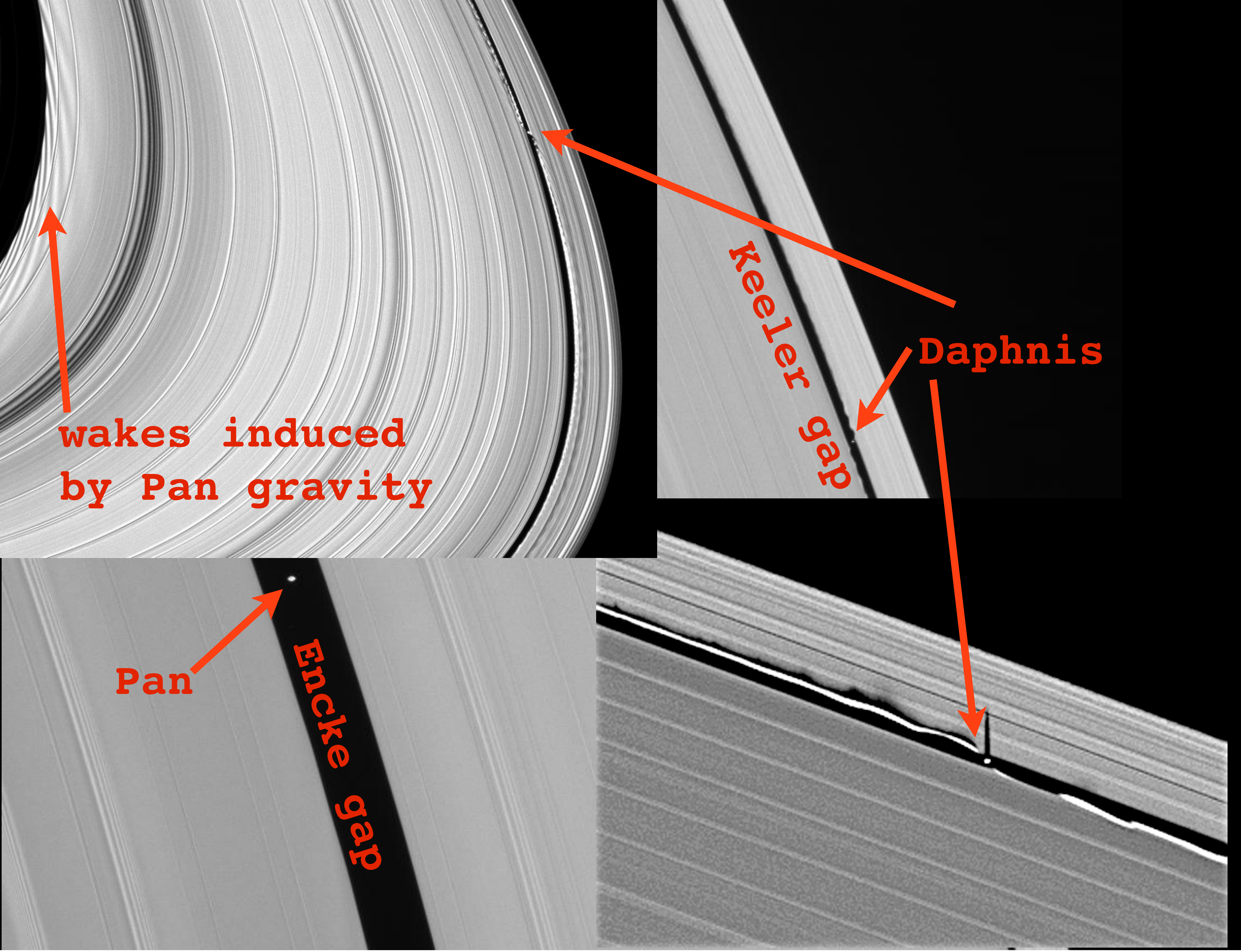
structure on all scales



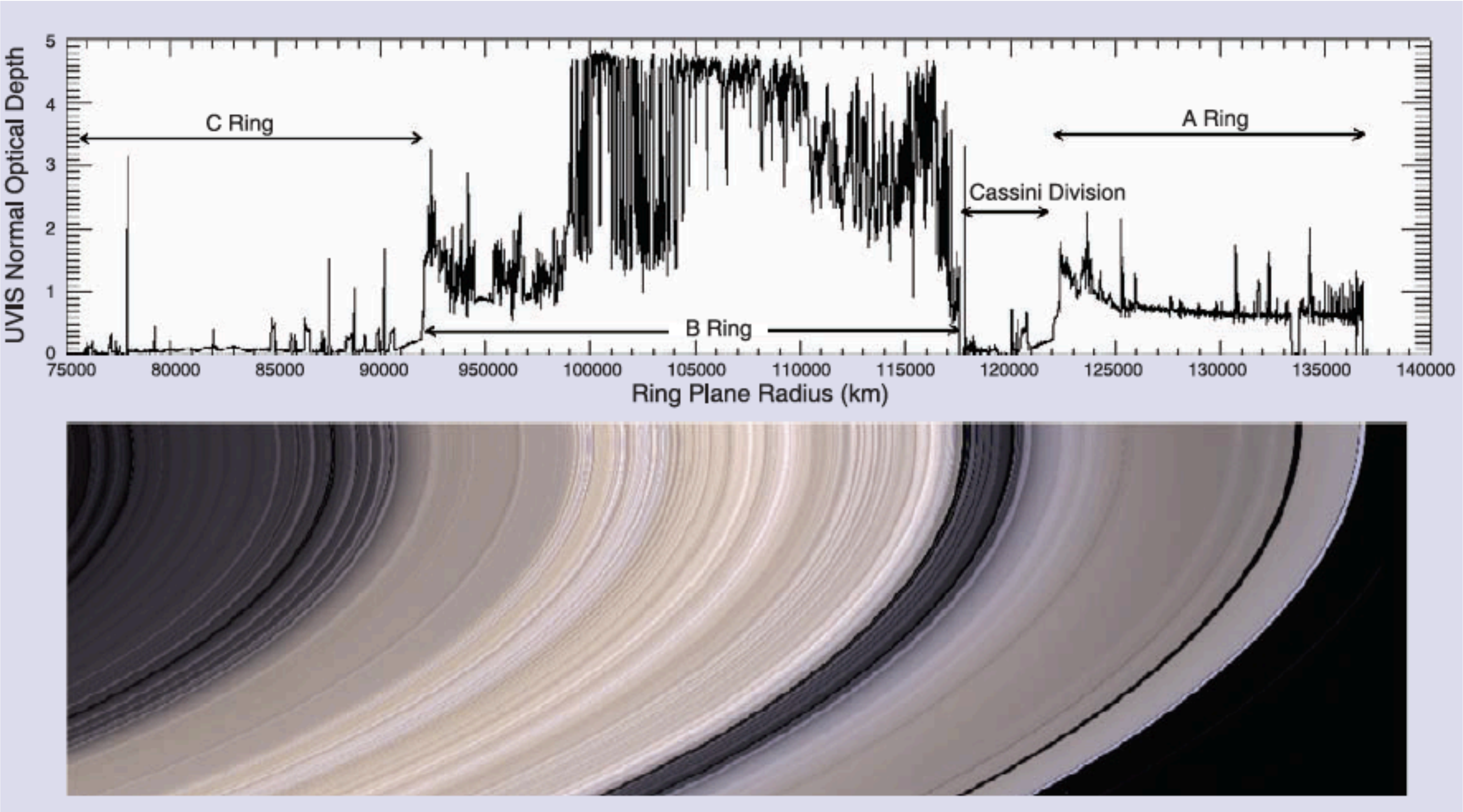
(from Cuzzi et al., Science, 2010)


Keeler gap
Daphnis
Pan
Encke gap


Keeler gap
Daphnis
Pan
Encke gap

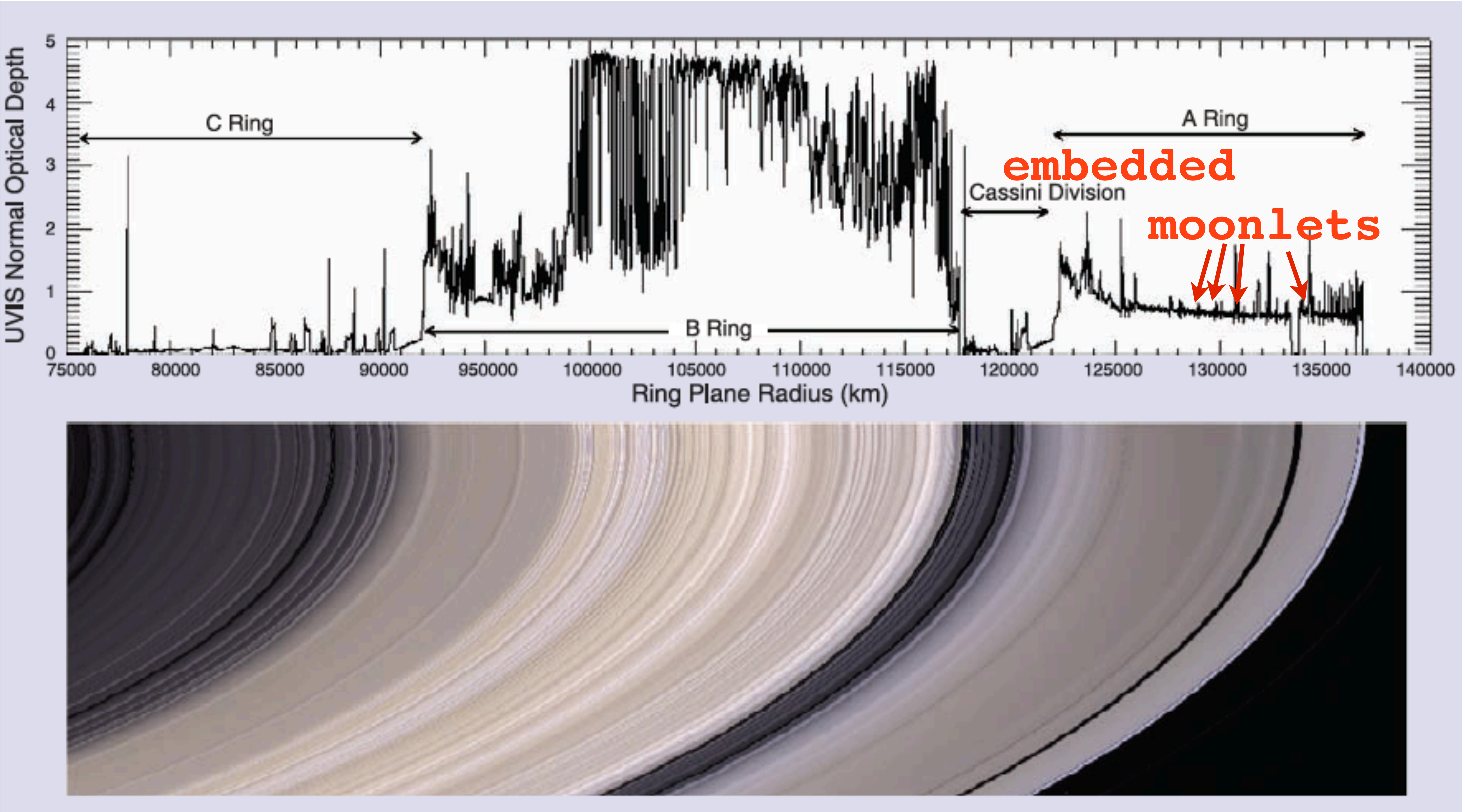

wakes induced by Pan gravity
Keeler gap
Daphnis
Pan
Encke gap

structure on all scales



(from Cuzzi et al., Science, 2010)

structure on all scales

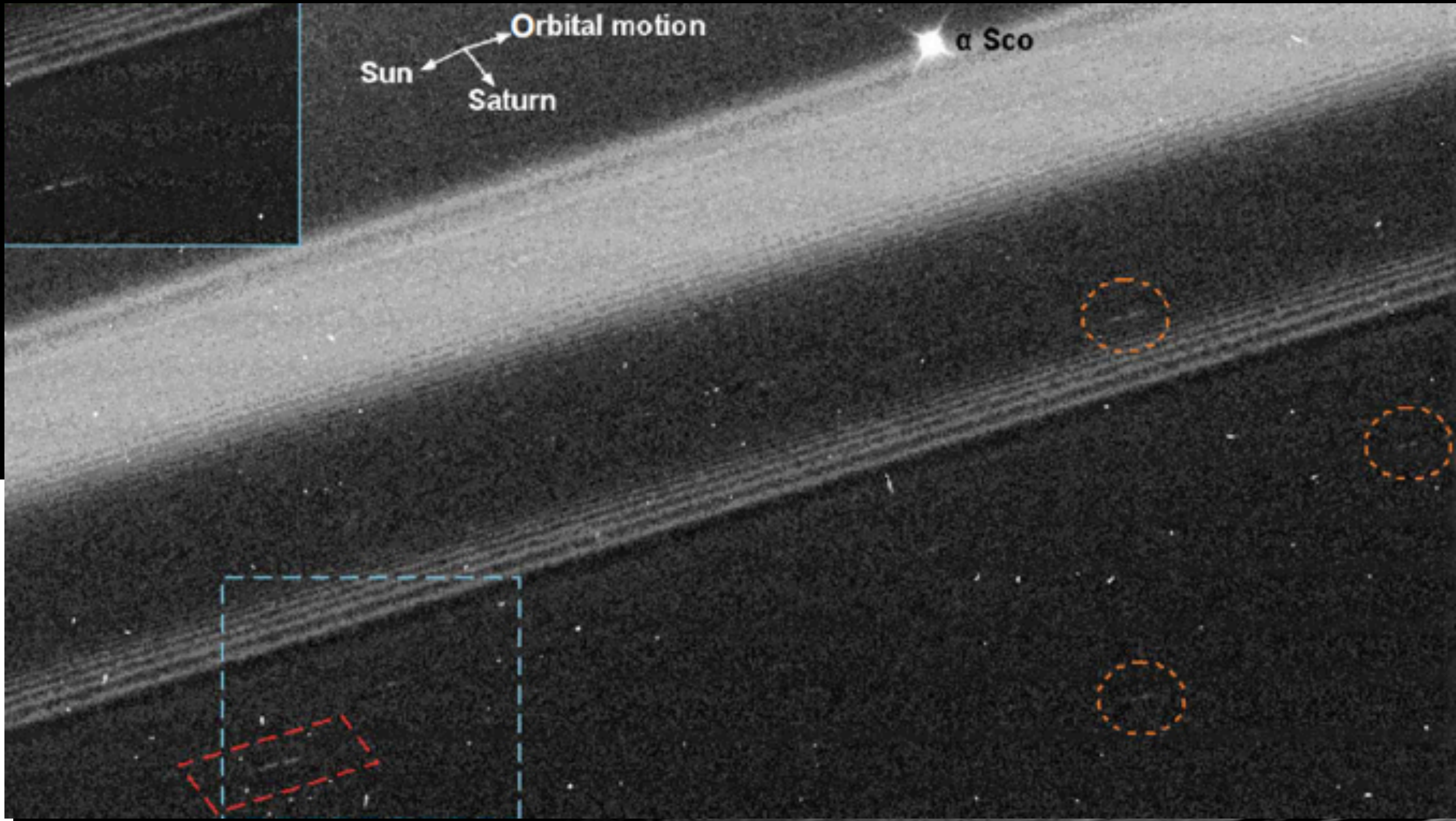


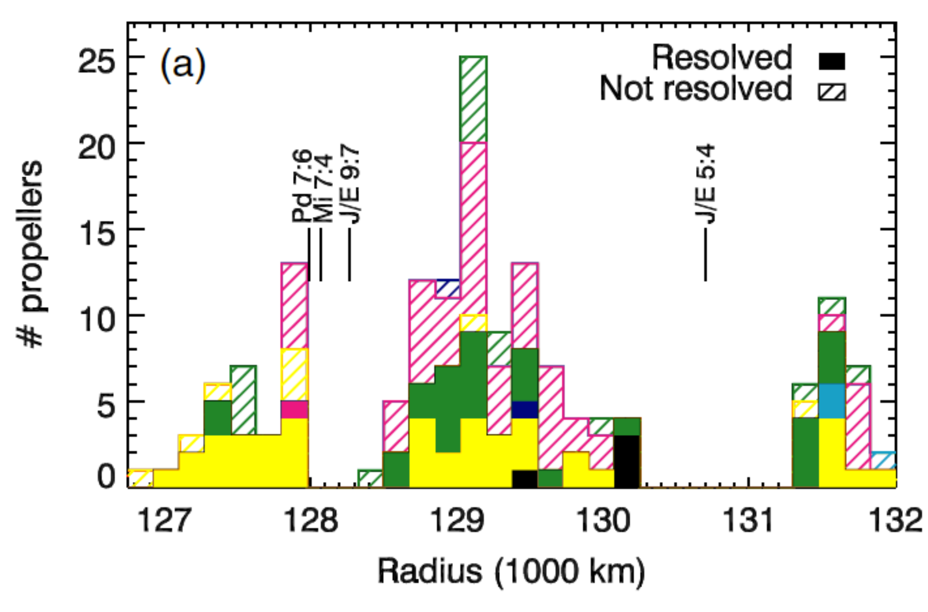
embedded moonlets

(from Cuzzi et al., Science, 2010)

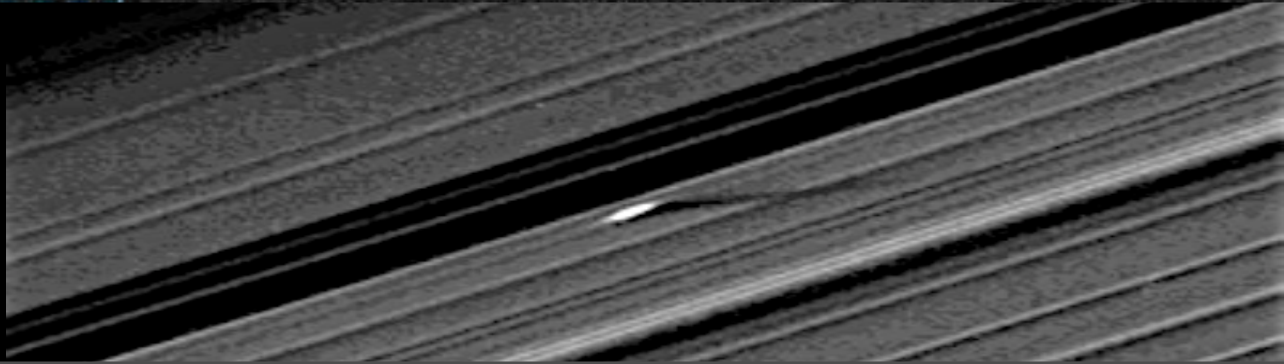
propellers

(Tiscareno et al., 2006, Nature, Sremcevic et al., 2007, Nature
Spahn & Sremcevic, 2000, A&A, Sremcevic et al, 2002, MNRS)

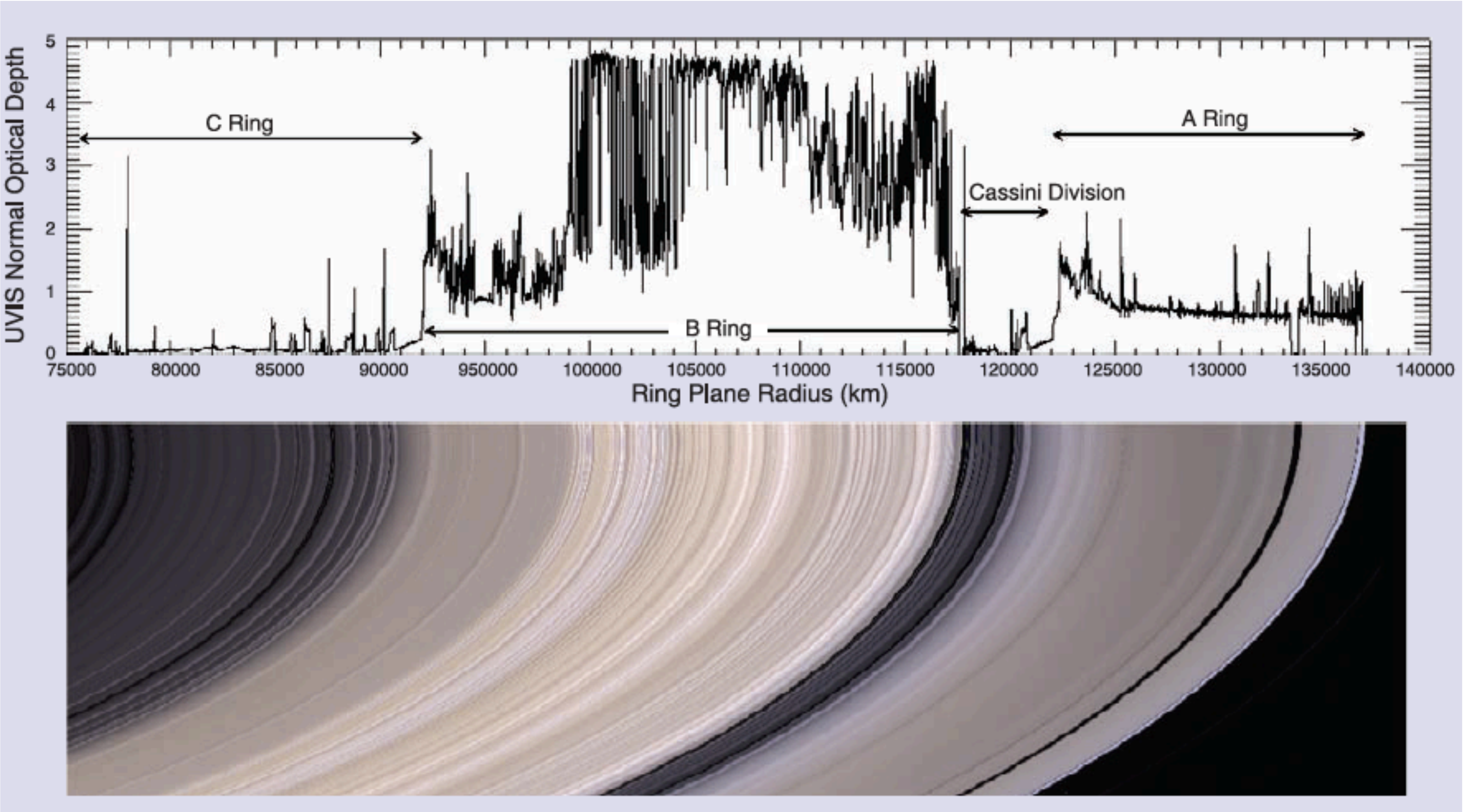




(Tiscareno et al., 2008, ApJ)

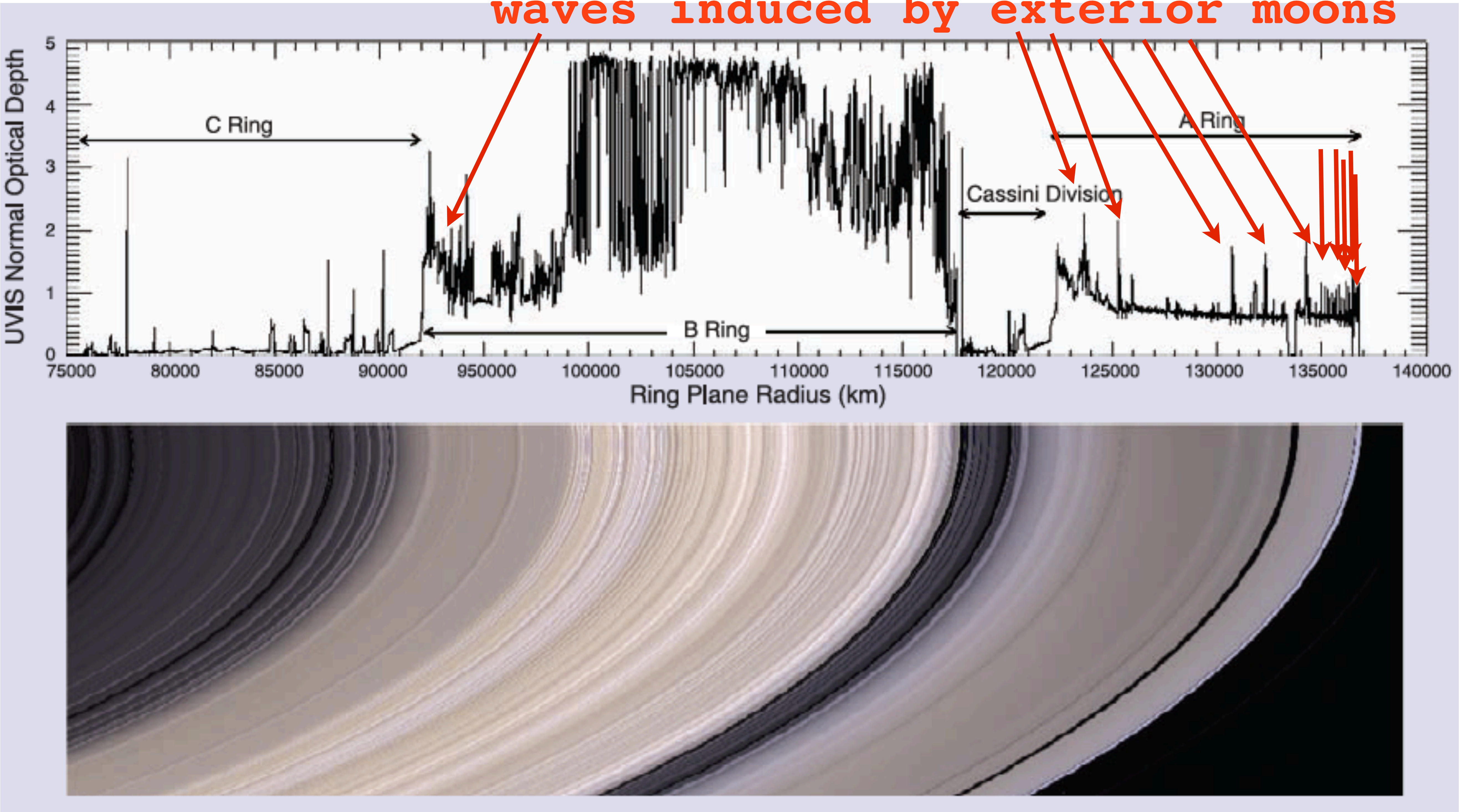


structure on all scales



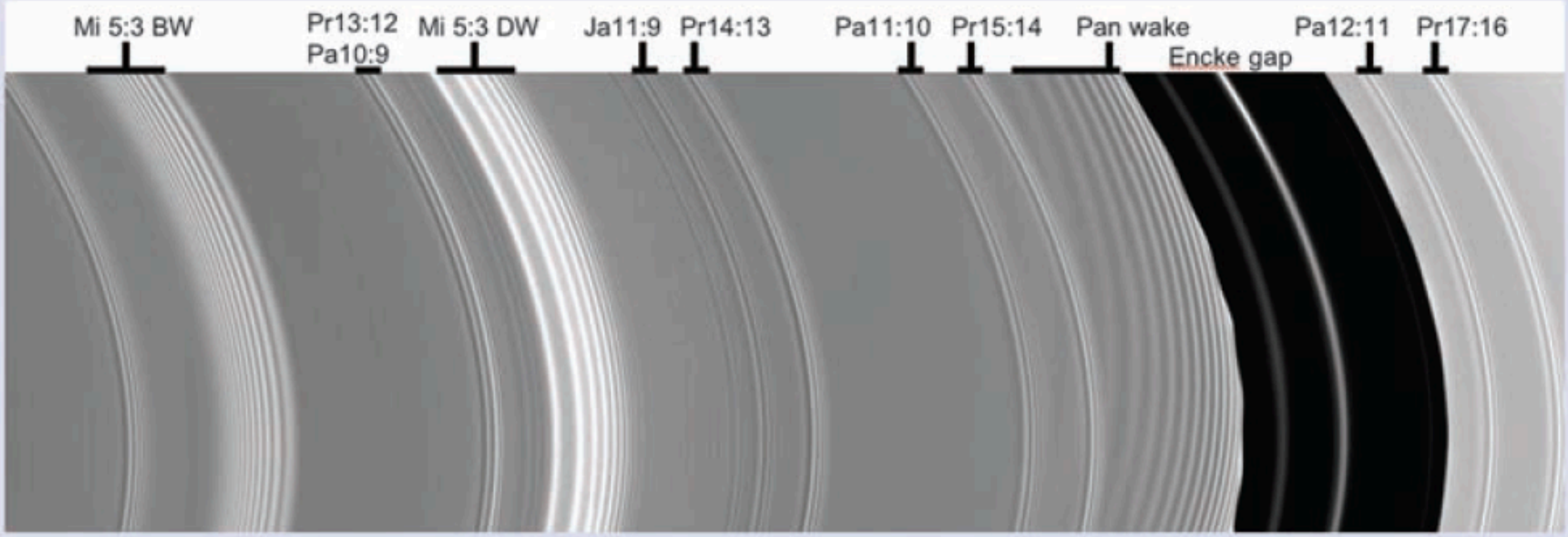
(from Cuzzi et al., Science, 2010)

structure on all scales



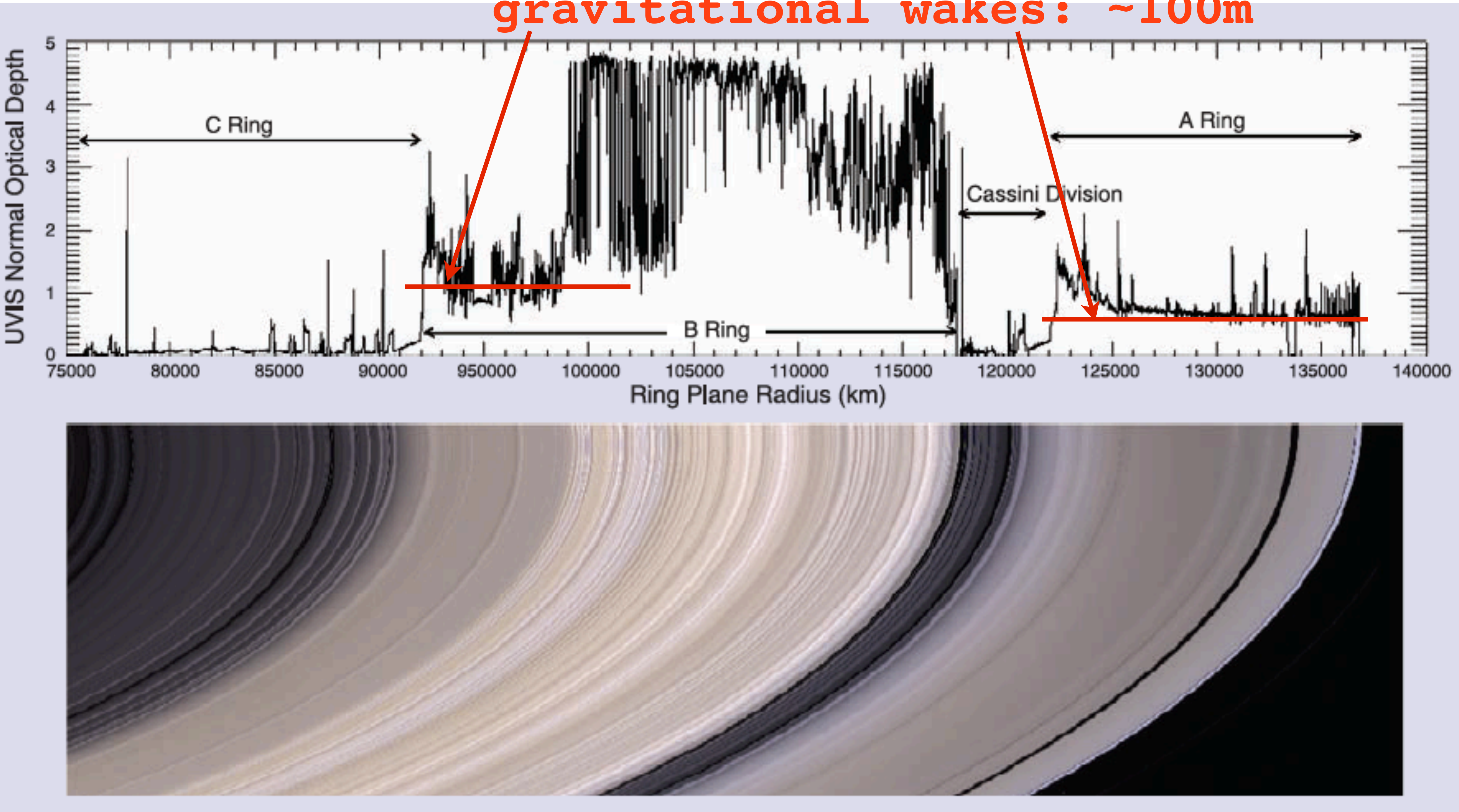
waves induced by exterior moons

(from Cuzzi et al., Science, 2010)



(from Cuzzi et al., Science, 2010)

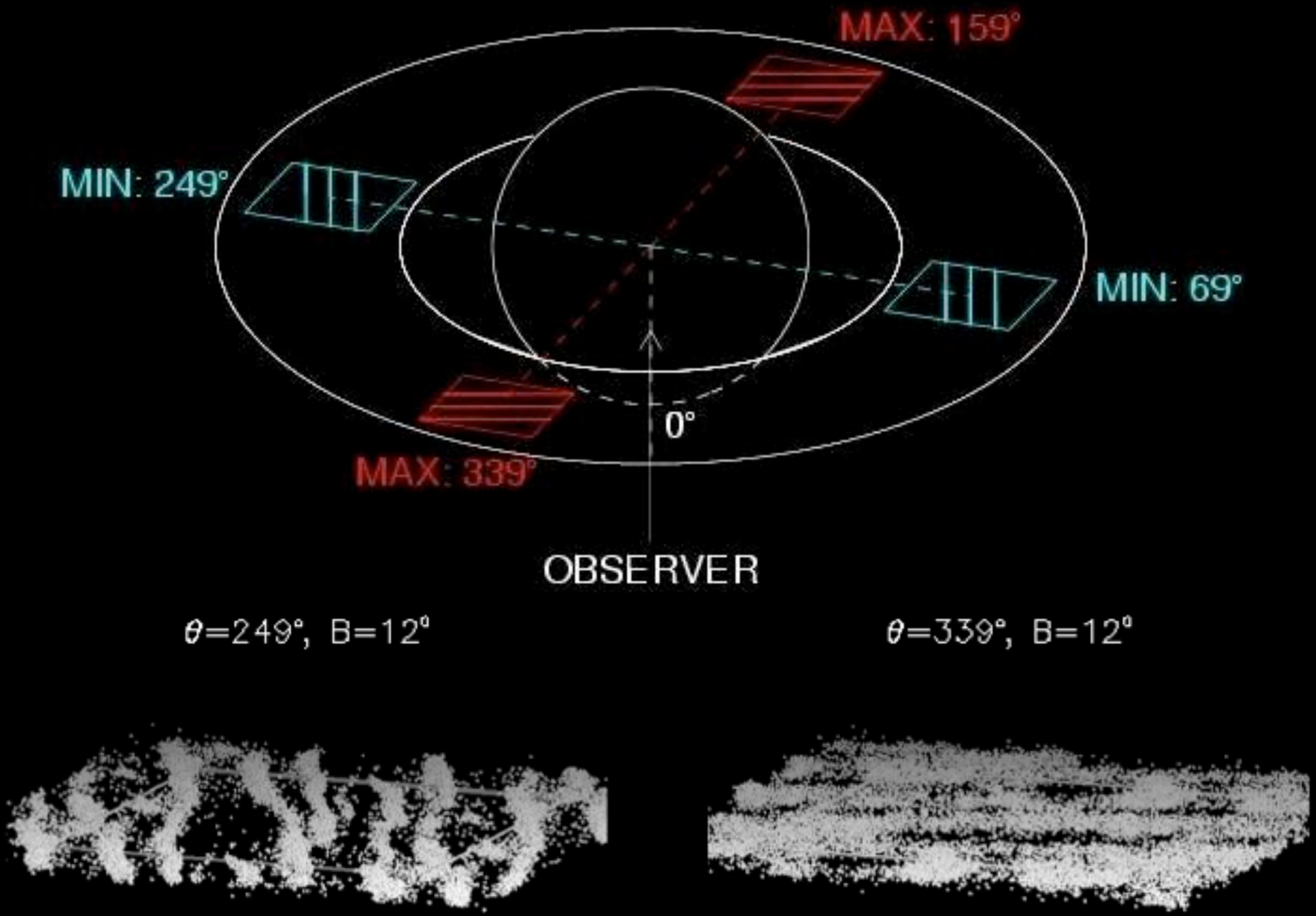
structure on all scales



gravitational wakes: ~100m

(from Cuzzi et al., Science, 2010)

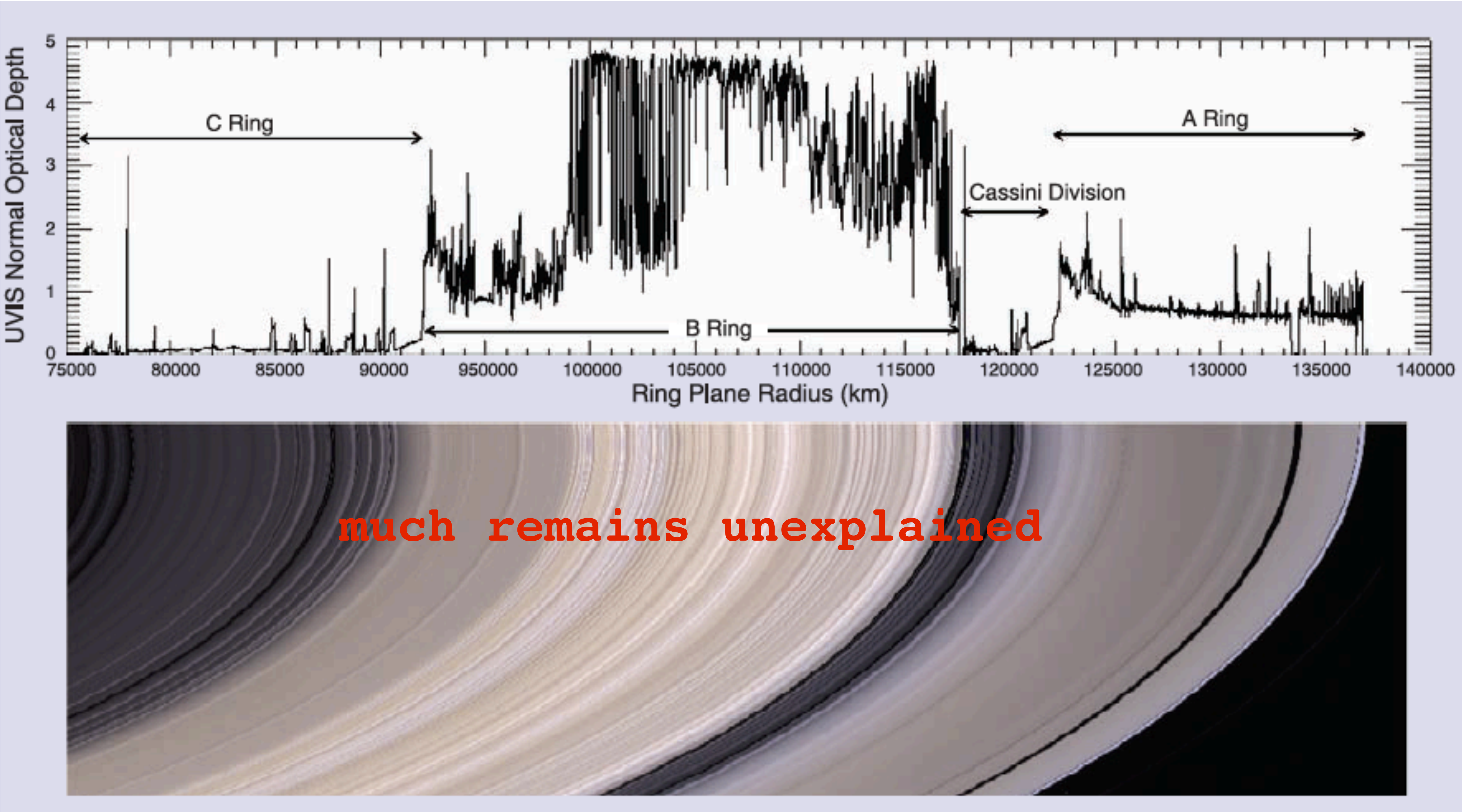
self-gravity wakes: brightness asymmetry



observation:

- Camichel 1958
  Franklin 1987
  Dones et al 1993
- HST
- CASSINI:
  VIMS, UVIS, ISS, RSS,
  CIRS

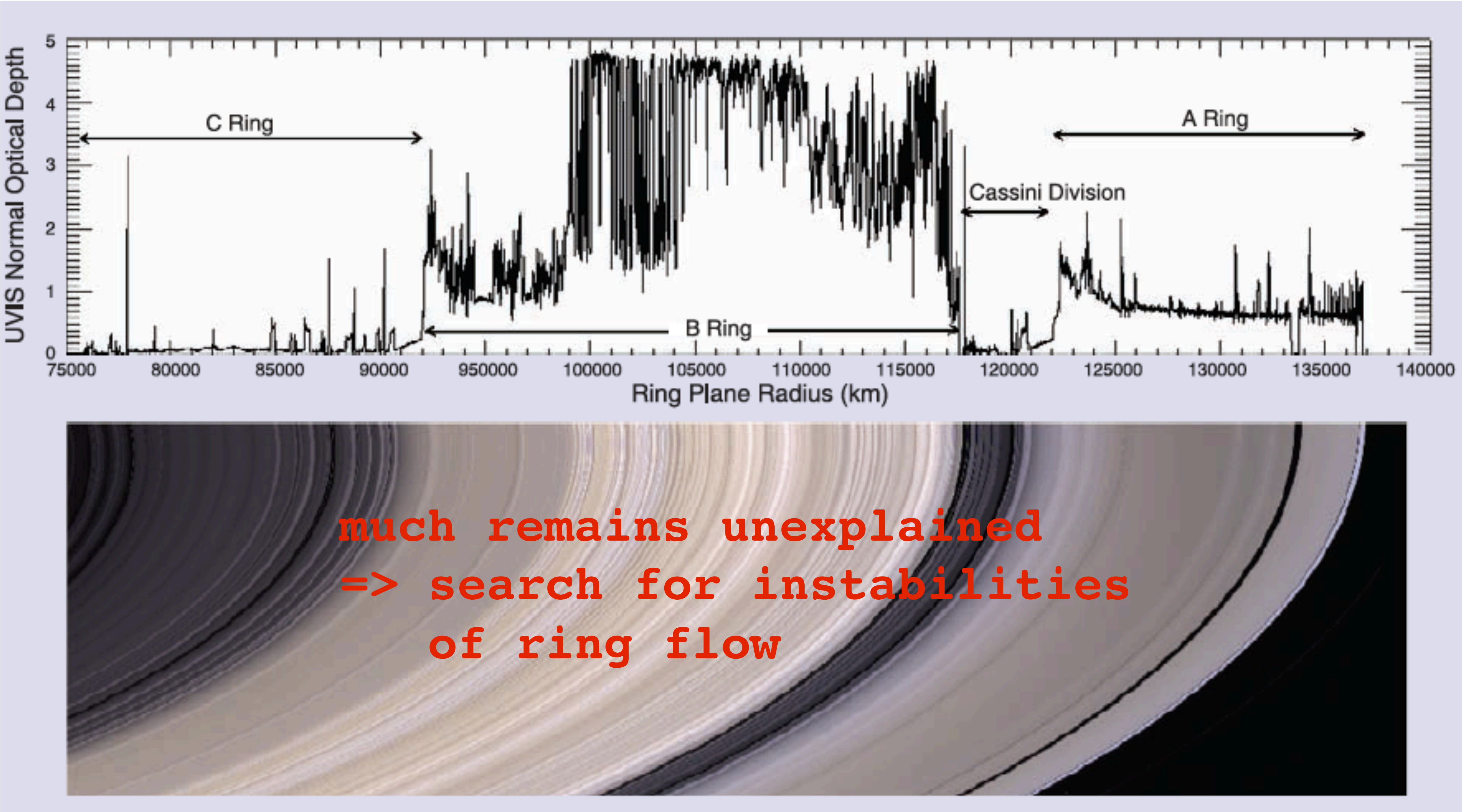
structure on all scales



much remains unexplained

(from Cuzzi et al., Science, 2010)

structure on all scales



much remains unexplained
=> search for instabilities
of ring flow

(from Cuzzi et al., Science, 2010)

Mass and Momentum Balance + Self Gravity

$$\begin{aligned}
\left(\frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r}\right)\sigma &= -\sigma \vec{\nabla}\cdot\vec{u} \\
\left(\frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r}\right)u_r - \frac{u_\varphi^2}{r} &= -\frac{\partial \Phi_{Planet}}{\partial r} - \frac{\partial \Phi_{Disk}}{\partial r} - \frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_r \\
\left(\frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r} + \frac{u_r}{r}\right)u_\varphi &= -\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_\varphi \\
\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi_{Disk}}{\partial r}\right) &= 4\pi G\,\sigma\,\delta(z)
\end{aligned}$$

Mass and Momentum Balance + Self Gravity

$$\begin{aligned}
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)\sigma &= -\sigma\vec{\nabla}\cdot\vec{u} \\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)u_r-\frac{u_\varphi^2}{r} &= -\frac{\partial\Phi_{Planet}}{\partial r}-\frac{\partial\Phi_{Disk}}{\partial r}-\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_r \\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}+\frac{u_r}{r}\right)u_\varphi &= -\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_\varphi \\
\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_{Disk}}{\partial r}\right) &= 4\pi G\,\sigma\,\delta(z)
\end{aligned}$$

linearize about $\Sigma = const, u = 0, v = 0$

$$u_\varphi \longrightarrow -\frac{3}{2}\Omega r + v$$

Mass and Momentum Balance + Self Gravity

$$\begin{aligned}
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)\sigma &= -\sigma\vec{\nabla}\cdot\vec{u}\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)u_r-\frac{u_\varphi^2}{r} &= -\frac{\partial\Phi_{Planet}}{\partial r}-\frac{\partial\Phi_{Disk}}{\partial r}-\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_r\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}+\frac{u_r}{r}\right)u_\varphi &= -\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_\varphi\\
\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_{Disk}}{\partial r}\right) &= 4\pi G\,\sigma\,\delta(z)
\end{aligned}$$

linearize about $\Sigma = const, u = 0, v = 0$

$$u_\varphi \longrightarrow -\frac{3}{2}\Omega r + v$$

$$\begin{aligned}
\dot{\sigma} &= -\Sigma\, u'\\
\dot{u} &= 2\Omega\, v - \left(\frac{1}{\Sigma}\left.\frac{\partial p}{\partial\sigma}\right|_0 - \frac{2\,\pi\, G}{|k|}\right)\sigma' + \frac{\alpha}{\Sigma}\eta_0 u''\\
\dot{v} &= -\frac{\Omega}{2}u + \frac{1}{\Sigma}\left(\eta_0\, v'' - \frac{3}{2}\Omega\left.\frac{\partial\eta}{\partial\sigma}\right|_0\sigma'\right)
\end{aligned}$$

Mass and Momentum Balance + Self Gravity

$$\begin{aligned}
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)\sigma &= -\sigma\vec{\nabla}\cdot\vec{u}\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)u_r-\frac{u_\varphi^2}{r} &= -\frac{\partial\Phi_{Planet}}{\partial r}-\frac{\partial\Phi_{Disk}}{\partial r}-\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_r\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}+\frac{u_r}{r}\right)u_\varphi &= -\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_\varphi\\
\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_{Disk}}{\partial r}\right) &= 4\pi G\,\sigma\,\delta(z)
\end{aligned}$$

linearize about $\Sigma = const, u = 0, v = 0$

radial modes

comoving rotating frame

$$\begin{aligned}
\dot{\sigma} &= -\Sigma\, u\\
\dot{u} &= 2\Omega\, v-\left(\frac{1}{\Sigma}\left.\frac{\partial p}{\partial\sigma}\right|_0-\frac{2\,\pi\,G}{|k|}\right)\sigma'+\frac{\alpha}{\Sigma}\eta_0 u''\\
\dot{v} &= -\frac{\Omega}{2}u+\frac{1}{\Sigma}\left(\eta_0\, v''-\frac{3}{2}\Omega\left.\frac{\partial\eta}{\partial\sigma}\right|_0\sigma'\right)
\end{aligned}$$

Mass and Momentum Balance + Self Gravity

$$\begin{aligned}
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)\sigma &= -\sigma\vec{\nabla}\cdot\vec{u}\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)u_r-\frac{u_\varphi^2}{r} &= -\frac{\partial\Phi_{Planet}}{\partial r}-\frac{\partial\Phi_{Disk}}{\partial r}-\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_r\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}+\frac{u_r}{r}\right)u_\varphi &= -\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_\varphi\\
\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_{Disk}}{\partial r}\right) &= 4\pi G\,\sigma\,\delta(z)
\end{aligned}$$

linearize about $\Sigma = const, u = 0, v = 0$

radial modes

comoving rotating frame

hydrodynamic (newtonian) stress, pressure

$$\begin{aligned}
\dot{\sigma} &= -\Sigma\,u'\\
\dot{u} &= 2\Omega\,v-\left(\frac{1}{\Sigma}\left.\frac{\partial p}{\partial\sigma}\right|_0-\frac{2\,\pi\,G}{|k|}\right)\sigma'+\frac{\alpha}{\Sigma}\eta_0 u''\\
\dot{v} &= -\frac{\Omega}{2}u+\frac{1}{\Sigma}\left(\eta_0\,v''-\frac{3}{2}\Omega\left.\frac{\partial\eta}{\partial\sigma}\right|_0\sigma'\right)
\end{aligned}$$

$$\begin{aligned}
P_{rr} &= p-2\eta\frac{\partial u_r}{\partial r}+\left(\frac{2}{3}\eta-\xi\right)\vec{\nabla}\cdot\vec{u}\\
P_{r\varphi} &= -\eta\left(\frac{\partial u_\varphi}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial\varphi}-\frac{u_\varphi}{r}\right)
\end{aligned}$$

Mass and Momentum Balance + Self Gravity

$$\begin{aligned}
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)\sigma &= -\sigma\vec{\nabla}\cdot\vec{u}\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)u_r-\frac{u_\varphi^2}{r} &= -\frac{\partial\Phi_{Planet}}{\partial r}-\frac{\partial\Phi_{Disk}}{\partial r}-\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_r\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}+\frac{u_r}{r}\right)u_\varphi &= -\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_\varphi\\
\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_{Disk}}{\partial r}\right) &= 4\pi G\,\sigma\,\delta(z)
\end{aligned}$$

linearize about $\Sigma = const, u = 0, v = 0$

radial modes

comoving rotating frame

hydrodynamic (newtonian) stress, pressure

$$\begin{aligned}
\dot{\sigma} &= -\Sigma\,u'\\
\dot{u} &= 2\Omega\,v-\left(\frac{1}{\Sigma}\left.\frac{\partial p}{\partial\sigma}\right|_0-\frac{2\,\pi\,G}{|k|}\right)\sigma'+\frac{\alpha}{\Sigma}\eta_0 u''\\
\dot{v} &= -\frac{\Omega}{2}u+\frac{1}{\Sigma}\left(\eta_0\,v''-\frac{3}{2}\Omega\left.\frac{\partial\eta}{\partial\sigma}\right|_0\sigma'\right)
\end{aligned}$$

$$\begin{aligned}
P_{rr} &= p-2\eta\frac{\partial u_r}{\partial r}+\left(\frac{2}{3}\eta-\xi\right)\vec{\nabla}\cdot\vec{u}\\
P_{r\varphi} &= -\eta\left(\frac{\partial u_\varphi}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial\varphi}-\frac{u_\varphi}{r}\right)
\end{aligned}$$

shear viscosity

bulk viscosity

Mass and Momentum Balance + Self Gravity

$$\begin{aligned}
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)\sigma &= -\sigma\vec{\nabla}\cdot\vec{u}\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)u_r-\frac{u_\varphi^2}{r} &= -\frac{\partial\Phi_{Planet}}{\partial r}-\frac{\partial\Phi_{Disk}}{\partial r}-\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_r\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}+\frac{u_r}{r}\right)u_\varphi &= -\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_\varphi\\
\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_{Disk}}{\partial r}\right) &= 4\pi G\,\sigma\,\delta(z)
\end{aligned}$$

linearize about $\Sigma = const, u = 0, v = 0$

radial modes

comoving rotating frame

hydrodynamic (newtonian) stress, pressure

$$\begin{aligned}
\dot{\sigma} &= -\Sigma\,u'\\
\dot{u} &= 2\Omega\,v-\left(\frac{1}{\Sigma}\left.\frac{\partial p}{\partial\sigma}\right|_0-\frac{2\,\pi\,G}{|k|}\right)\sigma'+\frac{\alpha}{\Sigma}\eta_0\,u''\\
\dot{v} &= -\frac{\Omega}{2}u+\frac{1}{\Sigma}\left(\eta_0\,v''-\frac{3}{2}\Omega\left.\frac{\partial\eta}{\partial\sigma}\right|_0\sigma'\right)
\end{aligned}$$

$$\begin{aligned}
P_{rr} &= p-2\eta\frac{\partial u_r}{\partial r}+\left(\frac{2}{3}\eta-\xi\right)\vec{\nabla}\cdot\vec{u}\\
P_{r\varphi} &= -\eta\left(\frac{\partial u_\varphi}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial\varphi}-\frac{u_\varphi}{r}\right)
\end{aligned}$$

$$\alpha=\frac{4}{3}+\frac{\xi_0}{\eta_0}=const$$

shear viscosity

bulk viscosity

Mass and Momentum Balance + Self Gravity

$$\begin{aligned}
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)\sigma &= -\sigma\vec{\nabla}\cdot\vec{u}\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)u_r-\frac{u_\varphi^2}{r} &= -\frac{\partial\Phi_{Planet}}{\partial r}-\frac{\partial\Phi_{Disk}}{\partial r}-\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_r\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}+\frac{u_r}{r}\right)u_\varphi &= -\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_\varphi\\
\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_{Disk}}{\partial r}\right) &= 4\pi G\,\sigma\,\delta(z)
\end{aligned}$$

Subscript 0:
steady state

linearize about $\Sigma = const, u = 0, v = 0$

radial modes
comoving rotating frame
hydrodynamic (newtonian) stress, pressure

$$\begin{aligned}
\dot{\sigma} &= -\Sigma\,u'\\
\dot{u} &= 2\Omega\,v-\left(\frac{1}{\Sigma}\left.\frac{\partial p}{\partial\sigma}\right|_0-\frac{2\,\pi\,G}{|k|}\right)\sigma'+\frac{\alpha}{\Sigma}\eta_0u''\\
\dot{v} &= -\frac{\Omega}{2}u+\frac{1}{\Sigma}\left(\eta_0\,v''-\frac{3}{2}\Omega\left.\frac{\partial\eta}{\partial\sigma}\right|_0\sigma'\right)
\end{aligned}$$

$$\begin{aligned}
P_{rr} &= p-2\eta\frac{\partial u_r}{\partial r}+\left(\frac{2}{3}\eta-\xi\right)\vec{\nabla}\cdot\vec{u}\\
P_{r\varphi} &= -\eta\left(\frac{\partial u_\varphi}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial\varphi}-\frac{u_\varphi}{r}\right)
\end{aligned}$$

$$\alpha=\frac{4}{3}+\frac{\xi_0}{\eta_0}=const$$

shear viscosity

bulk viscosity

Mass and Momentum Balance + Self Gravity

$$\begin{aligned}
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)\sigma &= -\sigma\vec{\nabla}\cdot\vec{u} \\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)u_r-\frac{u_\varphi^2}{r} &= -\frac{\partial\Phi_{Planet}}{\partial r}-\frac{\partial\Phi_{Disk}}{\partial r}-\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_r \\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}+\frac{u_r}{r}\right)u_\varphi &= -\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_\varphi \\
\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_{Disk}}{\partial r}\right) &= 4\pi G\,\sigma\,\delta(z)
\end{aligned}$$

linearize about $\Sigma = const, u = 0, v = 0$

radial modes

comoving rotating frame

hydrodynamic (newtonian) stress, pressure

$$\begin{aligned}
\dot{\sigma} &= -\Sigma\,u' \\
\dot{u} &= 2\Omega\,v-\left(\frac{1}{\Sigma}\left.\frac{\partial p}{\partial\sigma}\right|_0-\frac{2\,\pi\,G}{|k|}\right)\sigma'+\frac{\alpha}{\Sigma}\eta_0 u'' \\
\dot{v} &= -\frac{\Omega}{2}u+\frac{1}{\Sigma}\left(\eta_0\,v''-\frac{3}{2}\Omega\left.\frac{\partial\eta}{\partial\sigma}\right|_0\sigma'\right)
\end{aligned}$$

$$\begin{aligned}
P_{rr} &= p-2\eta\frac{\partial u_r}{\partial r}+\left(\frac{2}{3}\eta-\xi\right)\vec{\nabla}\cdot\vec{u} \\
P_{r\varphi} &= -\eta\left(\frac{\partial u_\varphi}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial\varphi}-\frac{u_\varphi}{r}\right)
\end{aligned}$$

$$\alpha = \frac{4}{3}+\frac{\xi_0}{\eta_0} = const$$

shear viscosity

bulk viscosity

Mass and Momentum Balance + Self Gravity

$$\begin{aligned}
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)\sigma &= -\sigma\vec{\nabla}\cdot\vec{u}\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)u_r-\frac{u_\varphi^2}{r} &= -\frac{\partial\Phi_{Planet}}{\partial r}-\frac{\partial\Phi_{Disk}}{\partial r}-\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_r\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}+\frac{u_r}{r}\right)u_\varphi &= -\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_\varphi\\
\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_{Disk}}{\partial r}\right) &= 4\pi G\,\sigma\,\delta(z)
\end{aligned}$$

linearize about $\Sigma = const, u = 0, v = 0$

radial modes

comoving rotating frame

hydrodynamic (newtonian) stress, pressure

$$\begin{aligned}
\dot{\sigma} &= -\Sigma\,u'\\
\dot{u} &= 2\Omega\,v-\left(\frac{1}{\Sigma}\left.\frac{\partial p}{\partial\sigma}\right|_0-\frac{2\,\pi\,G}{|k|}\right)\sigma'+\frac{\alpha}{\Sigma}\eta_0 u''\\
\dot{v} &= -\frac{\Omega}{2}u+\frac{1}{\Sigma}\left(\eta_0\,v''-\frac{3}{2}\Omega\left.\frac{\partial\eta}{\partial\sigma}\right|_0\sigma'\right)
\end{aligned}$$

$$\begin{aligned}
P_{rr} &= p-2\eta\frac{\partial u_r}{\partial r}+\left(\frac{2}{3}\eta-\xi\right)\vec{\nabla}\cdot\vec{u}\\
P_{r\varphi} &= -\eta\left(\frac{\partial u_\varphi}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial\varphi}-\frac{u_\varphi}{r}\right)
\end{aligned}$$

this term can trigger instabilities

$$\alpha = \frac{4}{3}+\frac{\xi_0}{\eta_0} = const$$

shear viscosity

bulk viscosity

Mass and Momentum Balance + Self Gravity

$$\begin{aligned}
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)\sigma &= -\sigma\vec{\nabla}\cdot\vec{u}\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}\right)u_r-\frac{u_\varphi^2}{r} &= -\frac{\partial\Phi_{Planet}}{\partial r}-\frac{\partial\Phi_{Disk}}{\partial r}-\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_r\\
\left(\frac{\partial}{\partial t}+u_r\frac{\partial}{\partial r}+\frac{u_r}{r}\right)u_\varphi &= -\frac{1}{\sigma}\left(\vec{\nabla}\cdot\vec{P}\right)_\varphi\\
\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_{Disk}}{\partial r}\right) &= 4\pi G\,\sigma\,\delta(z)
\end{aligned}$$

linearize about $\Sigma = const, u = 0, v = 0$

radial modes
comoving rotating frame
hydrodynamic (newtonian) stress, pressure
Poisson equation for thin sheet

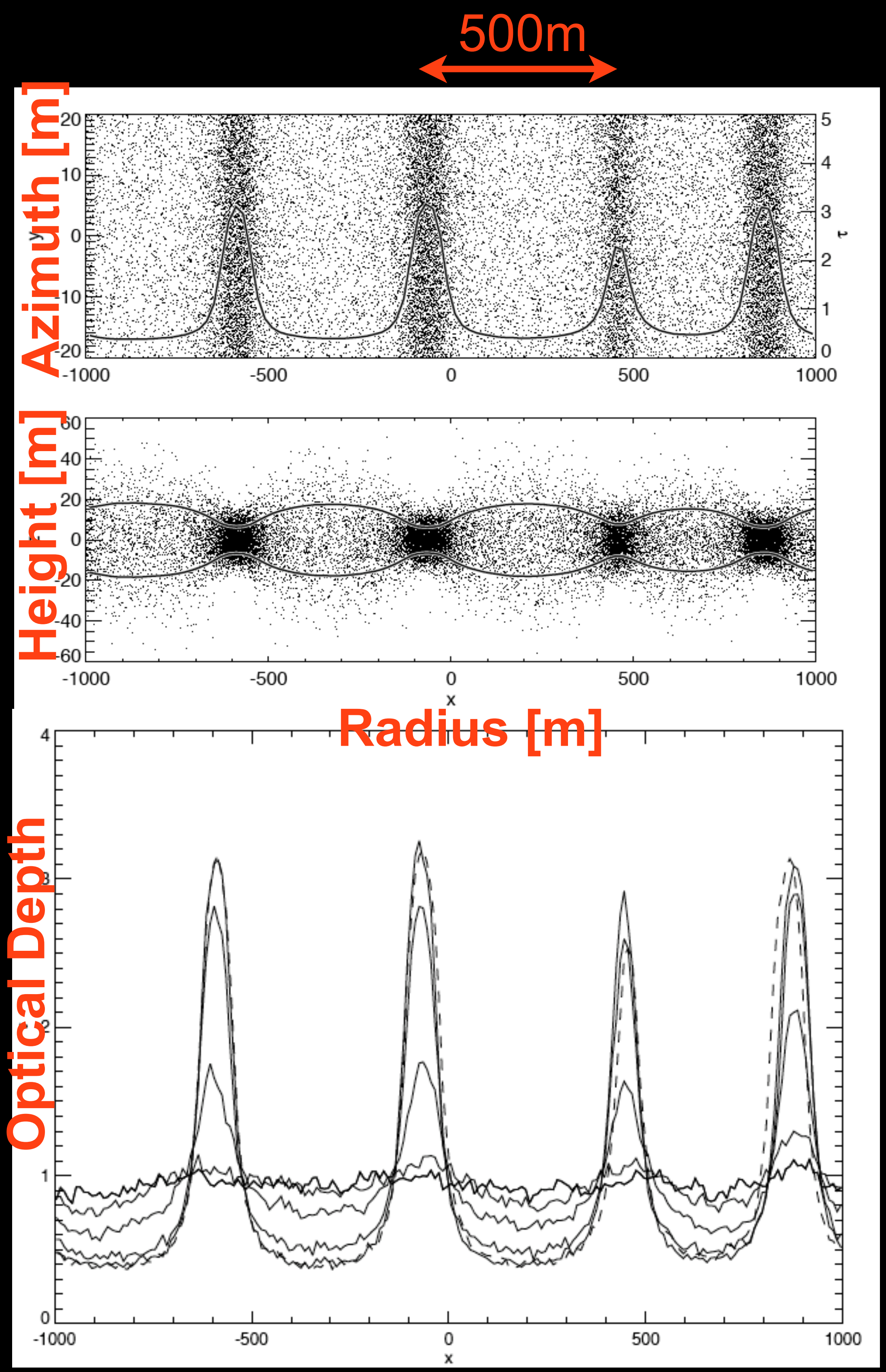
$$\begin{aligned}
\dot{\sigma} &= -\Sigma\,u'\\
\dot{u} &= 2\Omega\,v-\left(\frac{1}{\Sigma}\,\frac{\partial p}{\partial\sigma}\bigg|_0-\frac{2\,\pi\,G}{|k|}\right)\sigma'+\frac{\alpha}{\Sigma}\eta_0 u''\\
\dot{v} &= -\frac{\Omega}{2}u+\frac{1}{\Sigma}\left(\eta_0\,v''-\frac{3}{2}\Omega\,\frac{\partial\eta}{\partial\sigma}\bigg|_0\,\sigma'\right)
\end{aligned}$$

$$\begin{aligned}
\Phi_{Disk}(r,z) &= -\frac{2\pi G}{|k|}\sigma(r)\ \exp[-|k\,z|]\\
\sigma(r) &\propto \exp[i\,k\,r]
\end{aligned}$$

Viscous instability

Diffusion instabilty:
-> proposed in 80s
as explanation for B ring
irregular structure
-> discarded later:
conditions likely
not fulfilled in dense rings
-> but process itself works
-> would lead to bimodal
optical depth profile:
hot + low tau
cold + high tau
as in B2

Hämeen-Antilla78
Ward81
Lin&Bodenheimer81
Lukkari81



$$\begin{aligned}\dot\sigma &= -\Sigma\, u' \\ \dot u &= 2\Omega\, v - \left(\frac{1}{\Sigma}\,\frac{\partial p}{\partial \sigma}\bigg|_0 - \frac{2\,\pi\, G}{|k|}\right)\sigma' + \frac{\alpha}{\Sigma}\eta_0 u'' \\ \dot v &= -\frac{\Omega}{2}u + \frac{1}{\Sigma}\left(\eta_0\, v'' - \frac{3}{2}\Omega\,\frac{\partial \eta}{\partial \sigma}\bigg|_0 \sigma'\right)\end{aligned}$$

$$\dot\sigma = 3\,\frac{\partial \eta}{\partial \sigma}\bigg|_0\,\sigma''$$

$$\frac{\partial \eta}{\partial \sigma}\bigg|_0 > 0$$

realistic for Saturn’s rings

$$\eta \equiv \nu\sigma$$

$$\nu \propto c^2\,\frac{\omega_{col}}{\omega_{col}^2 + \Omega^2} + const. \times R^2\omega_{col} + const_2 \times \frac{\sigma^2\, G^2}{\Omega^3}$$

$$\omega_{col} \propto n_2$$

Oscillatory instability (overstability)

$$
\begin{aligned}
\dot{\sigma} &= -\Sigma\, u' \\
\dot{u} &= 2\Omega\, v - \left(\frac{1}{\Sigma} \left. \frac{\partial p}{\partial \sigma} \right|_0 - \frac{2\,\pi\, G}{|k|} \right) \sigma' + \frac{\alpha}{\Sigma} \eta_0 u'' \\
\dot{v} &= -\frac{\Omega}{2} u + \frac{1}{\Sigma} \left(\eta_0\, v'' - \frac{3}{2} \Omega \left. \frac{\partial \eta}{\partial \sigma} \right|_0 \sigma' \right)
\end{aligned}
$$

Oscillatory instability (overstability)

$$\begin{aligned}\dot{\sigma} &= -\Sigma\, u' \\ \dot{u} &= 2\Omega\, v - \left(\frac{1}{\Sigma}\,\frac{\partial p}{\partial \sigma}\bigg|_0 - \frac{2\,\pi\,G}{|k|}\right)\sigma' + \frac{\alpha}{\Sigma}\eta_0 u'' \\ \dot{v} &= -\frac{\Omega}{2}u + \frac{1}{\Sigma}\left(\eta_0\, v'' - \frac{3}{2}\Omega\,\frac{\partial \eta}{\partial \sigma}\bigg|_0\,\sigma'\right)\end{aligned}$$

$$\ddot{u} + u - \left(\frac{\partial p}{\partial \sigma}\bigg|_0 - \frac{2\pi G\,\Sigma}{|k|}\right)u'' \quad = \quad \nu_0(f(r,t) + \alpha\nu_0 u'''')$$

$\nu_0 \quad = \quad \frac{\eta_0}{\Sigma}$ (kinematic shear viscosity)

Oscillatory instability (overstability)

$$\begin{aligned}
\dot\sigma &= -\Sigma\, u' \\
\dot u &= 2\Omega\, v - \left(\frac{1}{\Sigma}\,\frac{\partial p}{\partial\sigma}\bigg|_0 - \frac{2\,\pi\,G}{|k|}\right)\sigma' + \frac{\alpha}{\Sigma}\eta_0 u'' \\
\dot v &= -\frac{\Omega}{2}u + \frac{1}{\Sigma}\left(\eta_0\, v'' - \frac{3}{2}\Omega\,\frac{\partial\eta}{\partial\sigma}\bigg|_0\,\sigma'\right)
\end{aligned}$$

$$\underbrace{\ddot u + u - \left(\frac{\partial p}{\partial\sigma}\bigg|_0 - \frac{2\pi G\,\Sigma}{|k|}\right)u''}_{\text{Acoustic inertial wave}} \quad = \quad \underbrace{\nu_0\big(f(r,t) + \alpha\nu_0 u''''\big)}_{\text{Viscous forcing}}$$

$\nu_0 = \dfrac{\eta_0}{\Sigma}$ (kinematic shear viscosity)

$$f(r,t) = (1+\alpha)\dot u'' + \int_{-\infty}^{t} d\tilde t\left[3\Omega^2\,\frac{\partial\ln\eta}{\partial\ln\sigma}\bigg|_0\,u'' - \left(\frac{\partial p}{\partial\sigma}\bigg|_0 - \frac{2\pi G\,\Sigma}{|k|}\right)u''''\right]$$

Viscously Forced Wave Equation

$$\begin{aligned}
\dot{\sigma} &= -\Sigma\, u' \\
\dot{u} &= 2\Omega\, v - \left(\frac{1}{\Sigma}\, \frac{\partial p}{\partial \sigma}\bigg|_0 - \frac{2\,\pi\, G}{|k|} \right) \sigma' + \frac{\alpha}{\Sigma} \eta_0 u'' \\
\dot{v} &= -\frac{\Omega}{2} u + \frac{1}{\Sigma} \left(\eta_0\, v'' - \frac{3}{2} \Omega\, \frac{\partial \eta}{\partial \sigma}\bigg|_0 \sigma' \right)
\end{aligned}$$

$$\ddot{u} + u - \left(\frac{\partial p}{\partial \sigma}\bigg|_0 - \frac{2\pi G\, \Sigma}{|k|} \right) u'' \quad = \quad \nu_0 (f(r,t) + \alpha \nu_0 u'''')$$

rapid oscillations

slow amplitude modulation

Multiscale expansion:

$$\begin{aligned}
u(r,t,\theta) &= A(\theta)\, u_0(r,t) \\
\frac{\partial}{\partial t} &\rightarrow \frac{\partial}{\partial t} + \nu_0 \frac{\partial}{\partial \theta}
\end{aligned}$$

$$\begin{aligned} \ddot{u} + u - \left(\left.\frac{\partial p}{\partial \sigma}\right|_0 - \frac{2\pi G\,\Sigma}{|k|} \right) u'' &= \nu_0 (f(r,t) + \alpha \nu_0 u'''') \\ u(r,t,\theta) &= A(\theta)\, u_0(r,t) \end{aligned}$$

$$\frac{\partial^2}{\partial t^2} u_0 + u_0 - \left(\left.\frac{\partial p}{\partial \sigma}\right|_0 - \frac{2\pi G}{|k|} \right) u_0'' = 0$$

at zeroth order in ν_0

$$\begin{aligned} \ddot{u} + u - \left(\left.\frac{\partial p}{\partial \sigma}\right|_0 - \frac{2\pi G\,\Sigma}{|k|} \right) u'' &= \nu_0 (f(r,t) + \alpha \nu_0 u'''') \\ u(r,t,\theta) &= A(\theta)\, u_0(r,t) \end{aligned}$$

$$\begin{aligned} \frac{\partial^2}{\partial t^2} u_0 + u_0 - \left(\left.\frac{\partial p}{\partial \sigma}\right|_0 - \frac{2\pi G}{|k|} \right) u_0'' &= 0 \\ u_0 &= \exp(i\,\omega\,t + i\,k\,x) \\ \omega &= \pm\sqrt{\Omega^2 - 2\pi G\,\Sigma\,|k| + \left.\frac{\partial p}{\partial \sigma}\right|_0 k^2} \end{aligned}$$

at zeroth order in ν_0

$$\begin{aligned} \ddot{u} + u - \left(\left.\frac{\partial p}{\partial \sigma}\right|_0 - \frac{2\pi G\,\Sigma}{|k|} \right) u'' &= \nu_0 (f(r,t) + \alpha \nu_0 u'''') \\ u(r,t,\theta) &= A(\theta)\, u_0(r,t) \end{aligned}$$

$$\begin{aligned} \frac{\partial^2}{\partial t^2} u_0 + u_0 - \left(\left.\frac{\partial p}{\partial \sigma}\right|_0 - \frac{2\pi G}{|k|} \right) u_0'' &= 0 \qquad \text{at zeroth order in } \nu_0 \\ u_0 &= \exp(i\,\omega\, t + i\, k\, x) \\ \omega &= \pm\sqrt{\Omega^2 - 2\pi G\,\Sigma\,|k| + \left.\frac{\partial p}{\partial \sigma}\right|_0 k^2} \end{aligned}$$

at first order in ν_0

$$\frac{\partial}{\partial \theta} A = -\frac{3}{2} k^2 \left(\frac{1+\alpha}{3} - \left.\frac{\partial \ln \eta}{\partial \ln \sigma}\right|_0 \right) A + O(k^3)$$

$$\begin{aligned} \ddot{u} + u - \left(\left.\frac{\partial p}{\partial \sigma}\right|_0 - \frac{2\pi G\,\Sigma}{|k|} \right) u'' &= \nu_0 (f(r,t) + \alpha \nu_0 u'''') \\ u(r,t,\theta) &= A(\theta)\, u_0(r,t) \end{aligned}$$

$$\begin{aligned} \frac{\partial^2}{\partial t^2} u_0 + u_0 - \left(\left.\frac{\partial p}{\partial \sigma}\right|_0 - \frac{2\pi G}{|k|} \right) u_0'' &= 0 \\ u_0 &= \exp(i\,\omega\, t + i\, k\, x) \\ \omega &= \pm\sqrt{\Omega^2 - 2\pi G\,\Sigma\,|k| + \left.\frac{\partial p}{\partial \sigma}\right|_0 k^2} \end{aligned}$$

at zeroth order in ν_0

at first order in ν_0

$$\frac{\partial}{\partial \theta} A = -\frac{3}{2} k^2 \underbrace{\left(\frac{1+\alpha}{3} - \left.\frac{\partial \ln \eta}{\partial \ln \sigma}\right|_0 \right)} A + O(k^3)$$

Exponential growth of amplitude for $\left.\frac{\partial \ln \eta}{\partial \ln \sigma}\right|_0 > \frac{1+\alpha}{3}$

$$\left.\frac{\partial \ln \eta}{\partial \ln \sigma}\right|_0 > \frac{1+\alpha}{3}$$

steep increase of η with increasing σ should be fulfilled

=> ring flow undergoes Hopf bifurcation

(Schmit&Tscharnuter, Icarus, 1996, 1999, Spahn et al, 2000, Salo et al, 2001, Schmidt et al, 2001)

=> traveling waves of 100 m wavelength

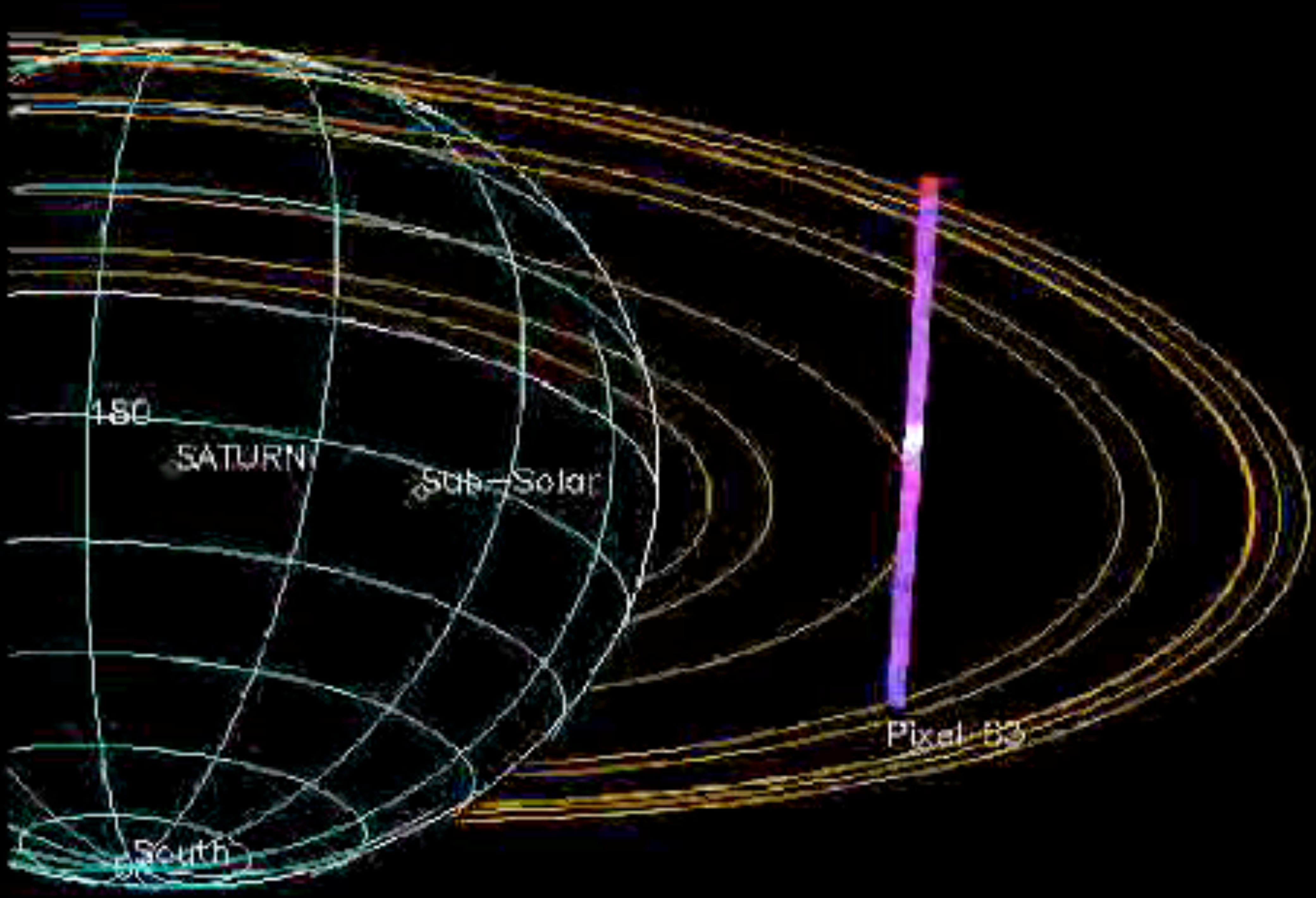
(Schmidt&Salo, PRL, 2003)

=> kinetic theory + hydrodynamic nonlinear wavetrain solutions

(Latter & Ogivie, Icarus, 2005, 2007, 2009)

U

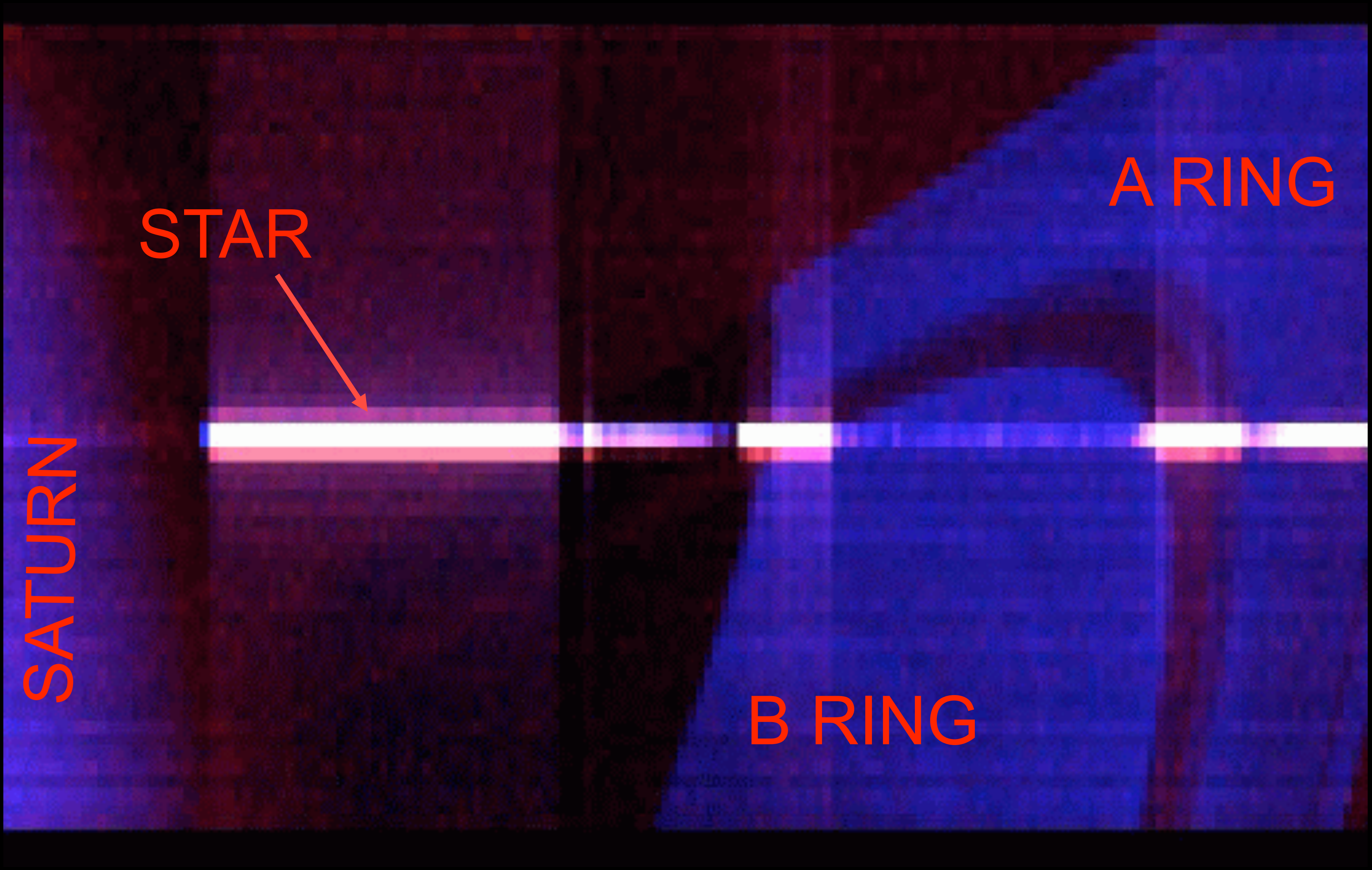
CASSINI UVIS stellar occultation



(Josh Colwell)

UVIS: Colwel et al 2007

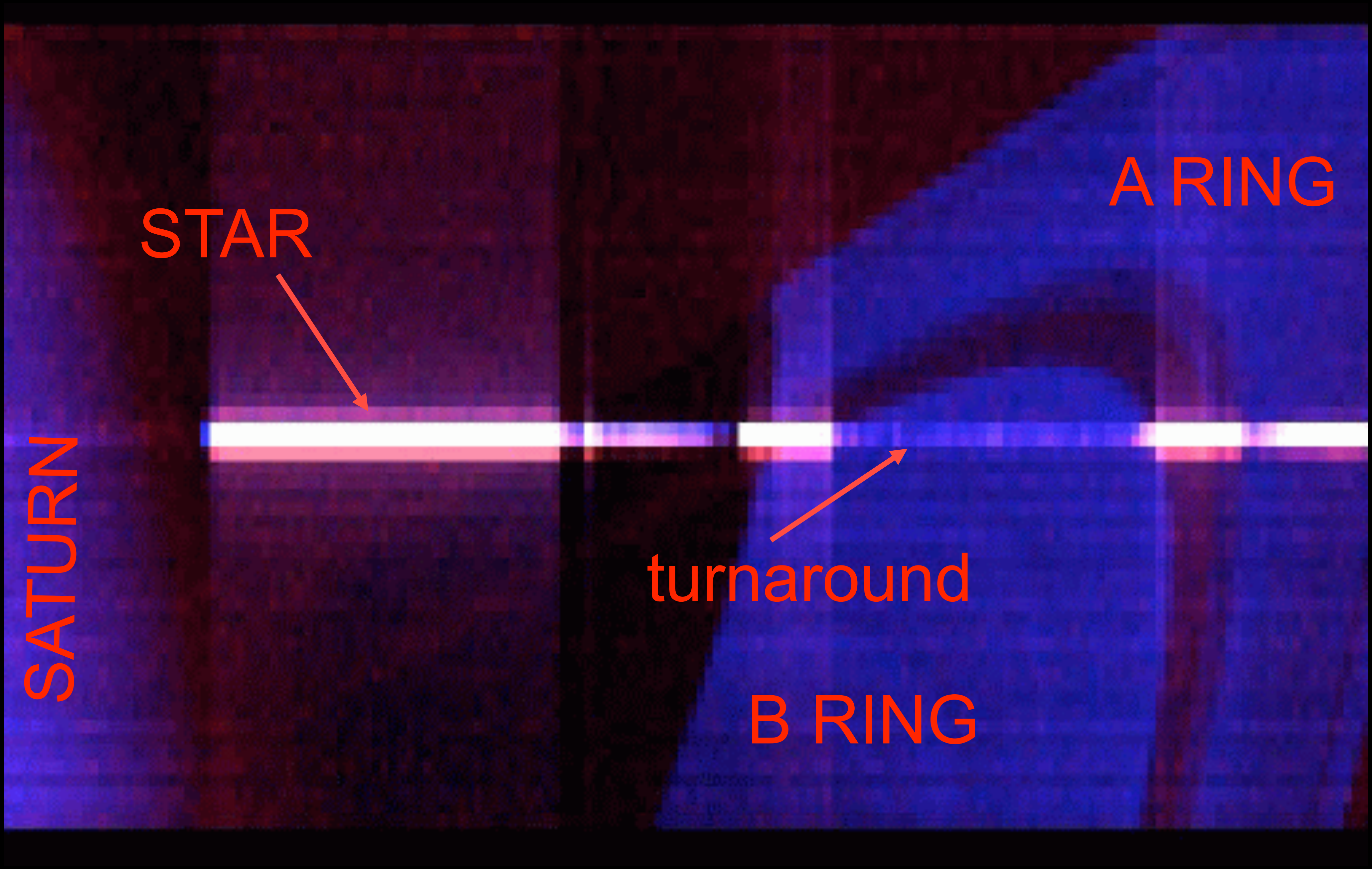
Ring Occultation by alpha-Leonis, UVIS FUV



From J.Colwell et all, ICARUS, 2007

UVIS: Colwel et al 2007

Ring Occultation by alpha-Leonis, UVIS FUV



From J.Colwell et all, ICARUS, 2007

UVIS: Colwel et al 2007

At Turnaround:

* nearly azimuthal track

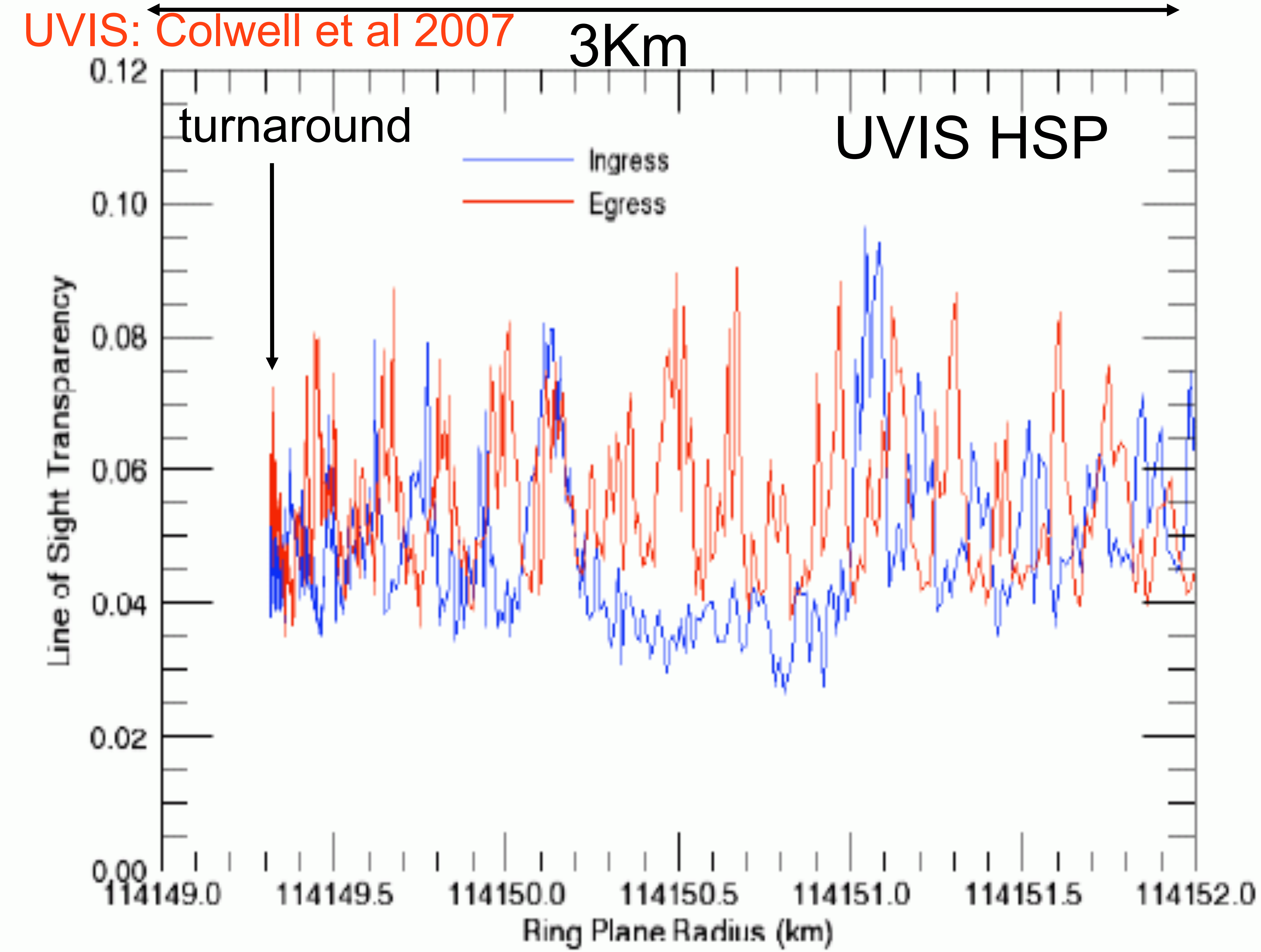
* small change in ring plane radius

-> drastic increase in radial resolution
1.5m per 2ms integration period
(HSP UVIS)

15m diffraction limited

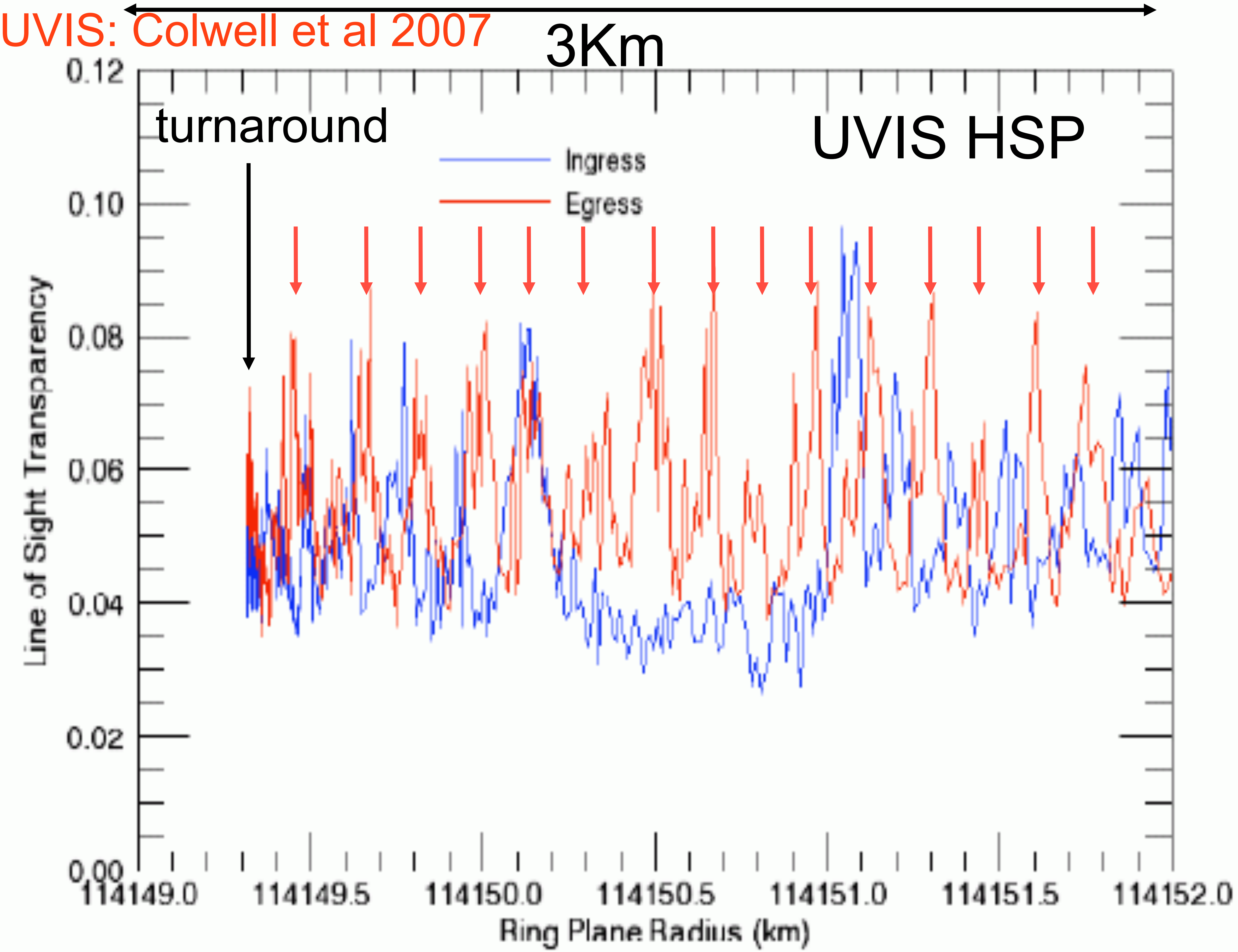
UVIS: Colwell et al 2007

3Km

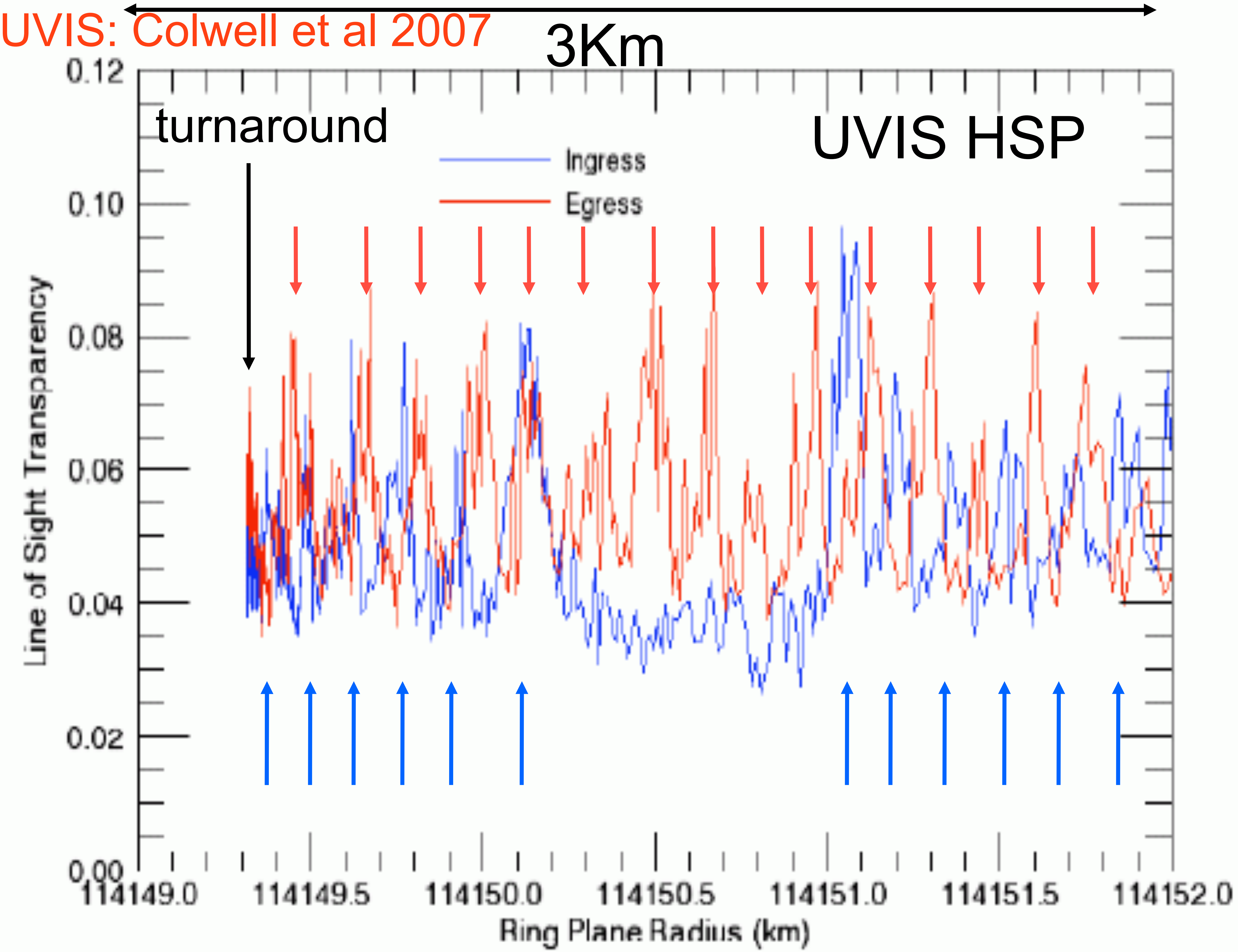


Tuesday, November 1, 2011

UVIS: Colwell et al 2007



UVIS: Colwell et al 2007



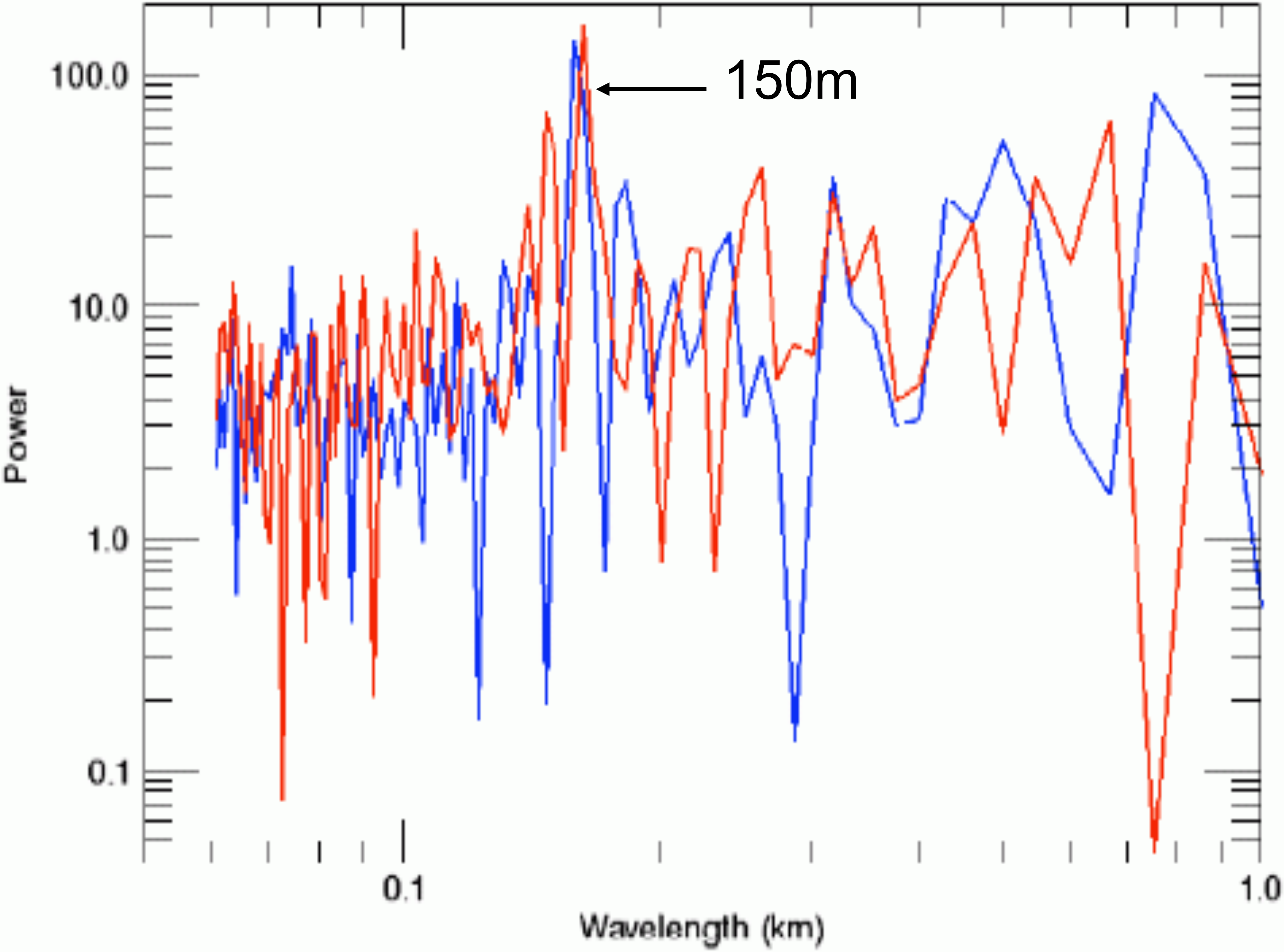
3Km

turnaround

UVIS HSP

UVIS: Colwell et al 2007

FFT of alpha Leo profiles



-> more observations:
CASSINI Radio Science Subsystem (RSS)
=> 150-200m axisymmetric waves
are in the inner A ring
and abundant in the B ring

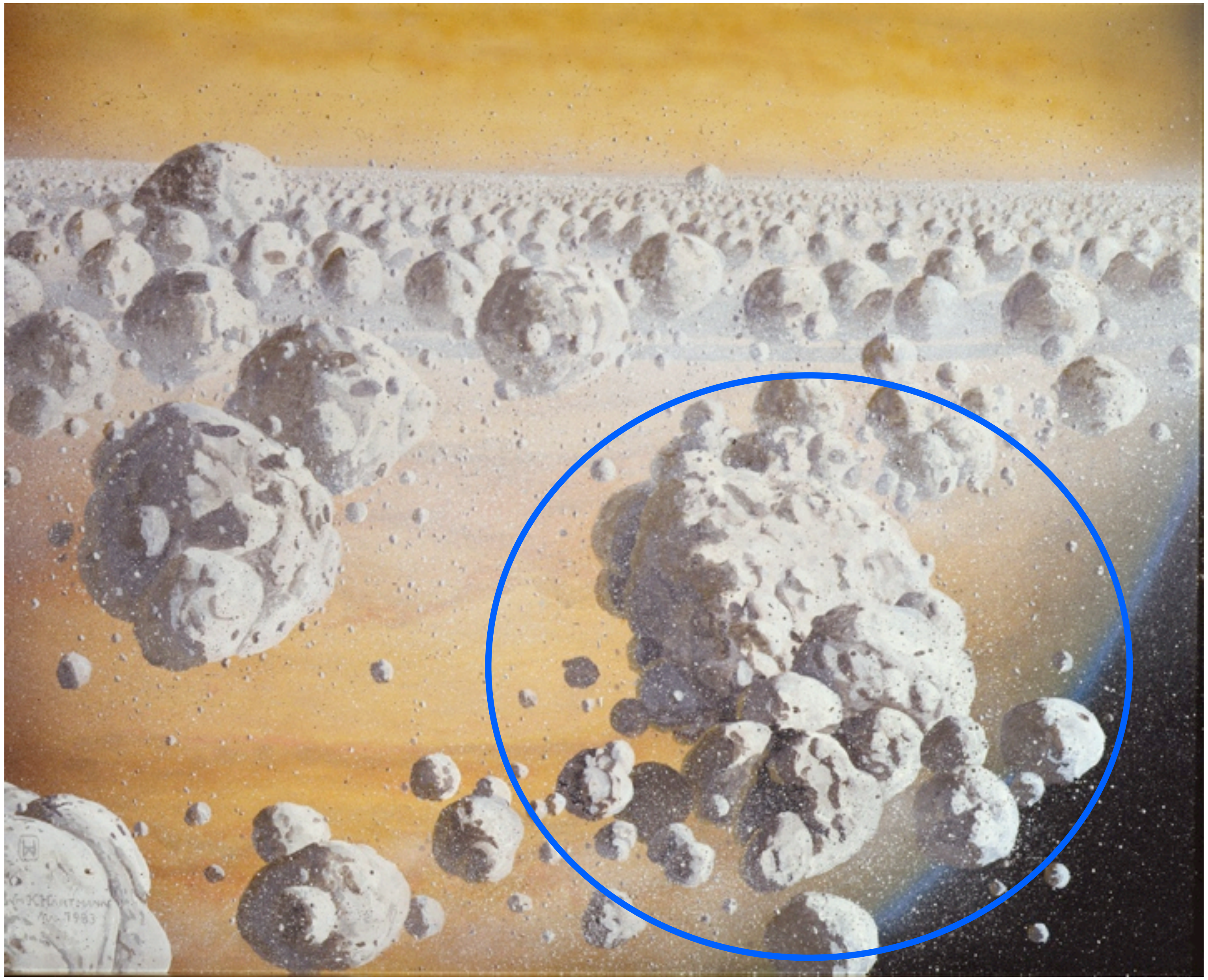
-> most likely interpretation:
viscous overstability

-> full nonlinear evolution TBD:
Complex Ginzburg Landau equation

-> can this process make larger structure
of several km?

size distribution of ring particles

- are ring particles metastable agglomerates (Davis *et al.*, 1984)?
- balance of coagulation and fragmentation?


(Bill Hartman)

Dynamic
Ephemeral
Bodies?
(Weidenschilling *et al.*,
see also Longaretti, 1989)

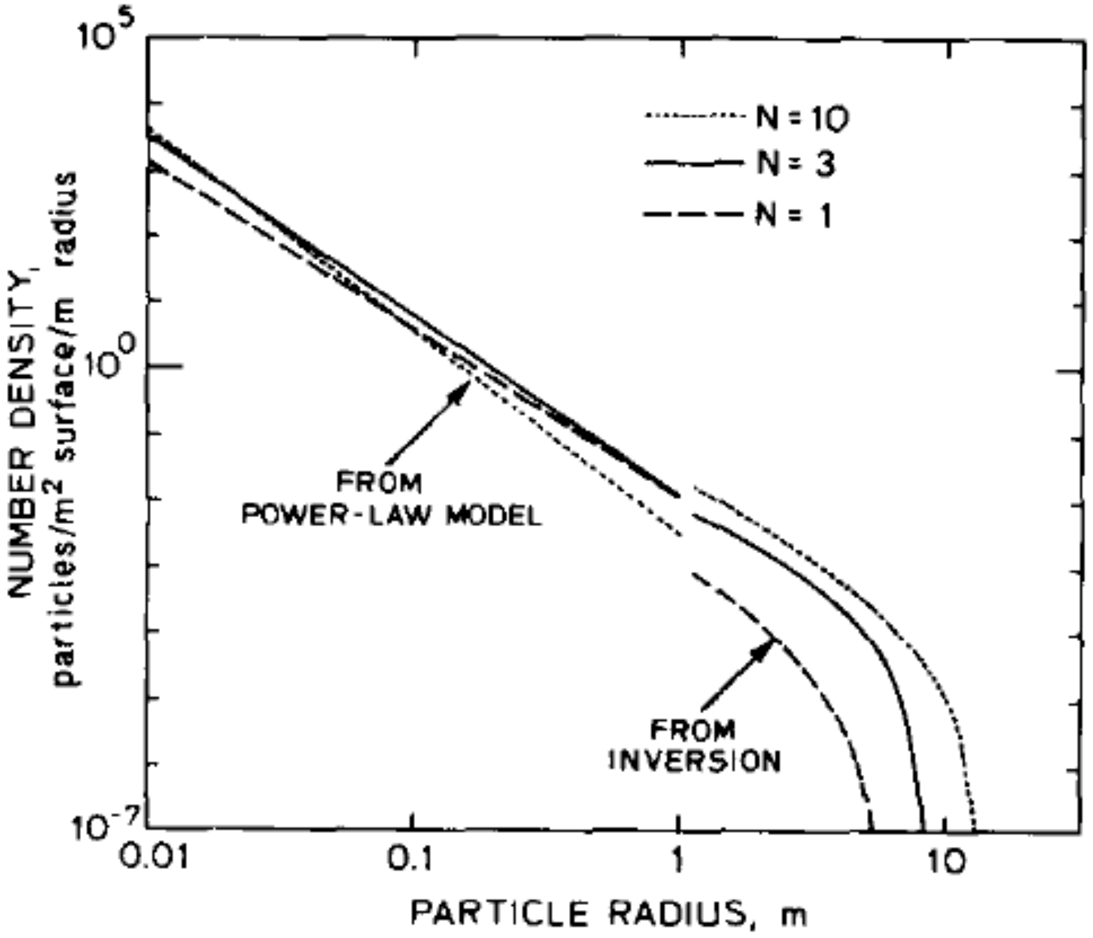


FIG. 2. Illustration of the dependence of the size distribution function $n(a)$ on parameter N. The supra-meter part is obtained from inversion of the scattered signal, the submeter part is obtained from opacity measured at 3.6- and 13-cm wavelengths and the assumption of a power-law model. Only the two parts for the case $N = 3$ form a nearly continuous and smooth transition at radius $a = 1$ m; we take this as the most likely form of the distribution.

(From: Zebker *et al.*, 1985)

Voyager Radio Science

(Zebker *et al.*, 1985):

-> power law:
cm < r < meters

-> knee/size-cut-off:
r > meters

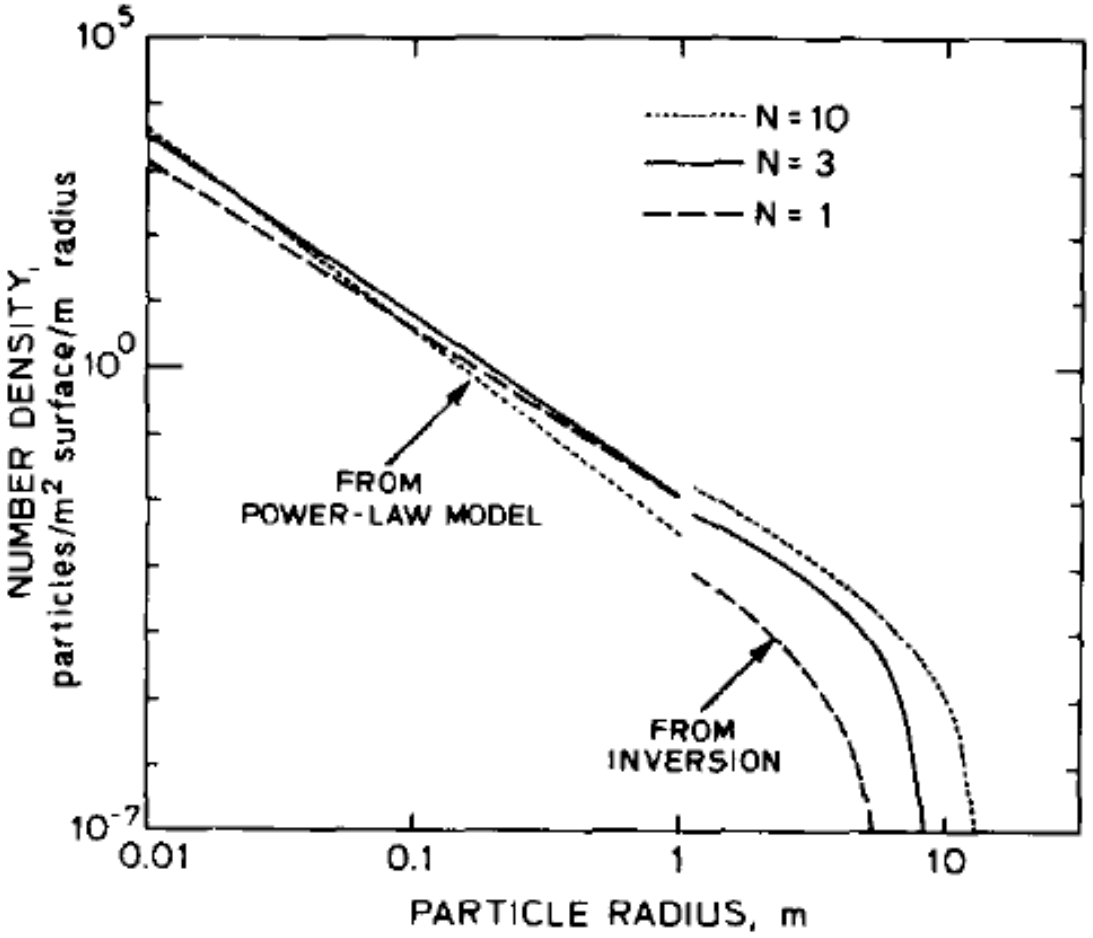


FIG. 2. Illustration of the dependence of the size distribution function $n(a)$ on parameter N. The supra-meter part is obtained from inversion of the scattered signal, the submeter part is obtained from opacity measured at 3.6- and 13-cm wavelengths and the assumption of a power-law model. Only the two parts for the case $N = 3$ form a nearly continuous and smooth transition at radius $a = 1$ m; we take this as the most likely form of the distribution.

(From: Zebker *et al.*, 1985)

Voyager Radio Science

(Zebker *et al.*, 1985):

-> power law:

cm < r < meters

-> knee/size-cut-off:

r > meters

stellar occ (28 Sgr)

observed from earth

(French & Nicholson, 2000),

+ Cassini radio science

(Marouf *et al.*, 2008,

Cuzzi *et al.*, 2009):

-> consistent results

-> kinetic model:

discrete model, ring-particles are clusters of primary, indestructible, identical, spheres of size r_0

-> kinetic model:
discrete model, ring-particles are clusters of primary, indestructible, identical, spheres of size r_0

-> evolution of cluster size distribution with sticking collisions (low speed) and disruptive collisions (high speed)

-> kinetic model:
discrete model, ring-particles are clusters of primary, indestructible, identical, spheres of size r_0

-> evolution of cluster size distribution with sticking collisions (low speed) and disruptive collisions (high speed)

-> analytical steady state solution:
simplified model - clusters (when disrupted) decay completely into primary particles, simplified collision kernels

-> kinetic model:
discrete model, ring-particles are clusters of primary, indestructible, identical, spheres of size r_0

-> evolution of cluster size distribution with sticking collisions (low speed) and disruptive collisions (high speed)

-> analytical steady state solution:
simplified model - clusters (when disrupted) decay completely into primary particles, simplified collision kernels

-> local model:
no self-gravity, no ring structure, no tidal force, Gaussian speed distribution

Boltzmann equation:

$$\frac{\partial}{\partial t} f_m(\vec{v}_m, t) = I_m^{agg} + I_m^{frag} + I_m^{reb} + I_m^{heat}$$

speed distribution, clusters of mass m

Boltzmann equation:

collision integrals

$$\frac{\partial}{\partial t} f_m(\vec{v}_m, t) = I_m^{agg} + I_m^{frag} + I_m^{reb} + I_m^{heat}$$

speed distribution, clusters of mass m

Boltzmann equation:

rebound:
dissipative
collisions

viscous
heating

collision
integrals

$$\frac{\partial}{\partial t} f_m(\vec{v}_m, t) = I_m^{agg} + I_m^{frag} + I_m^{reb} + I_m^{heat}$$

speed distribution, clusters of mass m

Boltzmann equation:

rebound:
dissipative
collisions

collision
integrals

viscous
heating

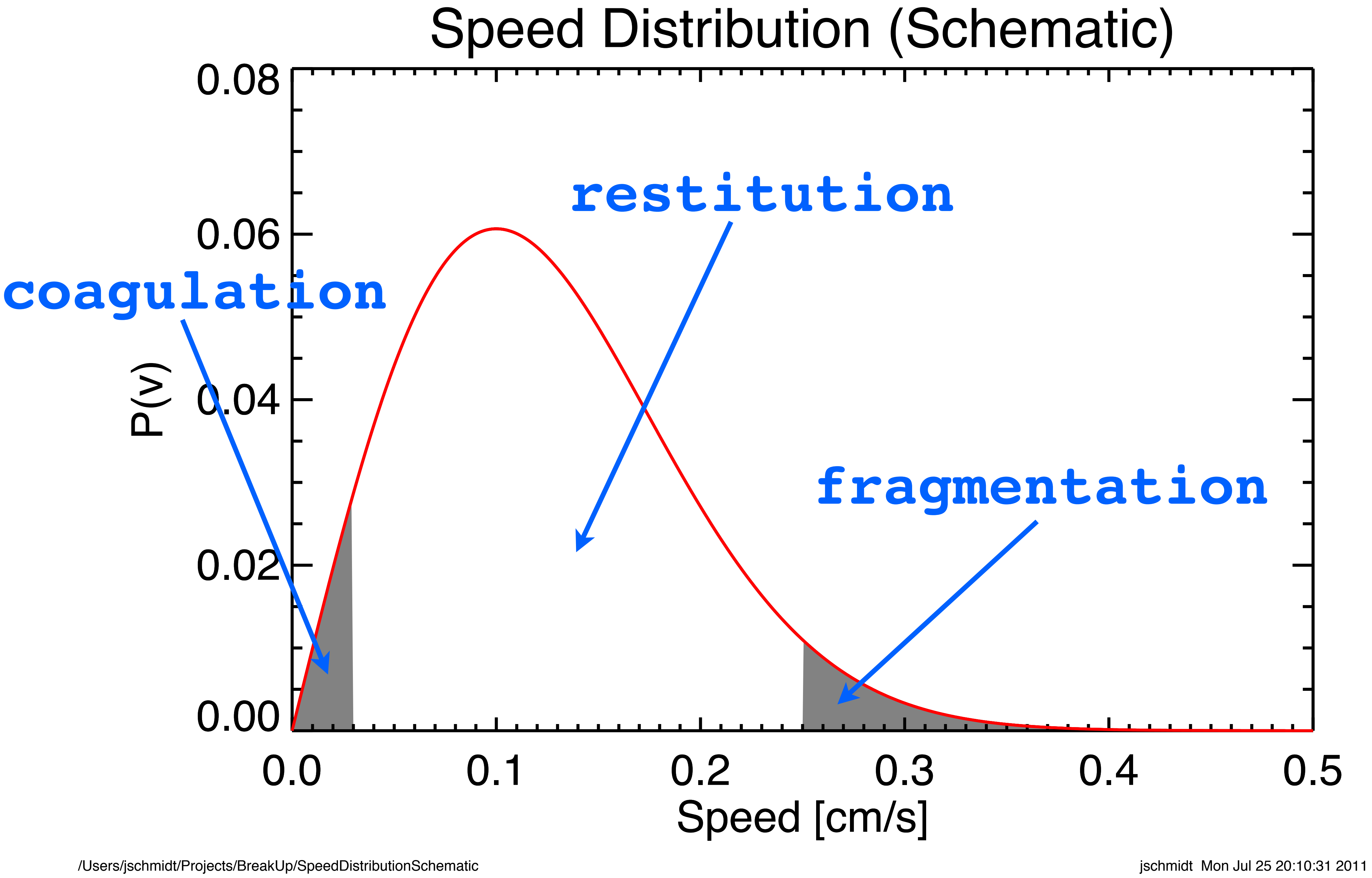
$$\frac{\partial}{\partial t} f_m(\vec{v}_m, t) = I_m^{agg} + I_m^{frag} + I_m^{reb} + I_m^{heat}$$

fragmentation:
disruptive collisions

aggregation:
sticking collisions

speed distribution, clusters of mass m

assumption:
fragmentation and coagulation energies are independent of cluster size



Speed Distribution (Schematic)

restitution

coagulation

fragmentation

n_k: **concentration of clusters containing k primary particles**

K_{ij}: **collision kernel (from Boltzmann equation)**

$K_{kj} n_j$: **frequency of collisions of clusters of size k with clusters of size j**

evolution equation for k>1:

$$\frac{dn_k}{dt} = \frac{1}{2} \sum_{i+j=k} K_{ij} n_i n_j - (1+\lambda) n_k \sum_{j \geq 1} K_{kj} n_j$$

evolution equation for k>1:

$$\frac{dn_k}{dt} = \frac{1}{2} \sum_{i+j=k} K_{ij} n_i n_j - (1+\lambda) n_k \sum_{j \geq 1} K_{kj} n_j$$

merging of clusters (Smoluchowski)

evolution equation for k>1:

$$\frac{dn_k}{dt} = \frac{1}{2}\sum_{i+j=k} K_{ij} n_i n_j - (1+\lambda) n_k \sum_{j\geq 1} K_{kj} n_j$$

merging of clusters
(Smoluchowski)

collisional decay of clusters into
primary particles, λ << 1

evolution equation for k>1:

$$\frac{dn_k}{dt} = \frac{1}{2}\sum_{i+j=k} K_{ij}n_in_j - (1+\lambda)n_k\sum_{j\geq 1} K_{kj}n_j$$

merging of clusters (Smoluchowski)

collisional decay of clusters into primary particles, λ << 1

evolution equation for k=1:

$$\begin{aligned}\frac{dn_1}{dt} &= -2n_1\sum_{j\geq 1} K_{1j}n_j \\ &+ \frac{\lambda}{2}\sum_{i,j\geq 2}(i+j)K_{ij}n_in_j + \lambda n_1\sum_{j\geq 2} jK_{1j}n_j\end{aligned}$$

Choice of Collision Kernel

(a) ballistic Kernel

$$K_{ij} = \underbrace{\left(i^{1/3} + j^{1/3}\right)^2}_{\text{cross section}} \underbrace{\sqrt{\frac{i+j}{ij}}}_{\text{relative speed}}$$

Choice of Collision Kernel

(a) ballistic Kernel

$$K_{ij} = \underbrace{\left(i^{1/3} + j^{1/3}\right)^2}_{\text{cross section}} \underbrace{\sqrt{\frac{i+j}{ij}}}_{\text{relative speed}}$$

(b) modified Kernel, better for rings

$$K_{ij} = \left(i^{1/3} + j^{1/3}\right)^2$$

Choice of Collision Kernel

(a) ballistic Kernel

$$K_{ij} = \underbrace{\left(i^{1/3} + j^{1/3}\right)^2}_{\text{cross section}} \underbrace{\sqrt{\frac{i+j}{ij}}}_{\text{relative speed}}$$

(b) modified Kernel, better for rings

$$K_{ij} = \left(i^{1/3} + j^{1/3}\right)^2$$

(c) general product Kernel

$$K_{ij} = (ij)^{\mu}$$

Choice of Collision Kernel

(a) ballistic Kernel

$$K_{ij} = \underbrace{\left(i^{1/3} + j^{1/3}\right)^2}_{\text{cross section}} \underbrace{\sqrt{\frac{i+j}{ij}}}_{\text{relative speed}}$$

(b) modified Kernel, better for rings

$$K_{ij} = \left(i^{1/3} + j^{1/3}\right)^2$$

(c) general product Kernel

$$K_{ij} = (ij)^{\mu}$$ **analytical solution**

Choice of Collision Kernel

(a) ballistic Kernel

$$K_{ij} = \underbrace{\left(i^{1/3}+j^{1/3}\right)^2}_{\text{cross section}}\ \underbrace{\sqrt{\frac{i+j}{ij}}}_{\text{relative speed}}$$

(b) modified Kernel, better for rings

$$K_{ij} = \left(i^{1/3}+j^{1/3}\right)^2$$

(c) general product Kernel

$$K_{ij} = (ij)^{\mu}$$ **analytical solution**

degree of homogeneity, κ:

$$K_{ai,aj} = a^{\kappa} K_{i,j}$$

Choice of Collision Kernel

(a) ballistic Kernel

$$K_{ij} = \underbrace{\left(i^{1/3}+j^{1/3}\right)^2}_{\text{cross section}} \underbrace{\sqrt{\frac{i+j}{ij}}}_{\text{relative speed}} \longleftarrow \mu = 1/12$$

(b) modified Kernel, better for rings

$$K_{ij} = \left(i^{1/3}+j^{1/3}\right)^2 \longleftarrow \mu = 1/3$$

(c) general product Kernel

$$K_{ij} = (ij)^{\mu}$$ **analytical solution**

same κ, if

degree of homogeneity, κ:
$K_{ai,aj} = a^{\kappa} K_{i,j}$

Choice of Collision Kernel

(a) ballistic Kernel

$$K_{ij} = \underbrace{\left(i^{1/3} + j^{1/3}\right)^2}_{\text{cross section}} \underbrace{\sqrt{\frac{i+j}{ij}}}_{\text{relative speed}}$$

$\mu = 1/12$

(b) modified Kernel, better for rings

$$K_{ij} = \left(i^{1/3} + j^{1/3}\right)^2$$

$\mu = 1/3$

(c) general product Kernel

$$K_{ij} = (ij)^{\mu}$$

analytical solution

same κ, if

degree of homogeneity, κ:
$K_{ai,aj} = a^{\kappa} K_{i,j}$

Choice of Collision Kernel

(a) ballistic Kernel

$$K_{ij} = \underbrace{\left(i^{1/3}+j^{1/3}\right)^2}_{\text{cross section}} \underbrace{\sqrt{\frac{i+j}{ij}}}_{\text{relative speed}}$$

equipartition: energies of random motion of different size groups

$\mu = 1/12$

(b) modified Kernel, better for rings

$$K_{ij} = \left(i^{1/3}+j^{1/3}\right)^2$$

$\mu = 1/3$

all size groups have the same dispersion velocity

(c) general product Kernel

$$K_{ij} = (ij)^{\mu}$$

analytical solution

same κ, if

degree of homogeneity, κ:

$K_{ai,aj} = a^{\kappa} K_{i,j}$

Choice of Collision Kernel

(a) ballistic Kernel

$$K_{ij} = \underbrace{\left(i^{1/3}+j^{1/3}\right)^2}_{\text{cross section}} \underbrace{\sqrt{\frac{i+j}{ij}}}_{\text{relative speed}}$$

$\mu = 1/12$

equipartition: energies of random motion of different size groups

(b) modified Kernel, better for rings

$$K_{ij} = \left(i^{1/3}+j^{1/3}\right)^2$$

$\mu = 1/3$

all size groups have the same dispersion velocity

(c) general product Kernel

$K_{ij} = (ij)^{\mu}$ analytical solution

degree of homogeneity, κ:
$K_{ai,aj} = a^{\kappa} K_{i,j}$

same κ, if

gas mixtures

rings

Solution for general product Kernel $K_{ij} = (ij)^{\mu}$

$$\boxed{n_k = \frac{F(\lambda)}{2\sqrt{\pi}}\ e^{-\lambda^2 k/4}\ k^{-3/2-\mu}} \quad (1 \ll k < \lambda^{-2})$$

Solution for general product Kernel $K_{ij} = (ij)^{\mu}$

$$\boxed{n_k = \frac{F(\lambda)}{2\sqrt{\pi}}\, e^{-\lambda^2 k/4}\; k^{-3/2-\mu}} \qquad (1 \ll k < \lambda^{-2})$$

low mass: power law

high mass: exponential cut-off

Solution for general product Kernel $K_{ij} = (ij)^{\mu}$

$$\boxed{n_k = \frac{F(\lambda)}{2\sqrt{\pi}}\; e^{-\lambda^2 k/4}\; k^{-3/2-\mu}} \qquad (1 \ll k < \lambda^{-2})$$

low mass: power law

high mass: exponential
cut-off

Size distribution

$$\boxed{F(R) \propto R^{-q} e^{-(R/R_c)^3}}, \quad q = 5/2 + 3\mu, \quad R_c^3 = 4r_0^3/\lambda^2$$

Solution for general product Kernel $K_{ij} = (ij)^{\mu}$

$$\boxed{n_k = \frac{F(\lambda)}{2\sqrt{\pi}}\; e^{-\lambda^2 k/4}\; k^{-3/2-\mu}} \qquad (1 \ll k < \lambda^{-2})$$

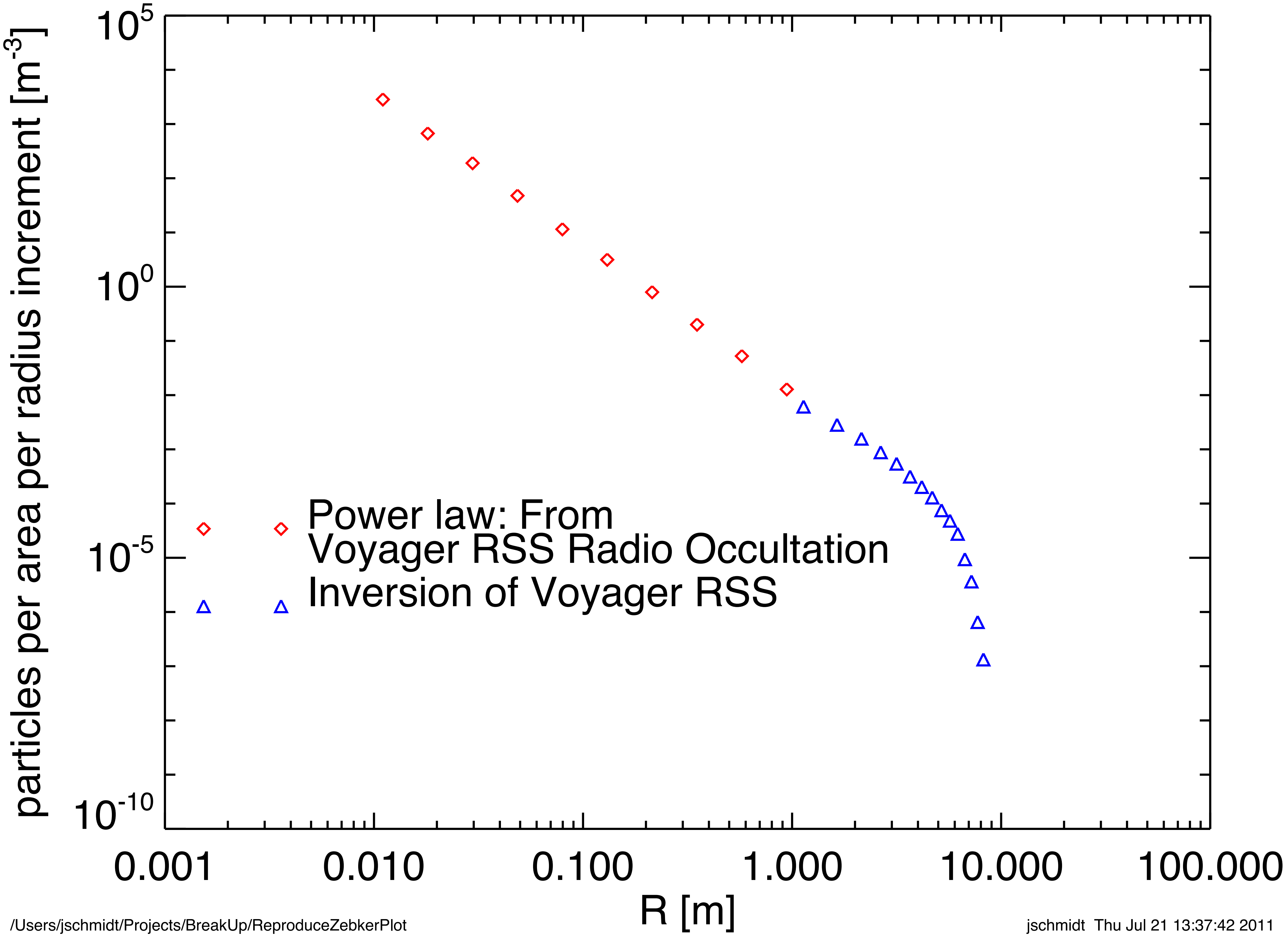
low mass: power law

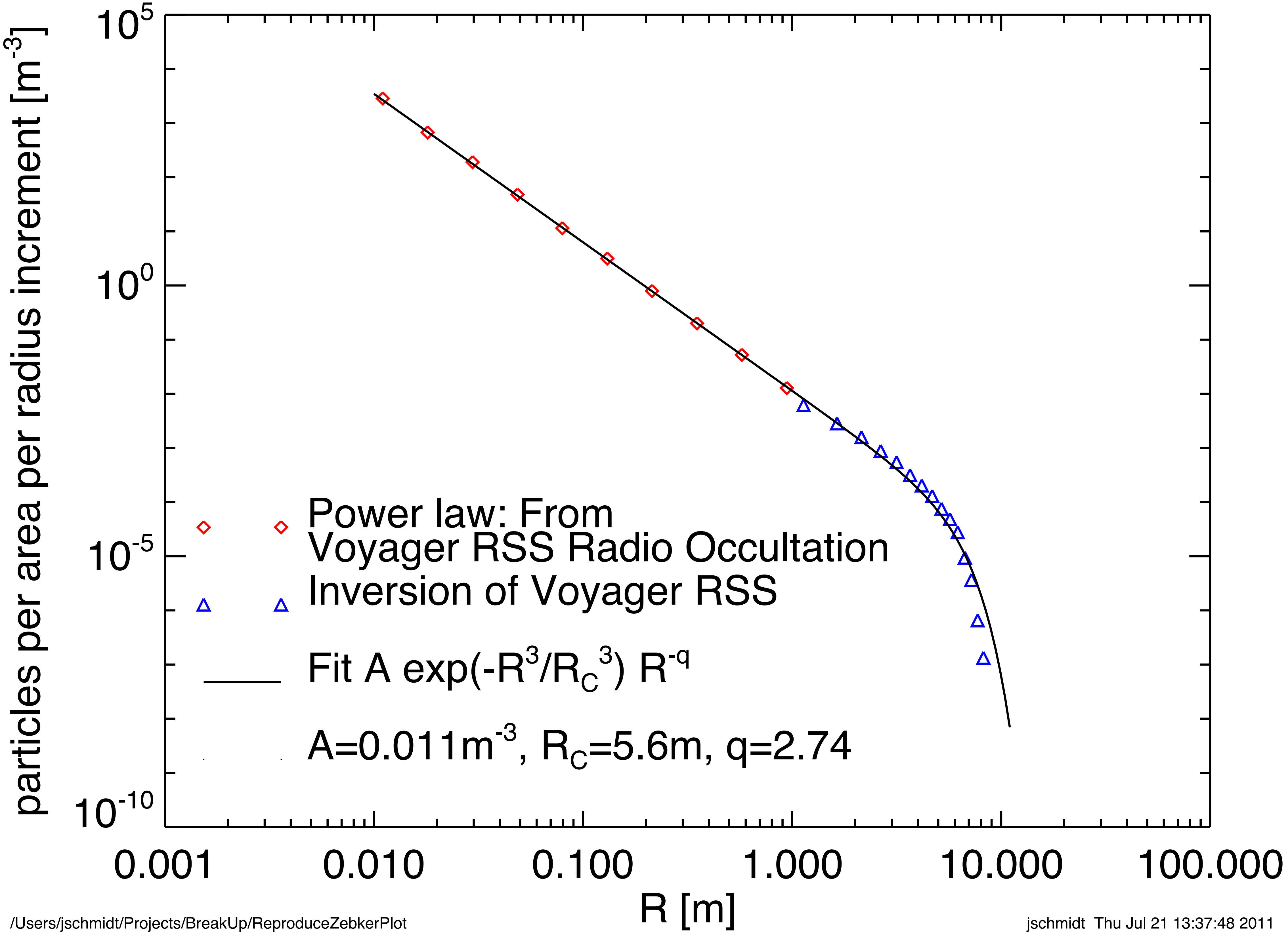
high mass: exponential cut-off

Size distribution

$$\boxed{F(R) \propto R^{-q} e^{-(R/R_c)^3}}, \qquad q = 5/2 + 3\mu, \qquad R_c^3 = 4 r_0^3/\lambda^2$$

$$1/12 \leq \mu \leq 1/3 \qquad \Rightarrow \qquad \boxed{2.75 \leq q \leq 3.5}$$

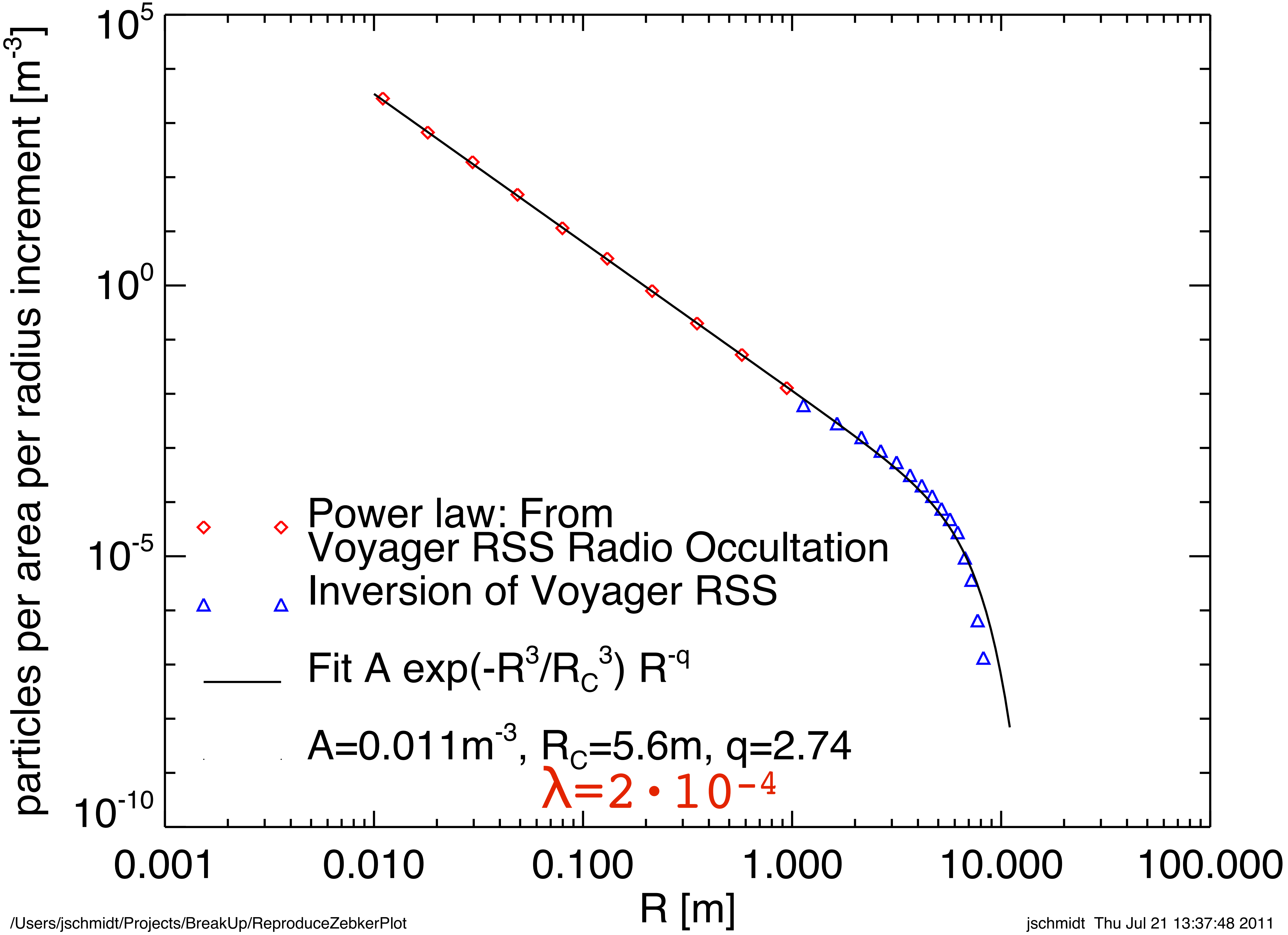

particles per area per radius increment [m^{-3}]
10^5
10^0
10^{-5}
10^{-10}
0.001
0.010
0.100
1.000
10.000
100.000
R [m]
◇ ◇ Power law: From Voyager RSS Radio Occultation
△ △ Inversion of Voyager RSS
/Users/jschmidt/Projects/BreakUp/ReproduceZebkerPlot
jschmidt Thu Jul 21 13:37:42 2011



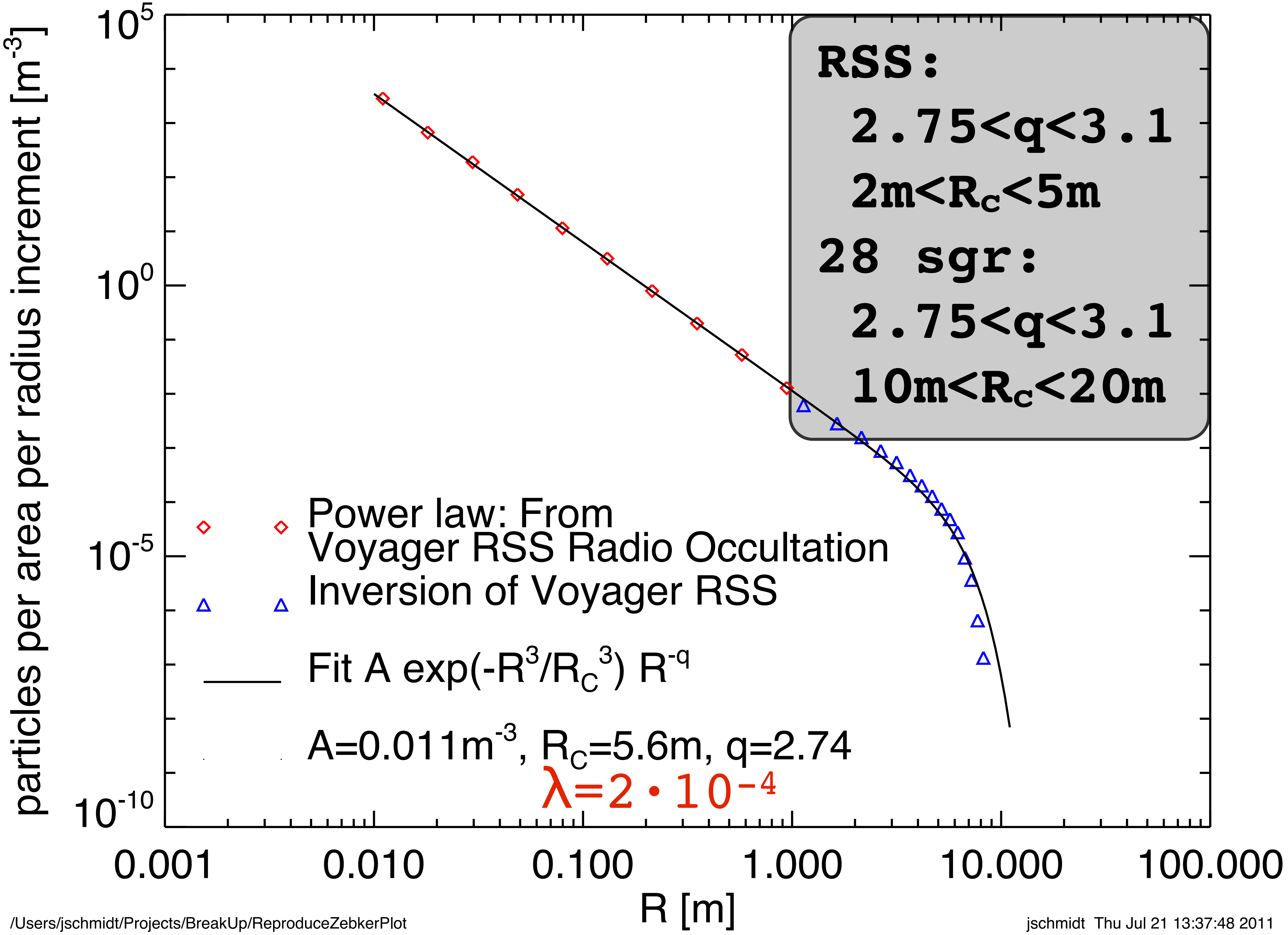
/Users/jschmidt/Projects/BreakUp/ReproduceZebkerPlot

jschmidt Thu Jul 21 13:37:48 2011

Tuesday, November 1, 2011



$\lambda=2\cdot10^{-4}$



RSS:

$2.75<q<3.1$

$2m<R_c<5m$

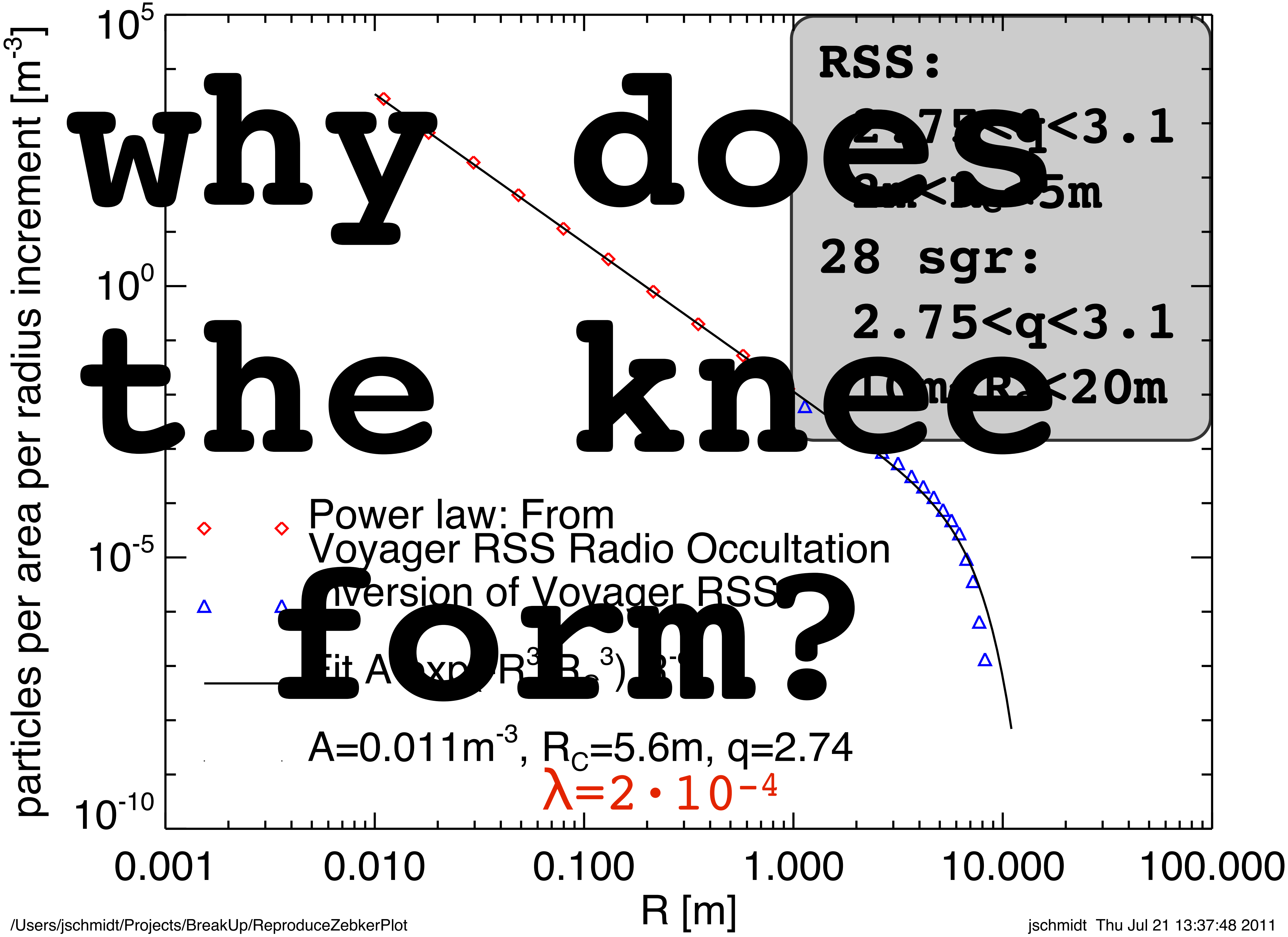
28 sgr:

$2.75<q<3.1$

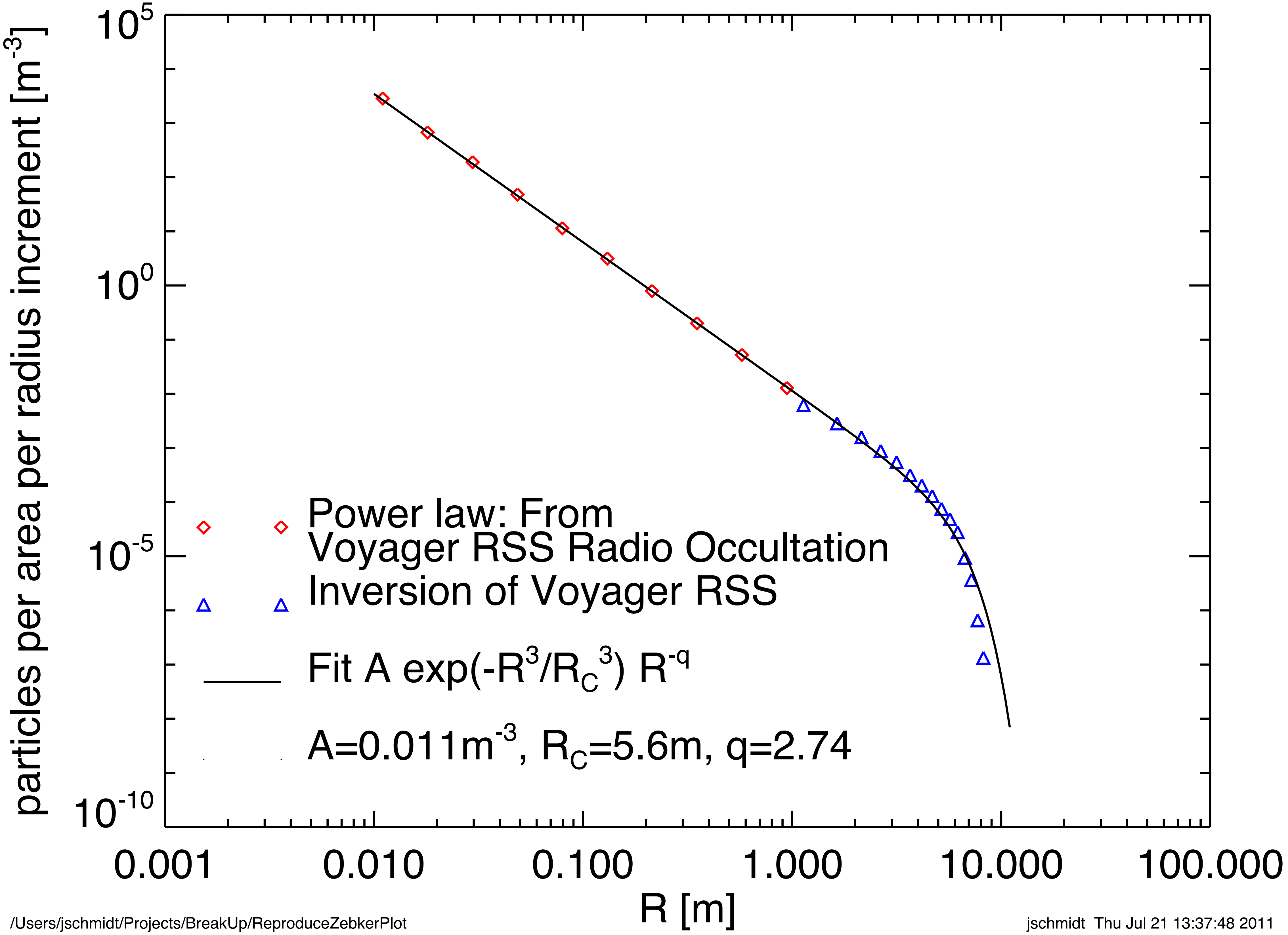
$10m<R_c<20m$

$\lambda=2\cdot10^{-4}$

why does the knee form?

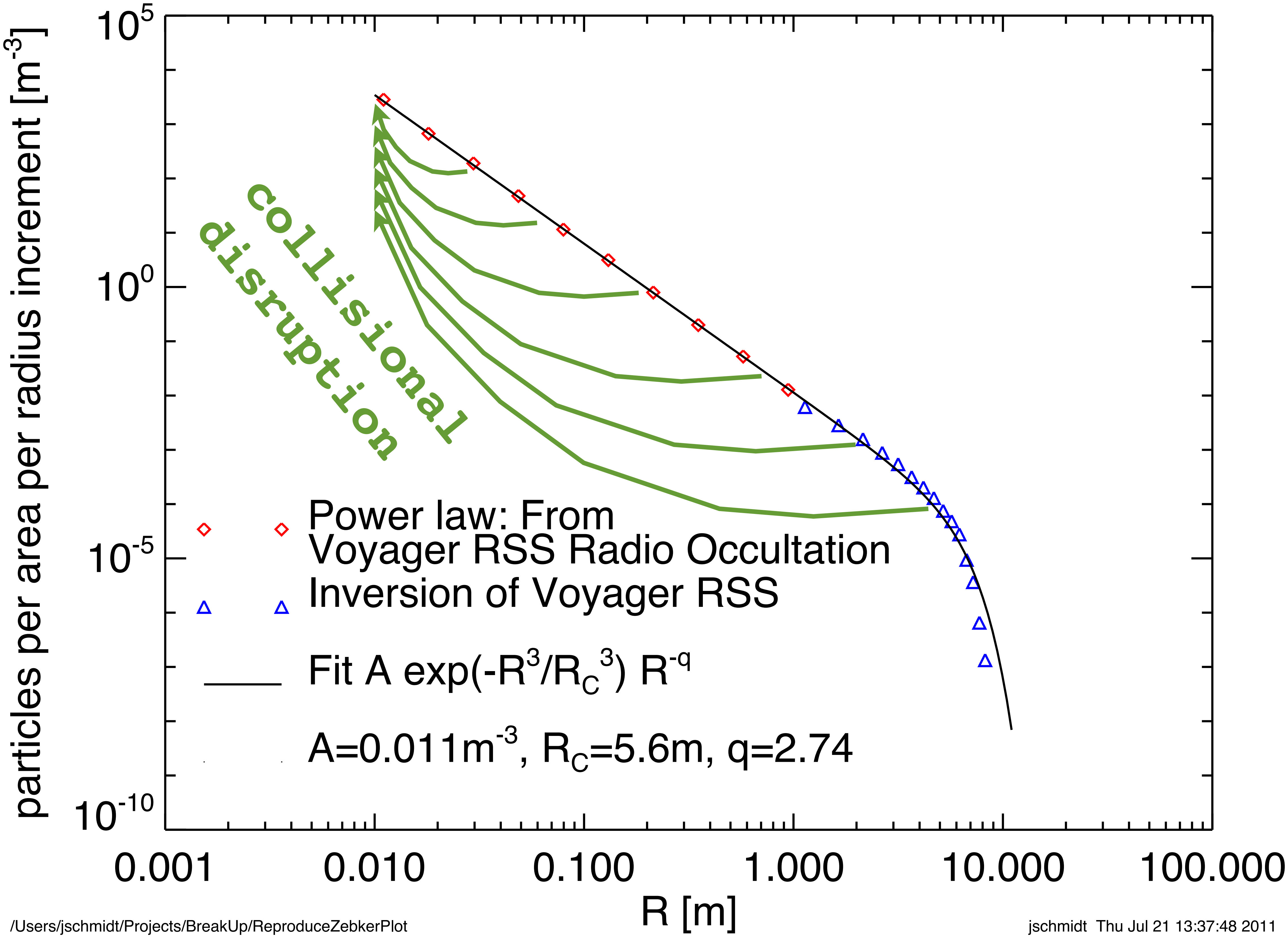


$\lambda=2\cdot10^{-4}$

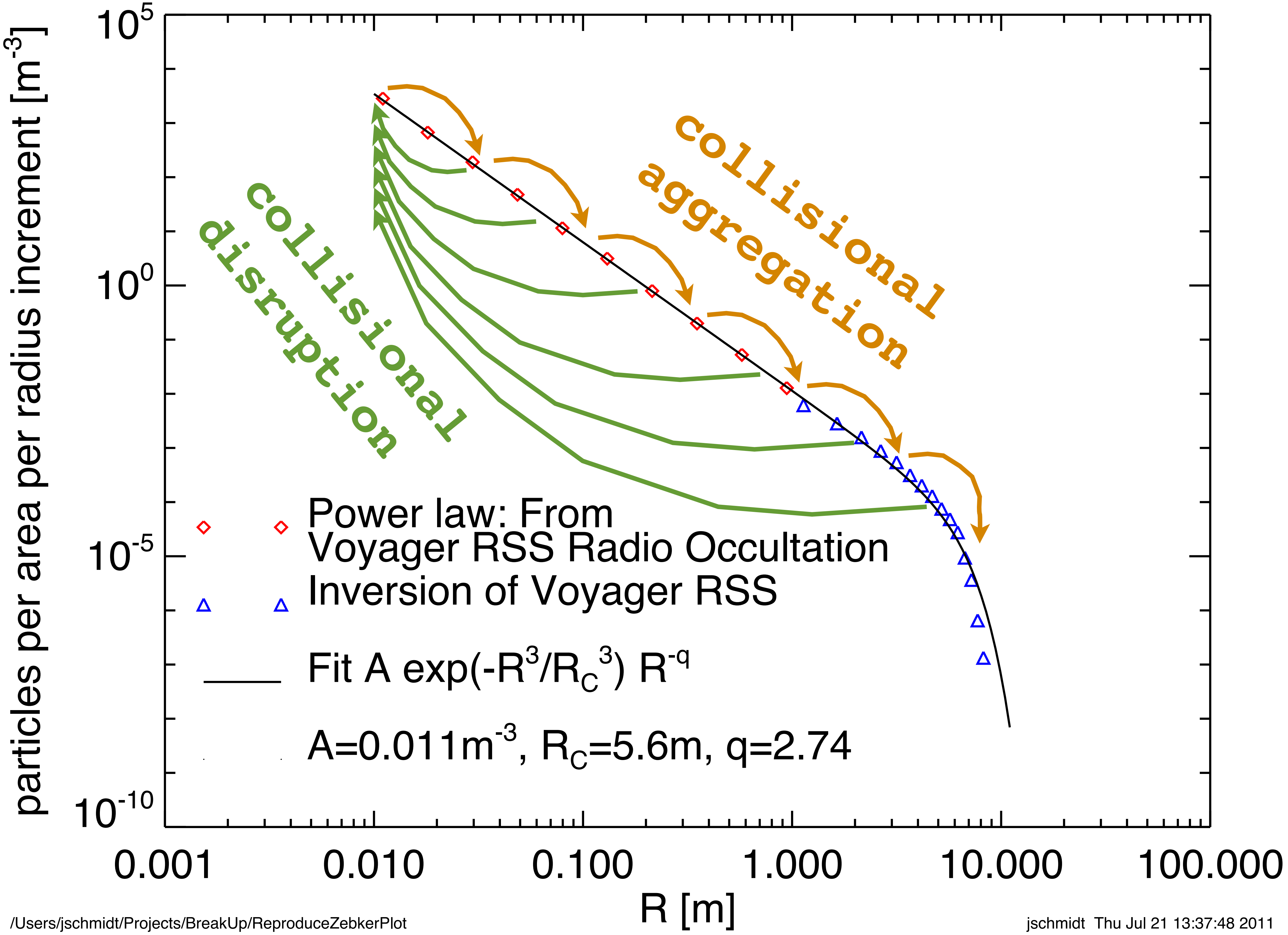


/Users/jschmidt/Projects/BreakUp/ReproduceZebkerPlot

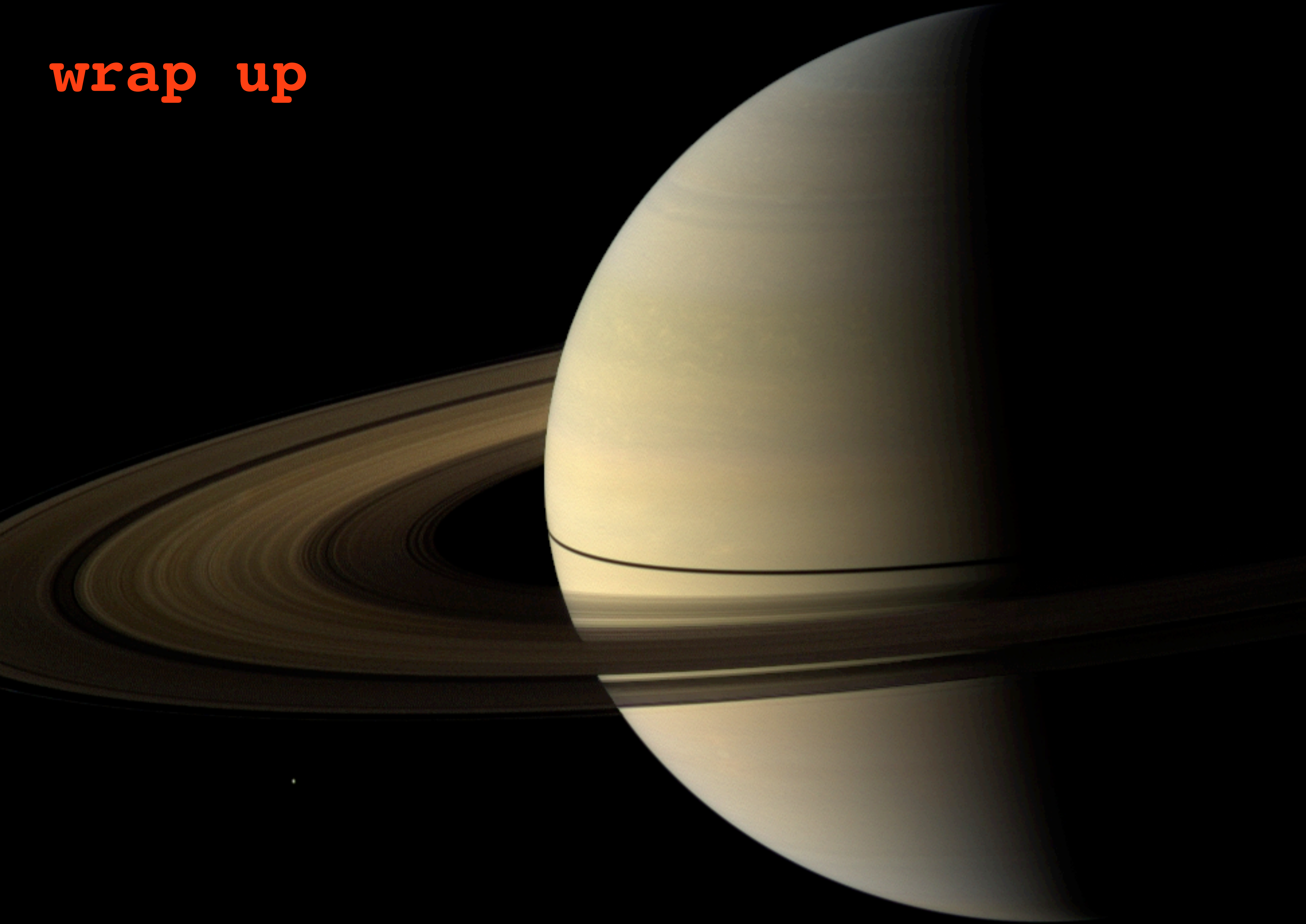
jschmidt Thu Jul 21 13:37:48 2011



Tuesday, November 1, 2011

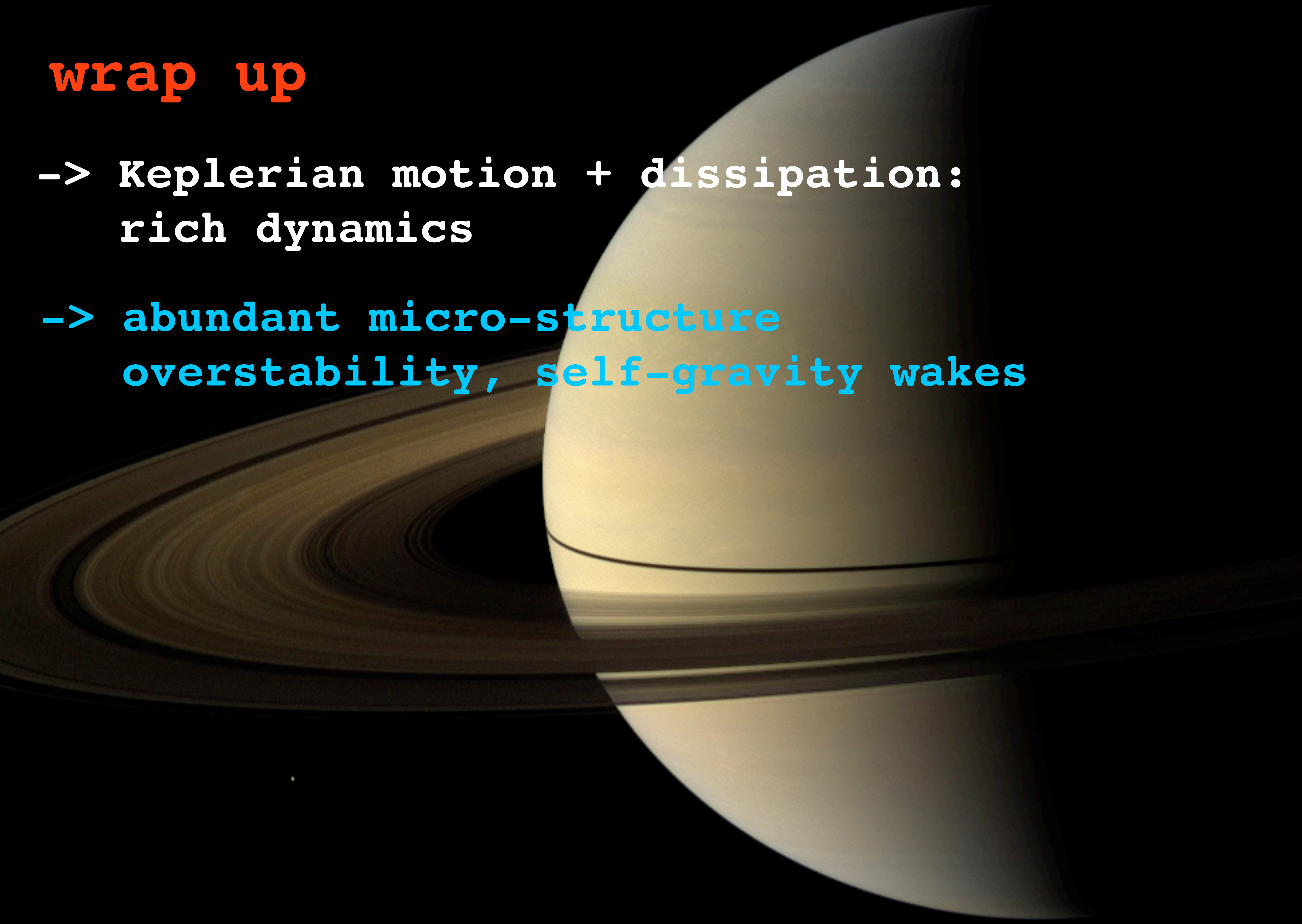

particles per area per radius increment [m^-3]
10^5
10^0
10^-5
10^-10
0.001
0.010
0.100
1.000
10.000
100.000
R [m]
collisional aggregation
collisional disruption
Power law: From Voyager RSS Radio Occultation
Inversion of Voyager RSS
Fit A exp(-R^3/R_C^3) R^-q
A=0.011m^-3, R_C=5.6m, q=2.74
/Users/jschmidt/Projects/BreakUp/ReproduceZebkerPlot
jschmidt Thu Jul 21 13:37:48 2011

wrap up



wrap up

-> Keplerian motion + dissipation:
rich dynamics

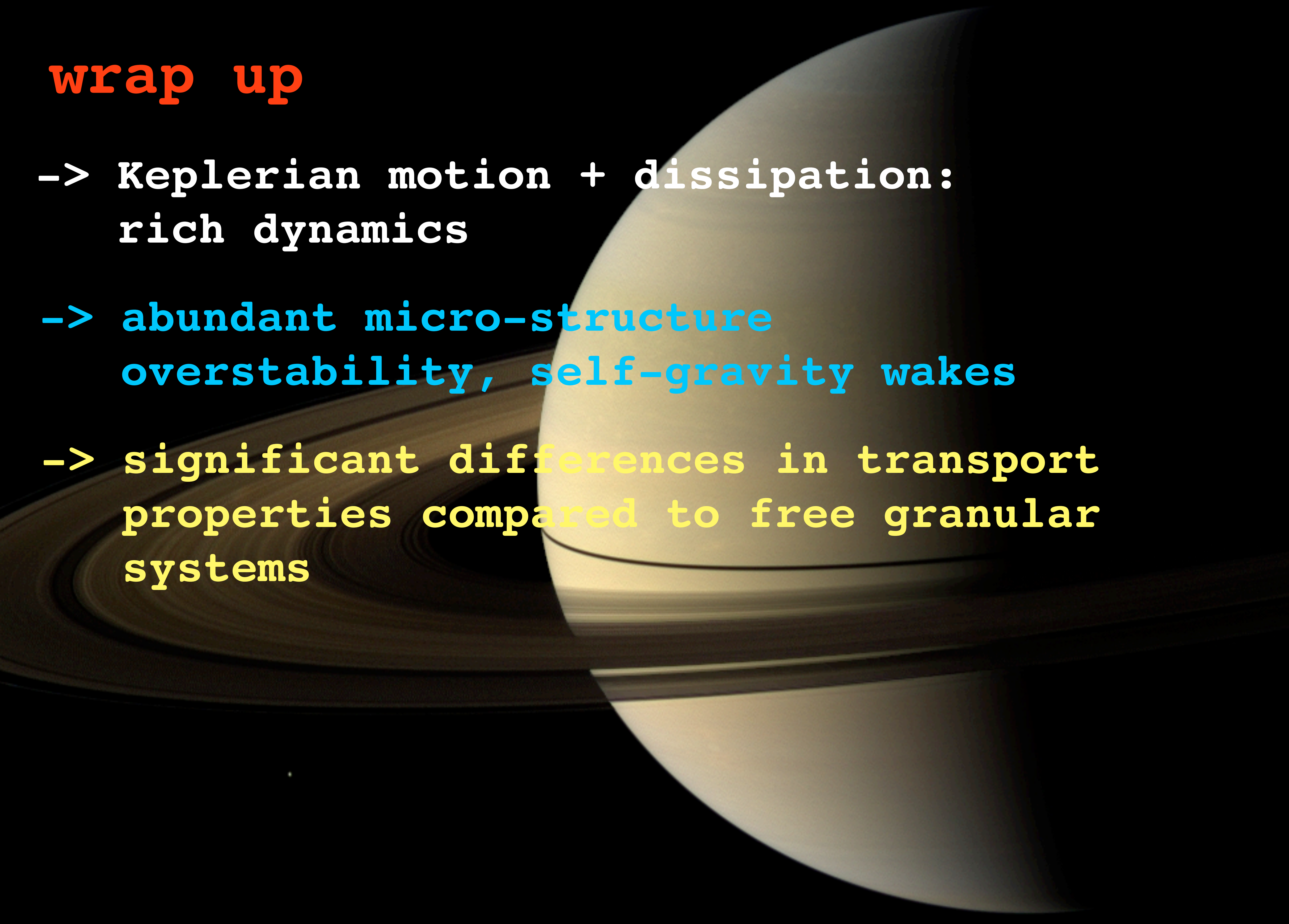


wrap up

-> Keplerian motion + dissipation:
rich dynamics

-> abundant micro-structure
overstability, self-gravity wakes

wrap up



- -> Keplerian motion + dissipation: rich dynamics
- -> abundant micro-structure overstability, self-gravity wakes
- -> significant differences in transport properties compared to free granular systems

wrap up

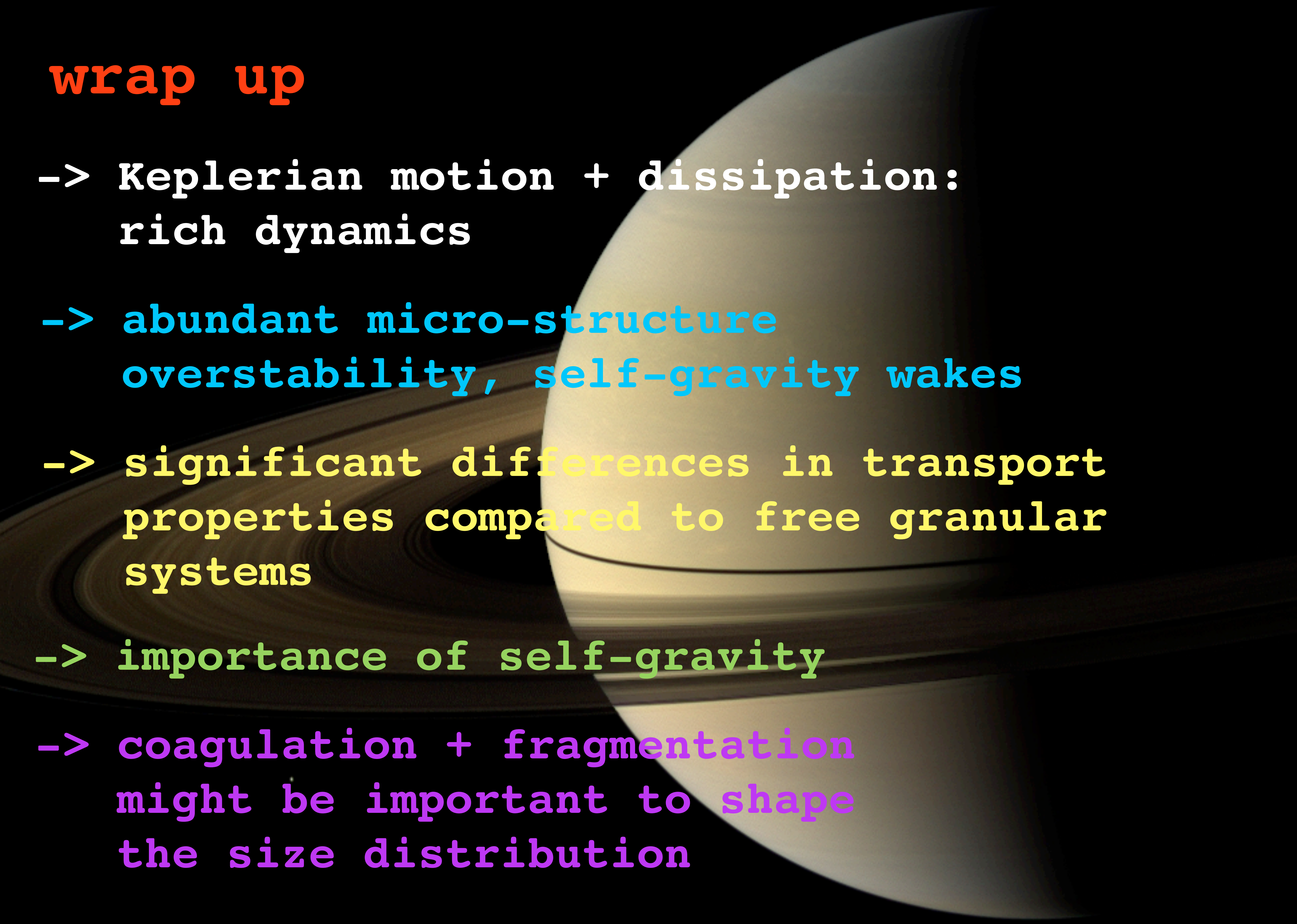
-> Keplerian motion + dissipation:
rich dynamics

-> abundant micro-structure
overstability, self-gravity wakes

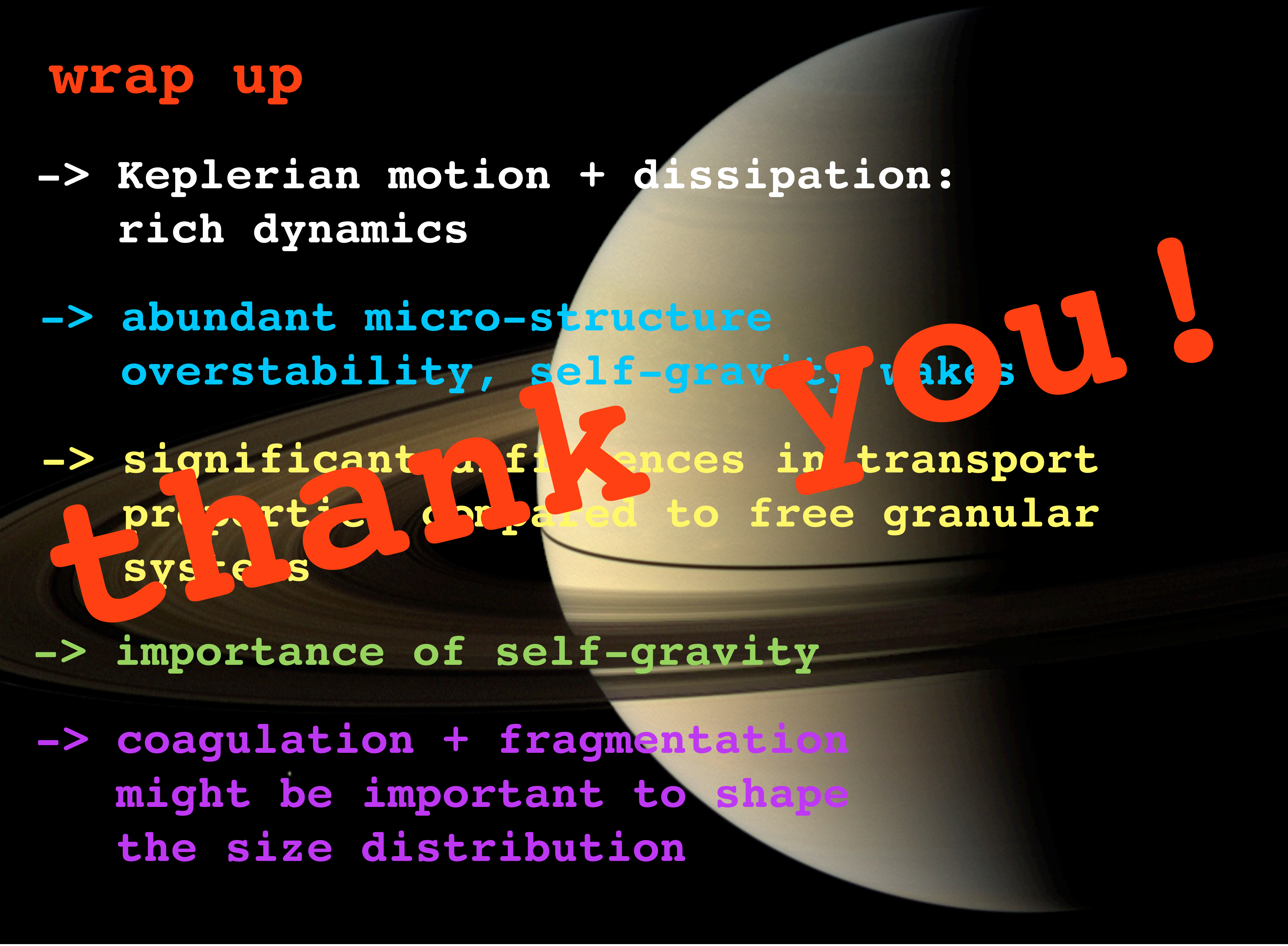
-> significant differences in transport properties compared to free granular systems

-> importance of self-gravity

wrap up



- -> Keplerian motion + dissipation: rich dynamics
- -> abundant micro-structure overstability, self-gravity wakes
- -> significant differences in transport properties compared to free granular systems
- -> importance of self-gravity
- -> coagulation + fragmentation might be important to shape the size distribution



wrap up

-> Keplerian motion + dissipation:
rich dynamics

-> abundant micro-structure
overstability, self-gravity wakes

-> significant differences in transport
properties compared to free granular
systems

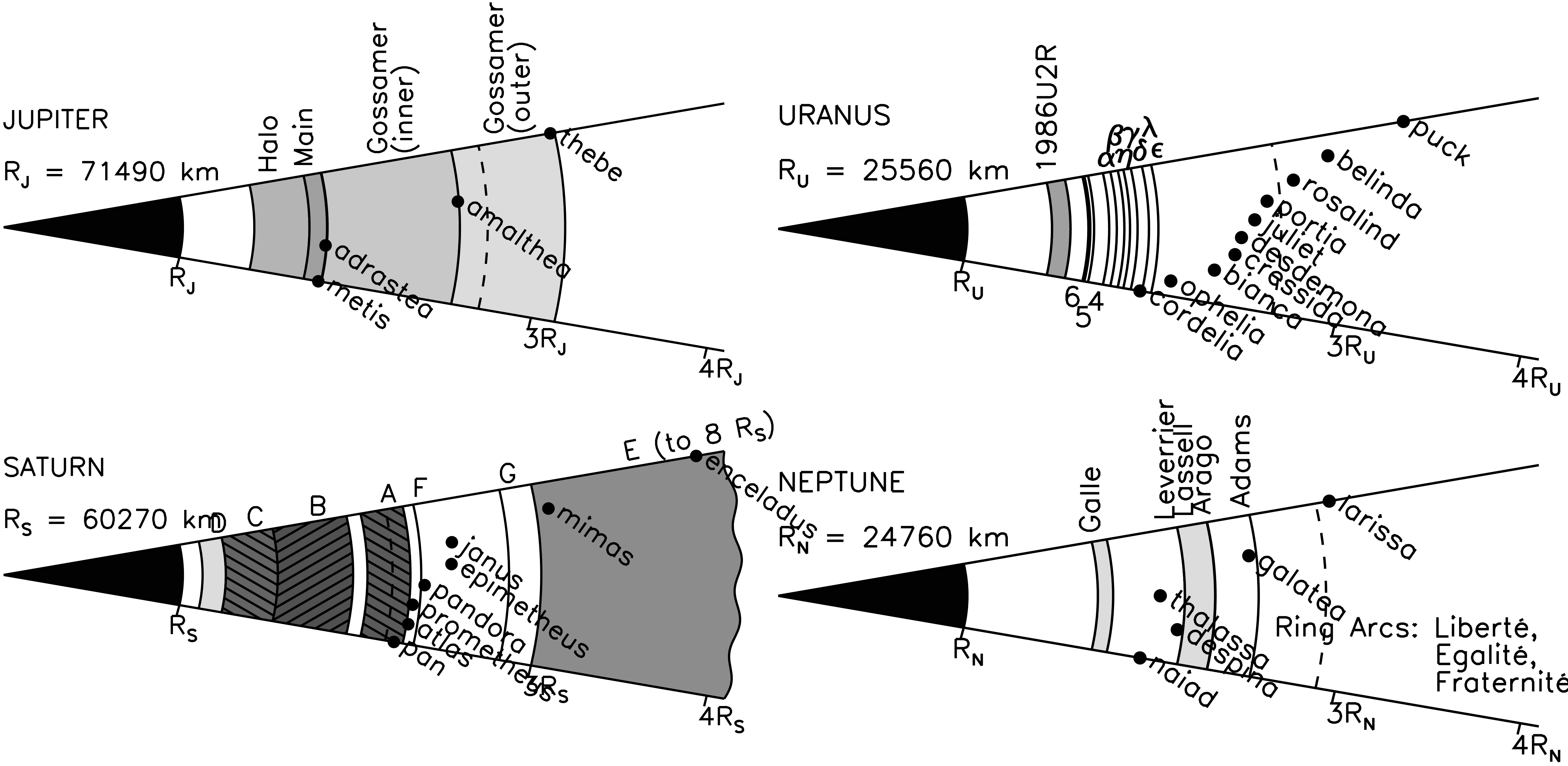
-> importance of self-gravity

-> coagulation + fragmentation
might be important to shape
the size distribution

thank you!

spare slides

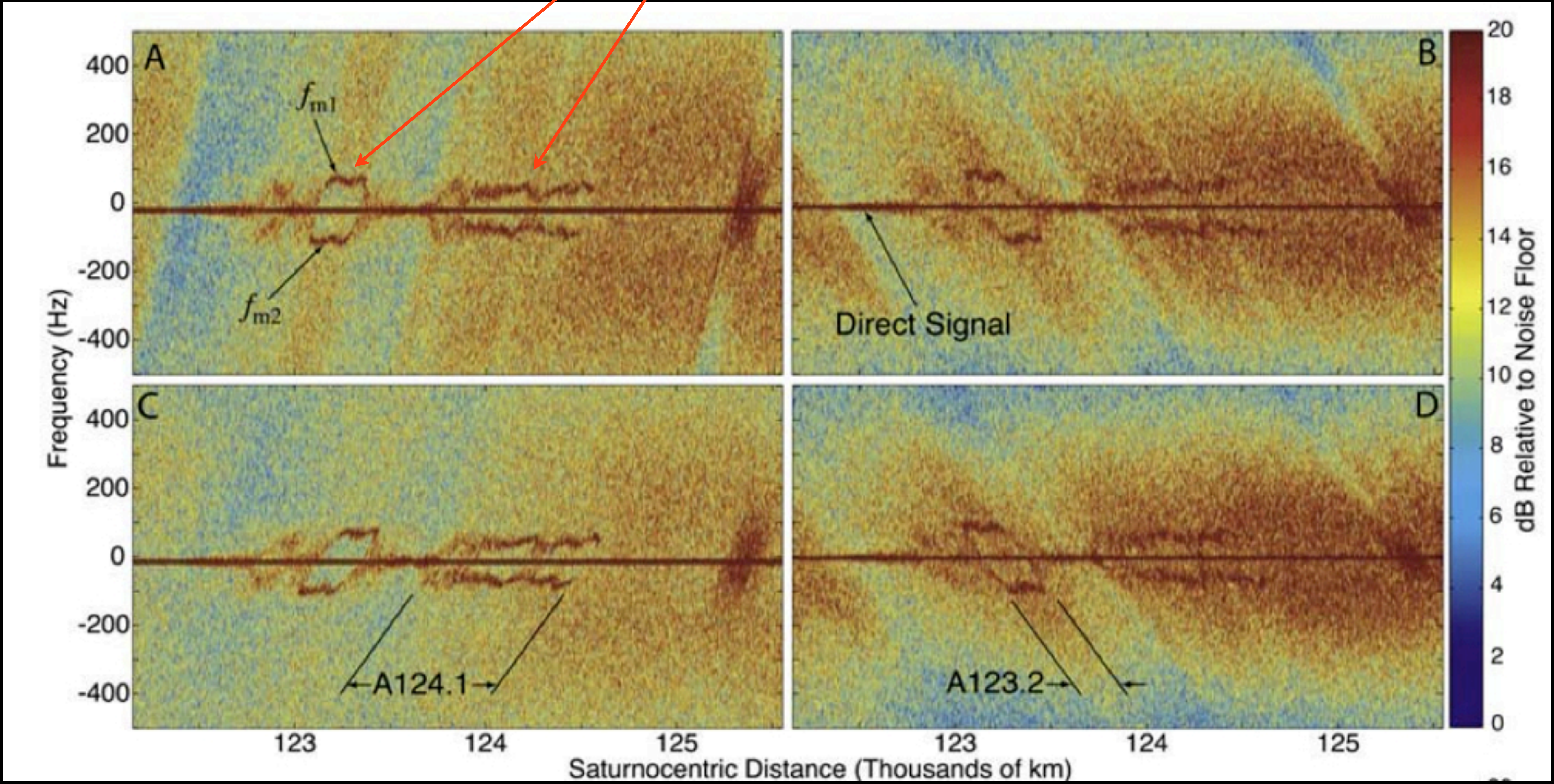
solar system ring map



RSS: Thompson et al 2007

In the A ring

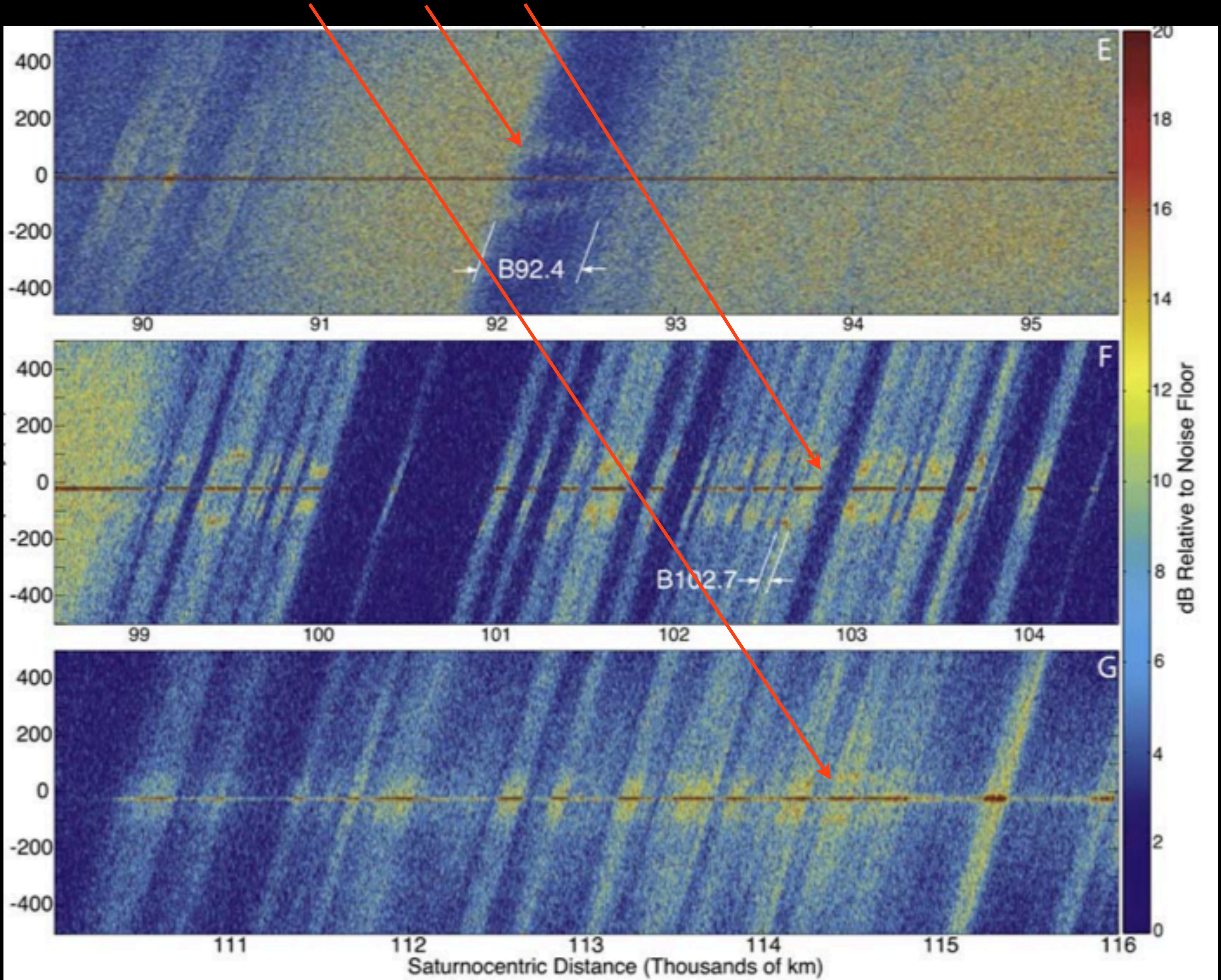
150m-200m radial wave

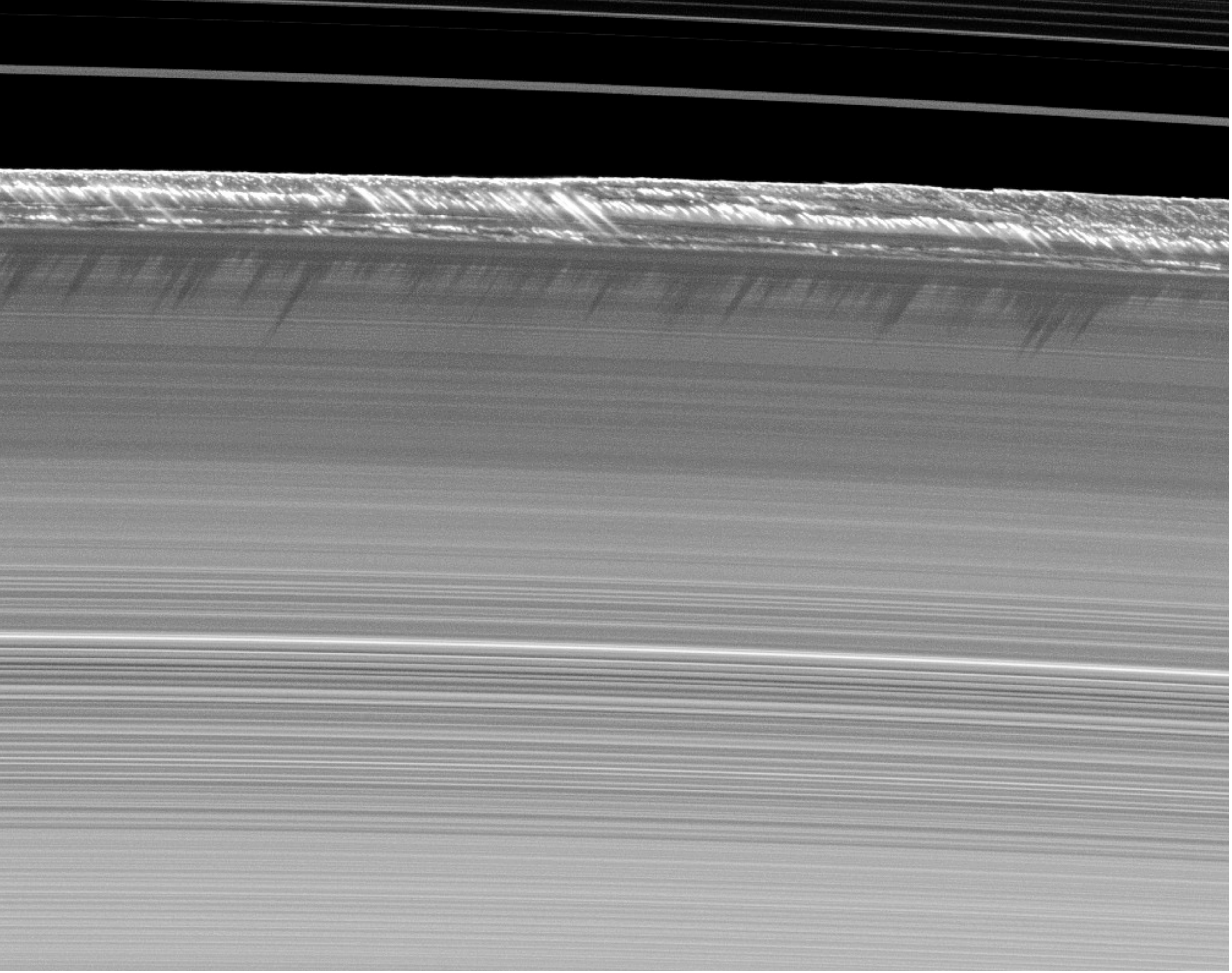


RSS: Thompson et al 2007

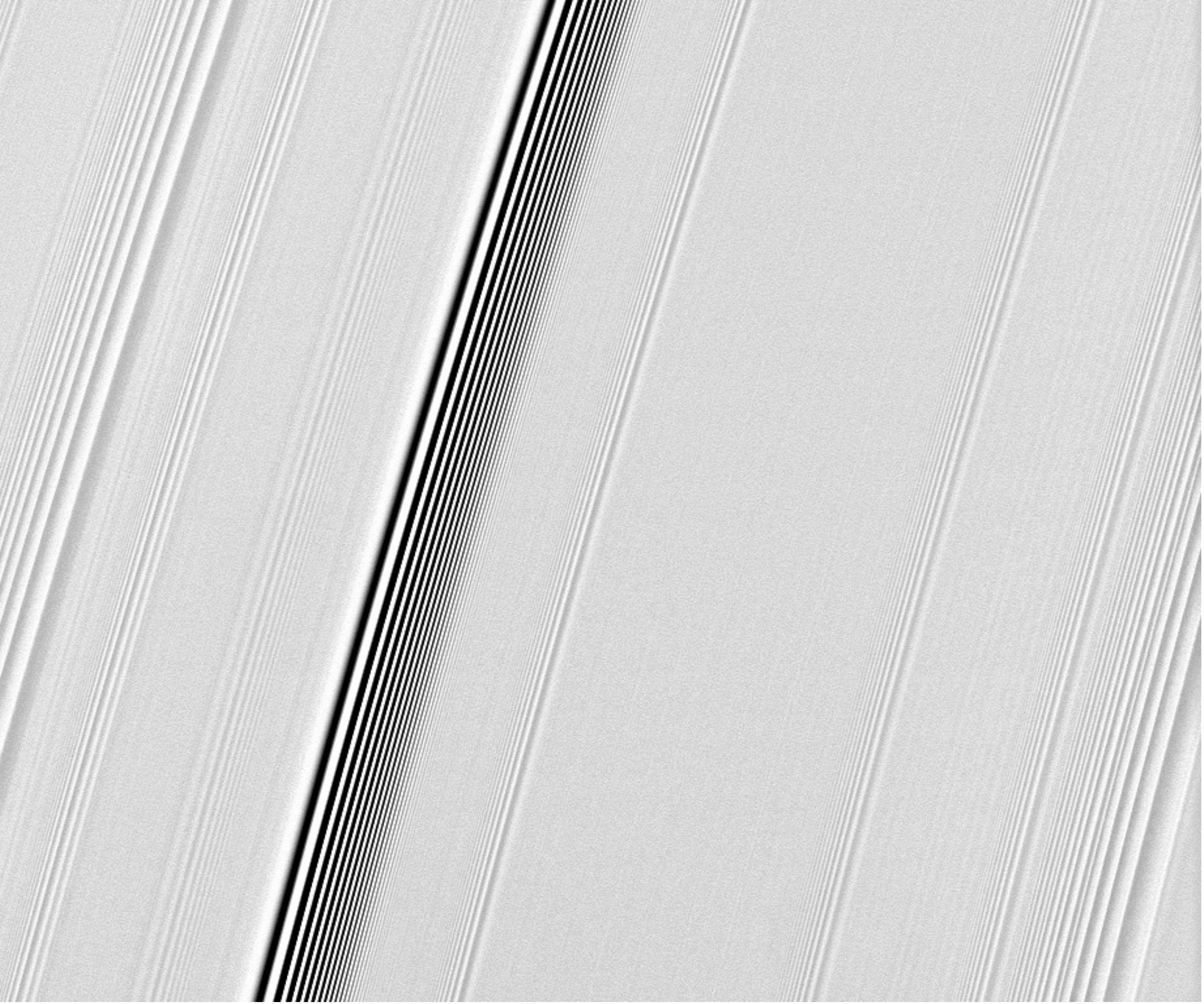
In the B ring

150m-200m radial waves

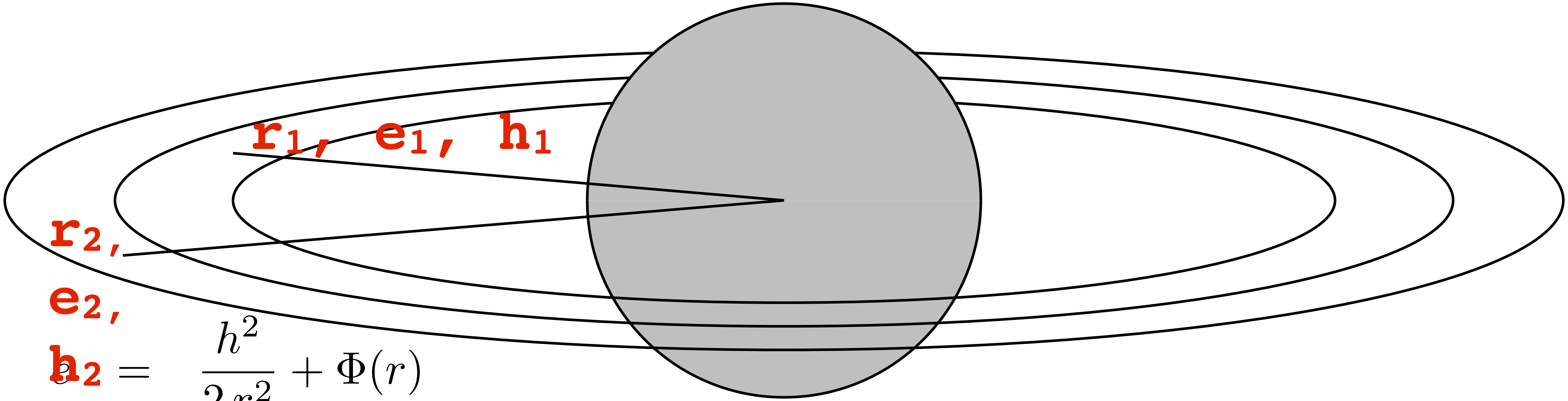




Tuesday, November 1, 2011



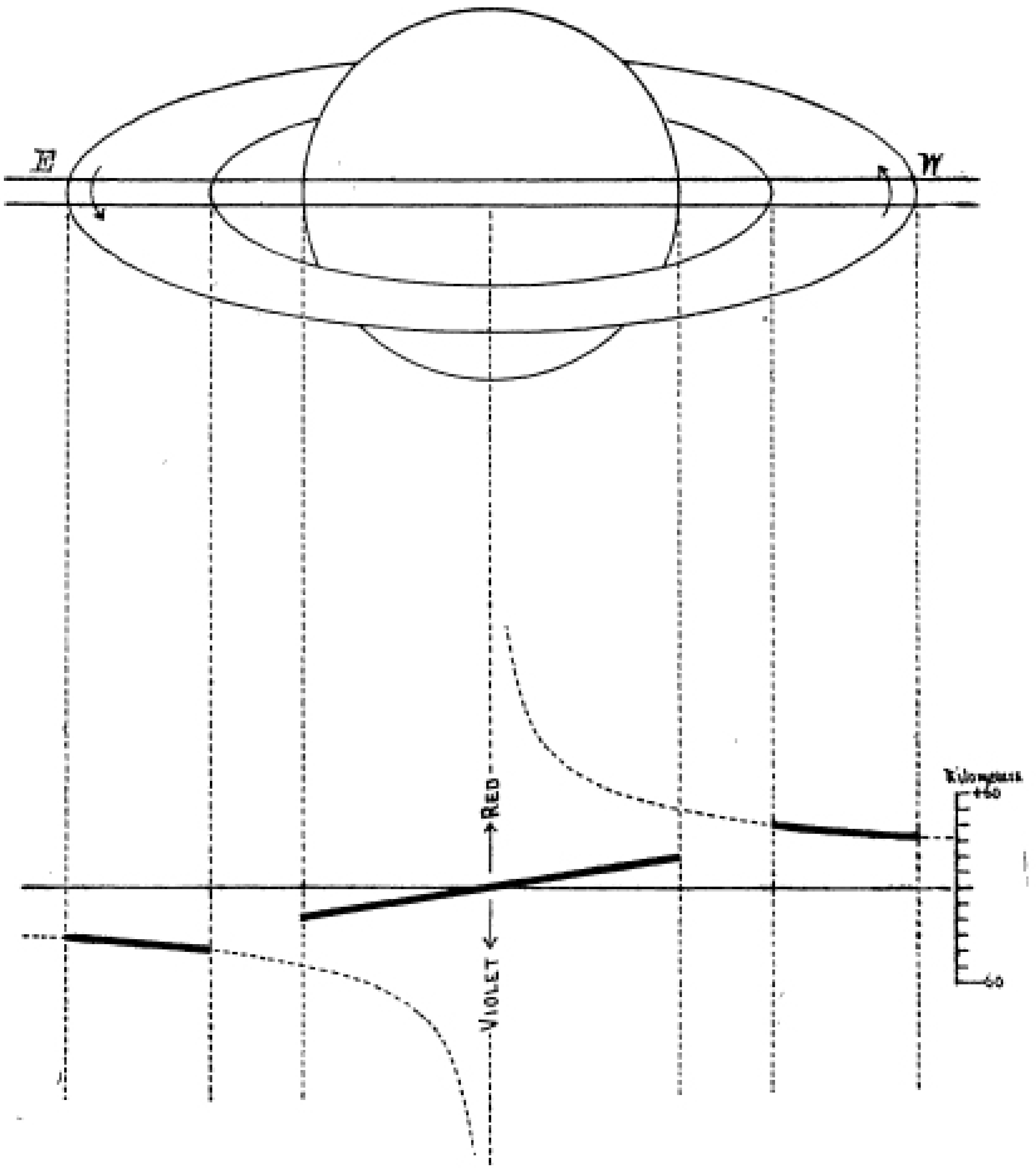
Global budget of energy and angular momentum



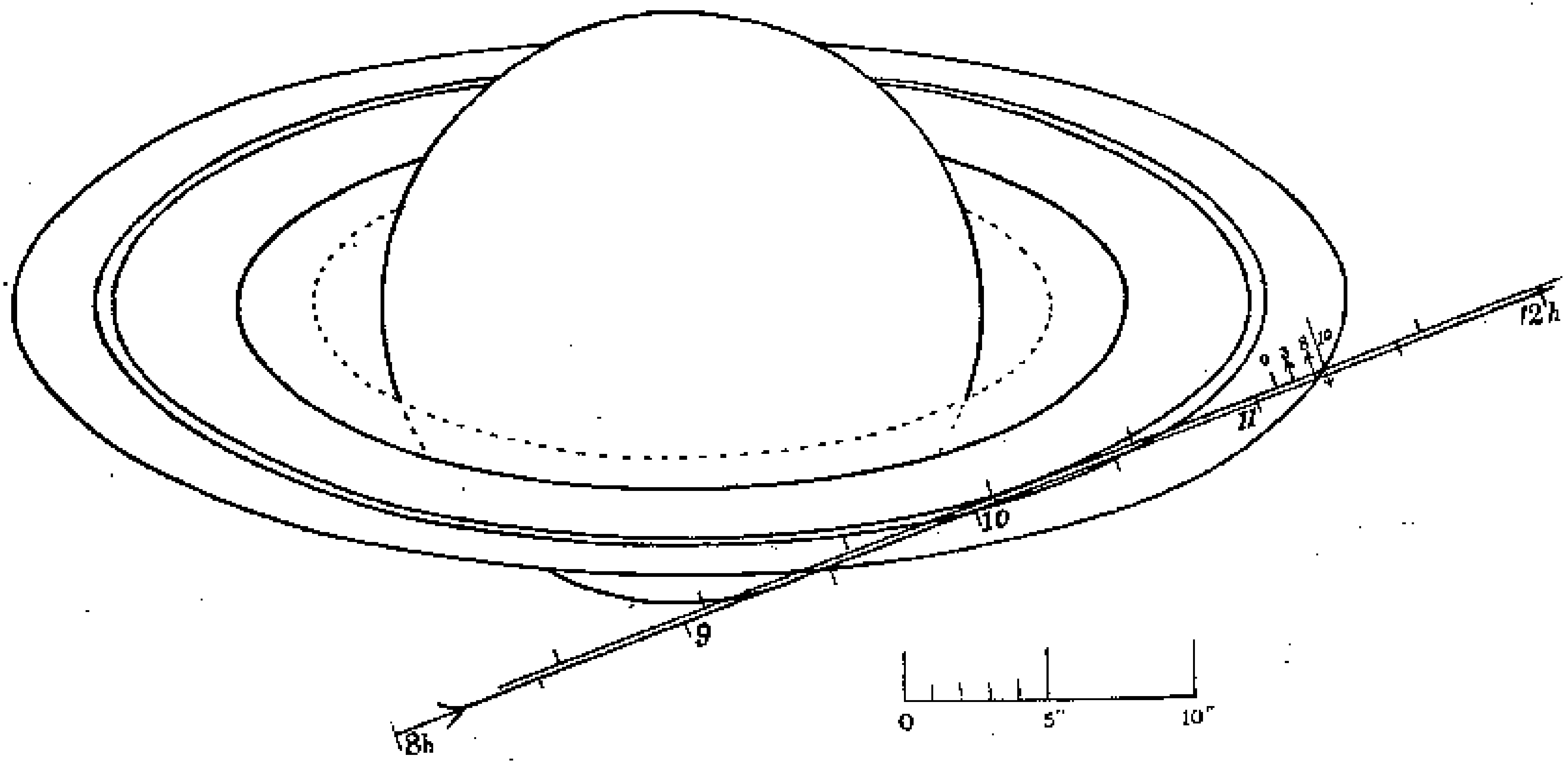
$$\begin{aligned} e &= \frac{h^2}{2\,r^2} + \Phi(r) \\ h &= \Omega\, r^2 \\ \delta E &= \delta(m_1 e_1) + \delta(m_2 e_2) \\ &= \delta m_1 e_1 + \delta m_2 e_2 + \Omega_1 m_1 \delta h_1 + \Omega_2 m_2 \delta h_2 \\ &= \delta m_1 \left[(e_1 - \Omega_1 h_1) - (e_2 - \Omega_2 h_2) \right] + \delta H_1 (\Omega_1 - \Omega_2) \end{aligned}$$

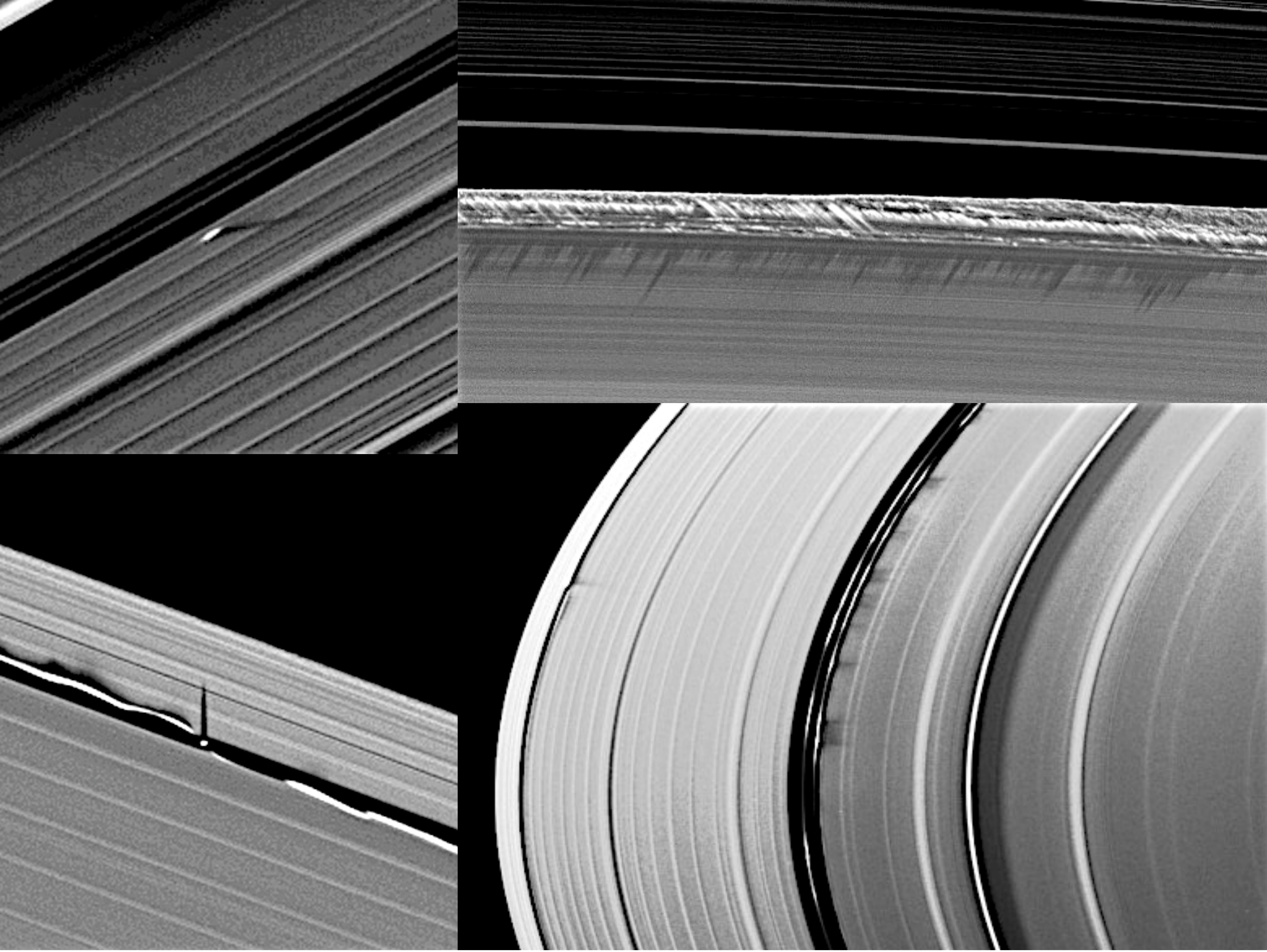
$$e - \Omega h = -\frac{3}{2}\Omega^2 r^2 \qquad \frac{de}{dh} = \Omega$$

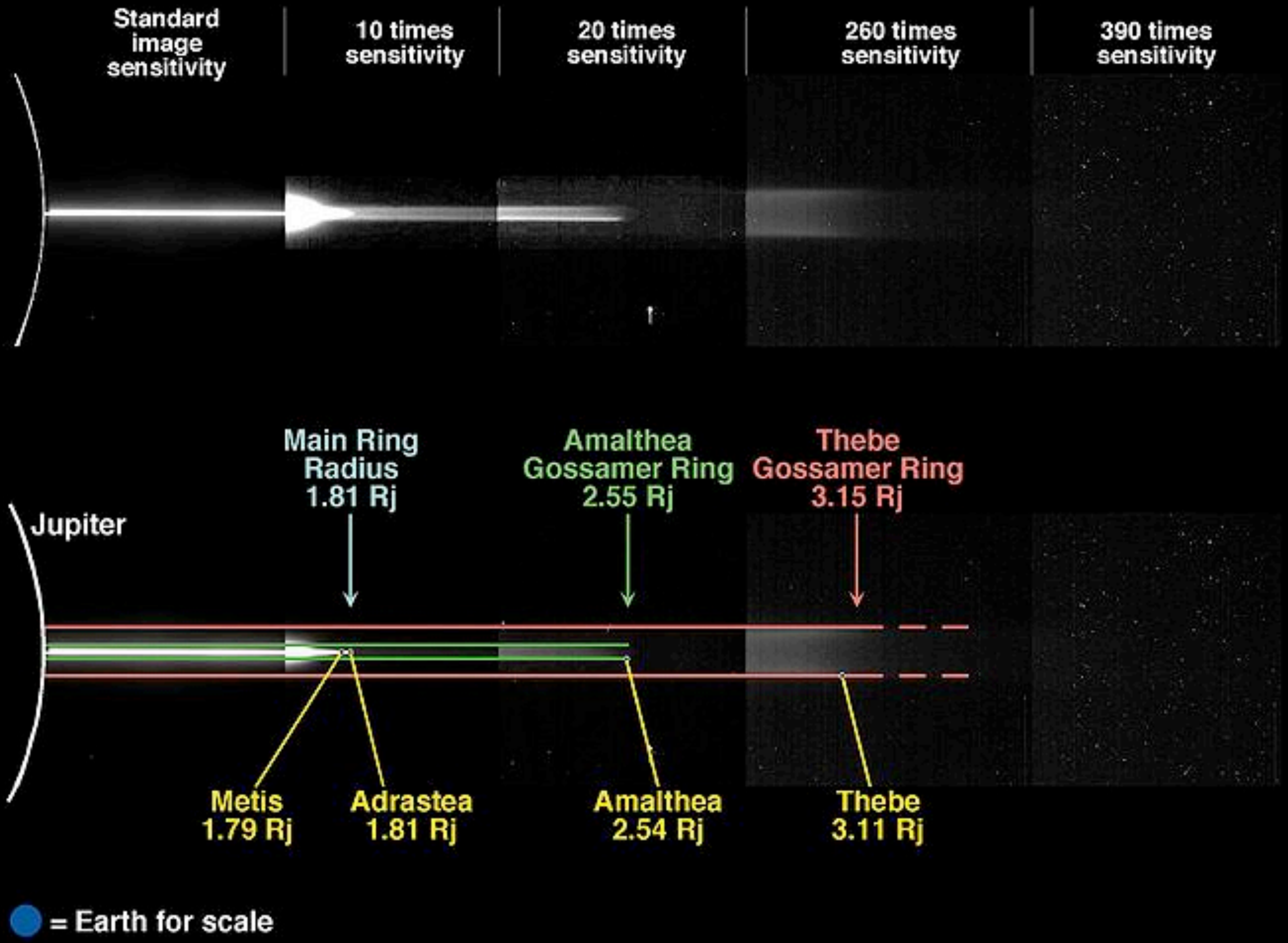
Some historical remarks



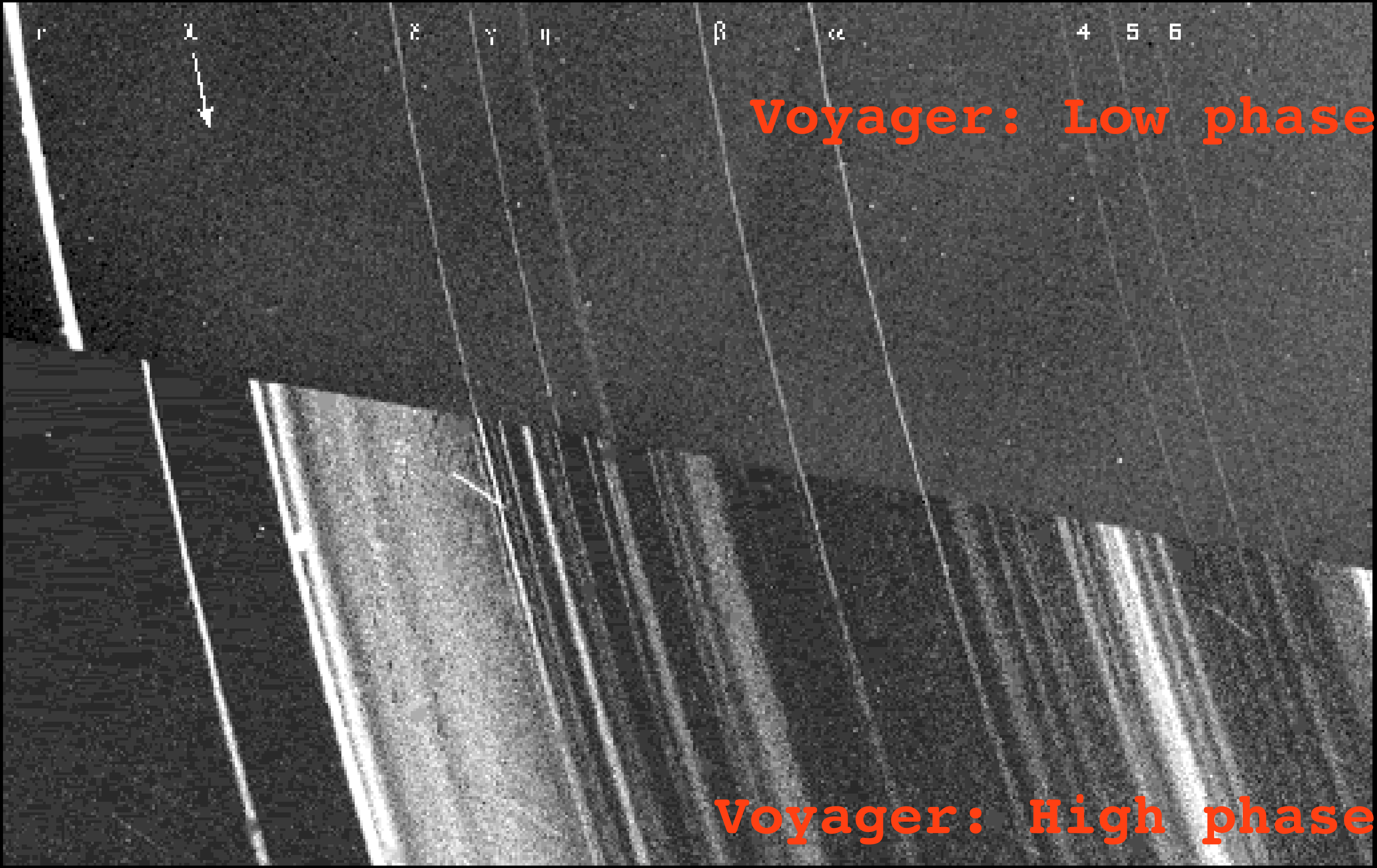
Some historical remarks



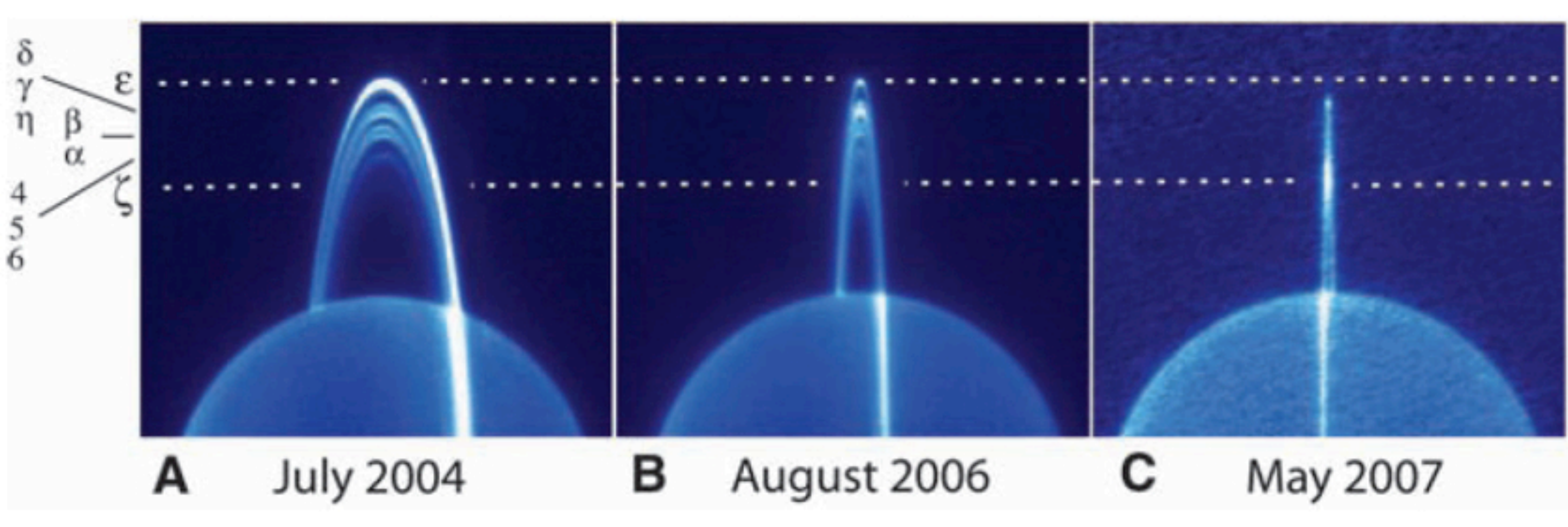



Standard image sensitivity
10 times sensitivity
20 times sensitivity
260 times sensitivity
390 times sensitivity
Main Ring Radius 1.81 Rj
Amalthea Gossamer Ring 2.55 Rj
Thebe Gossamer Ring 3.15 Rj
Jupiter
Metis 1.79 Rj
Adrastea 1.81 Rj
Amalthea 2.54 Rj
Thebe 3.11 Rj
● = Earth for scale

δ λ ε γ η β α 4 5 6



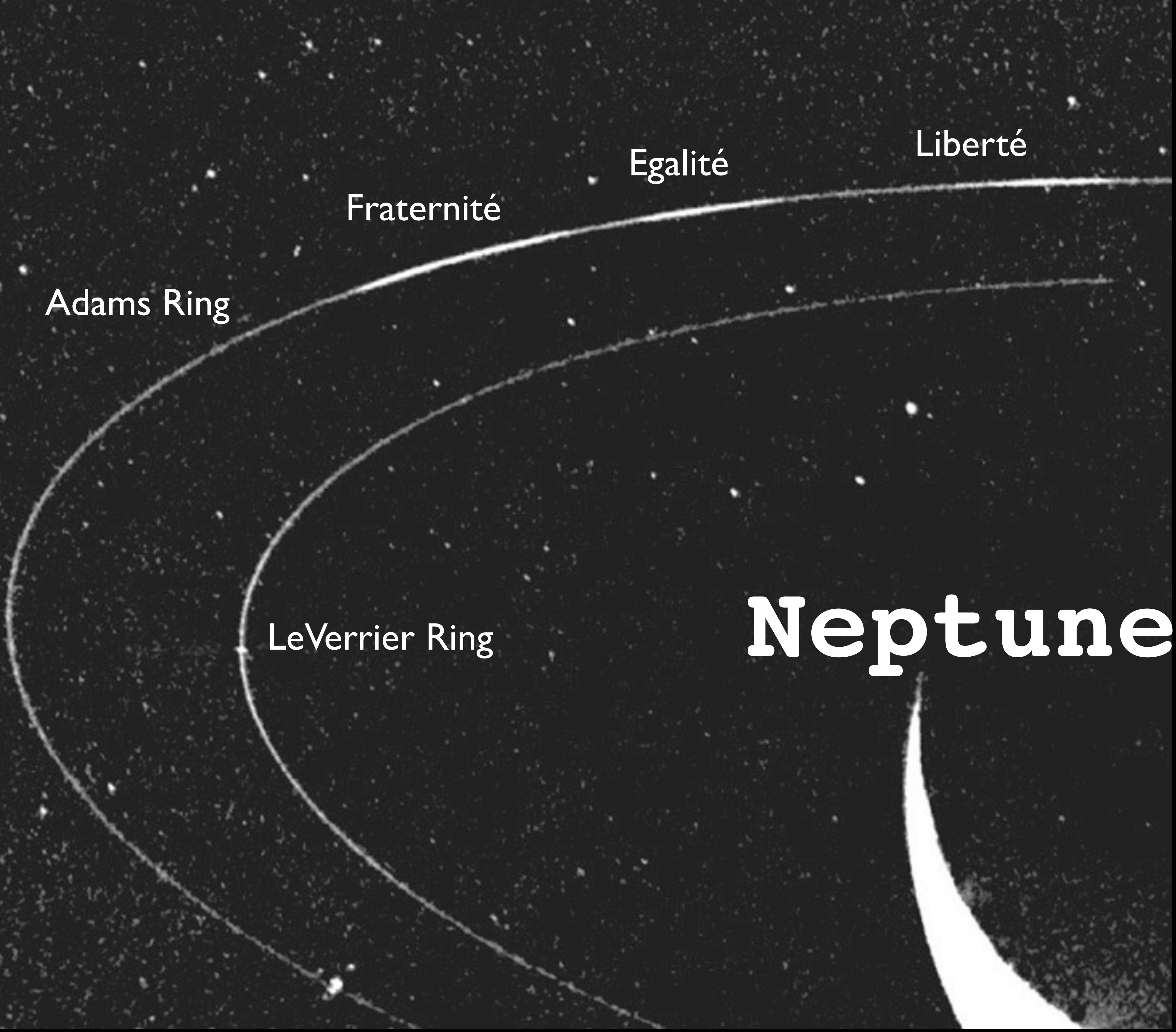
Keck observations of Uranus ring plane crossing

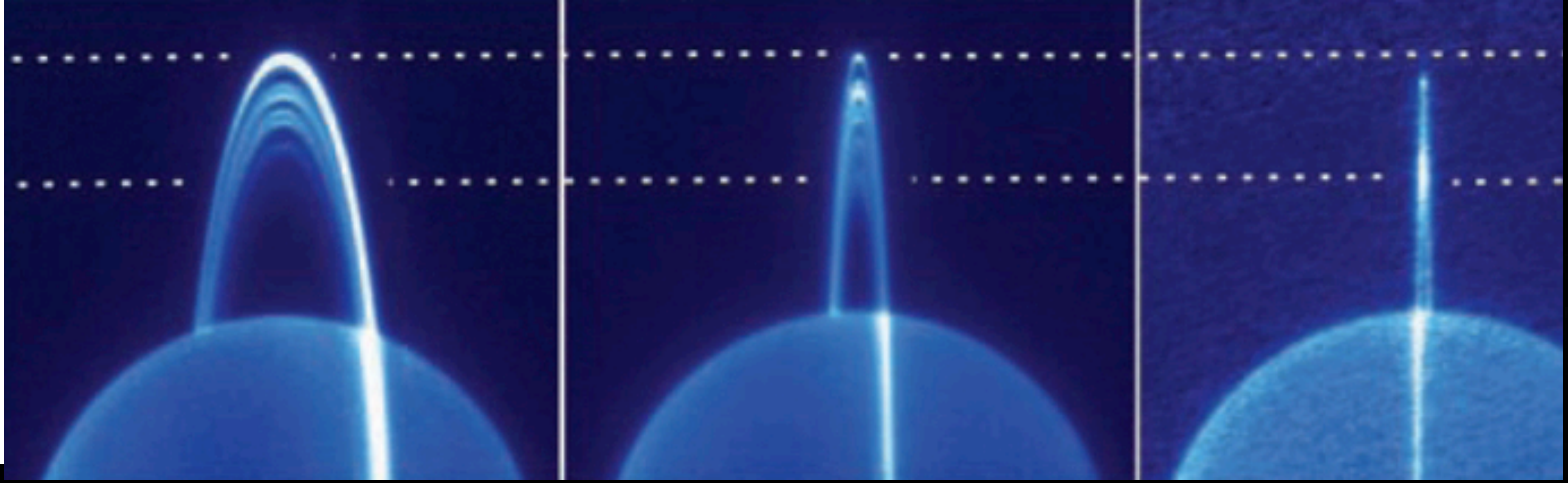


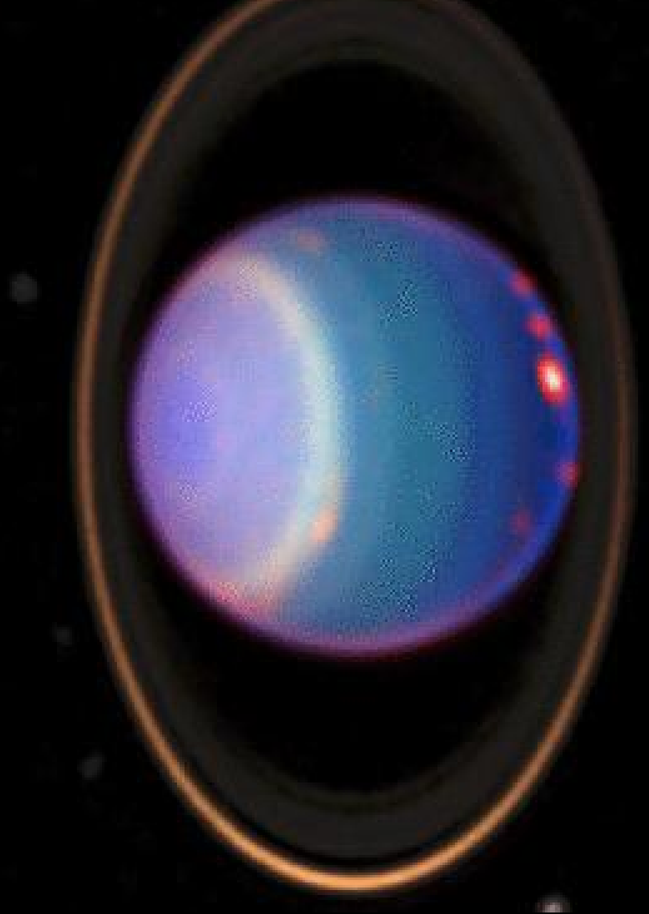
(De Pater et al, Science, 2007)

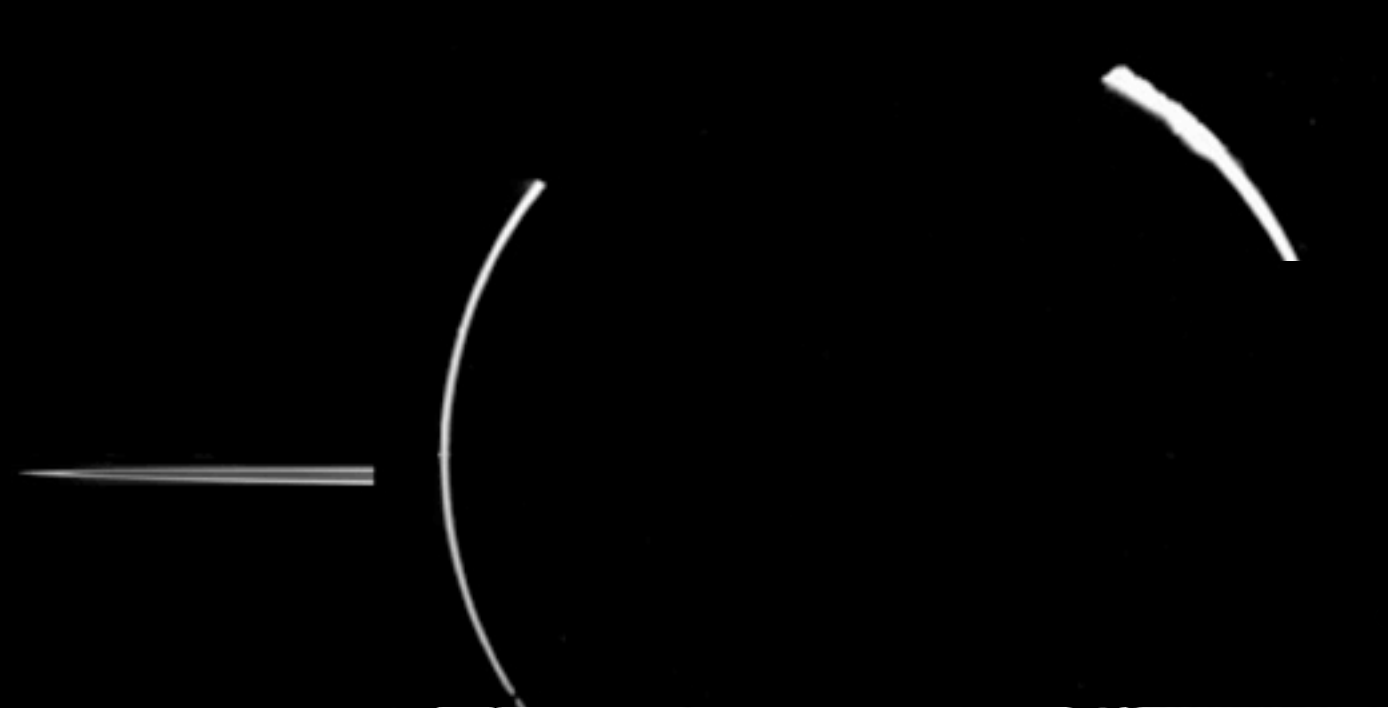
Edge-On:
Brightening of
dust rings

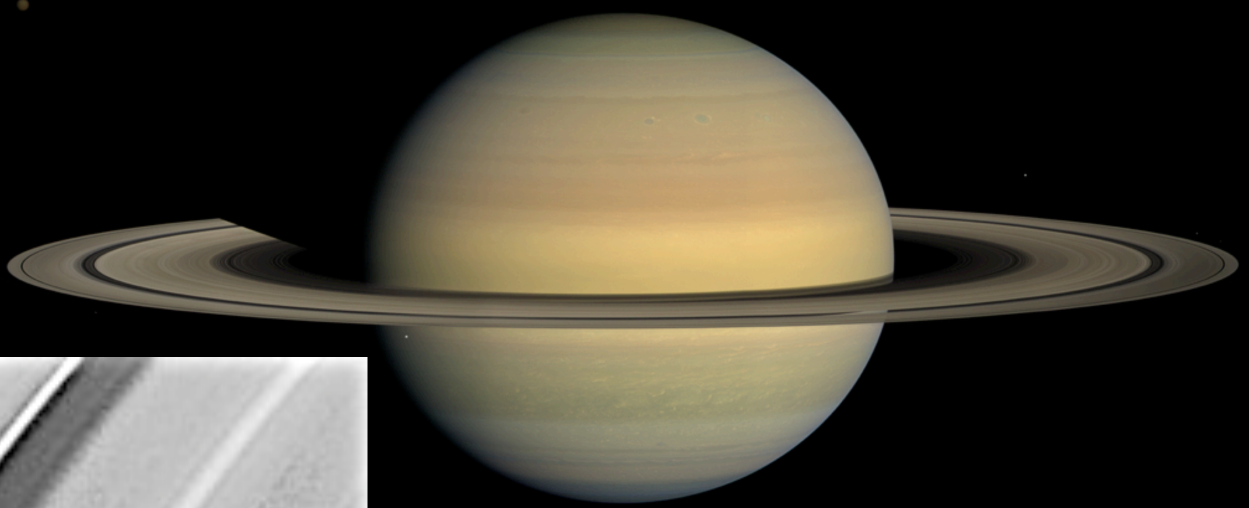
Neptune

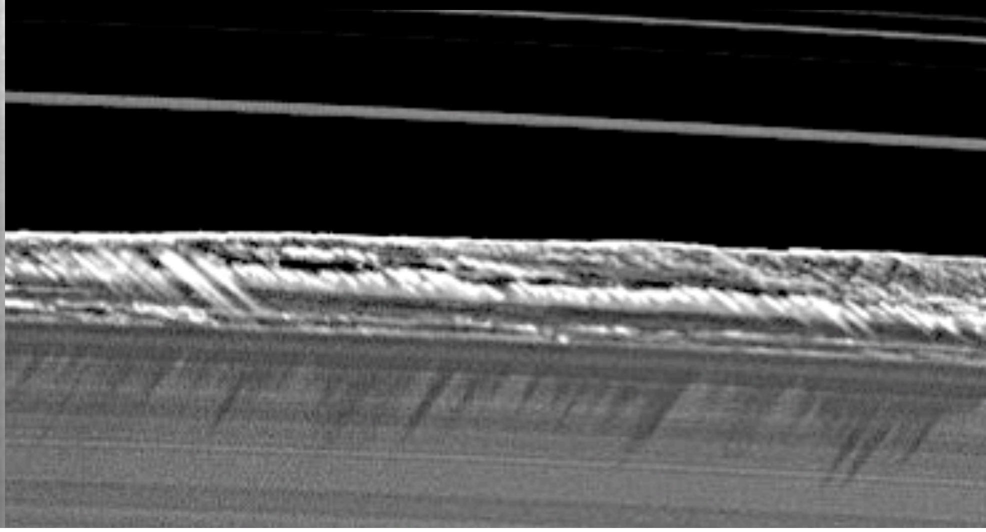

Liberté
Egalité
Fraternité
Adams Ring
LeVerrier Ring
Neptune







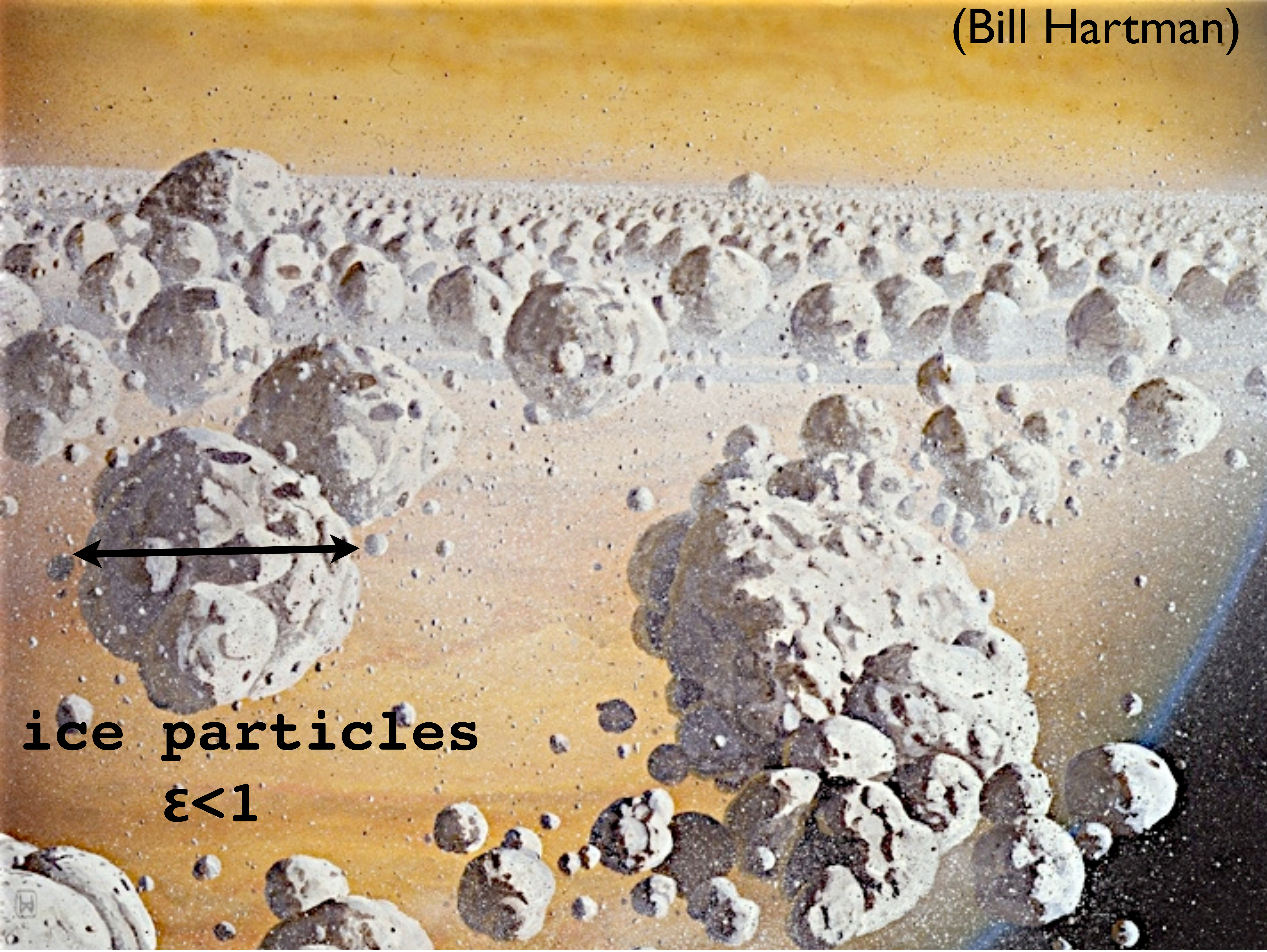




ring creation?
ring re-creation?

© W.K Hartmann

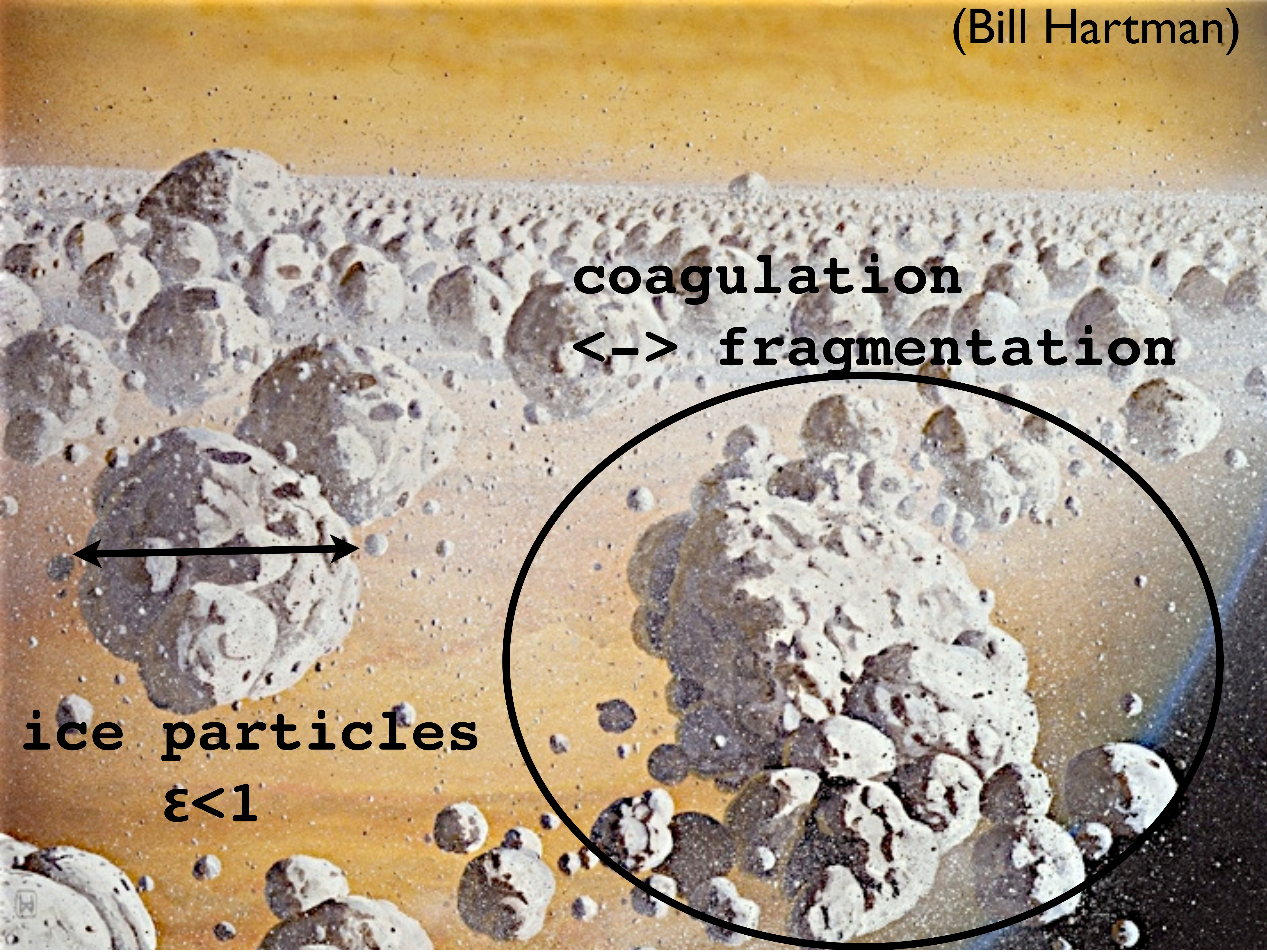
(Bill Hartman)



(Bill Hartman)

ice particles

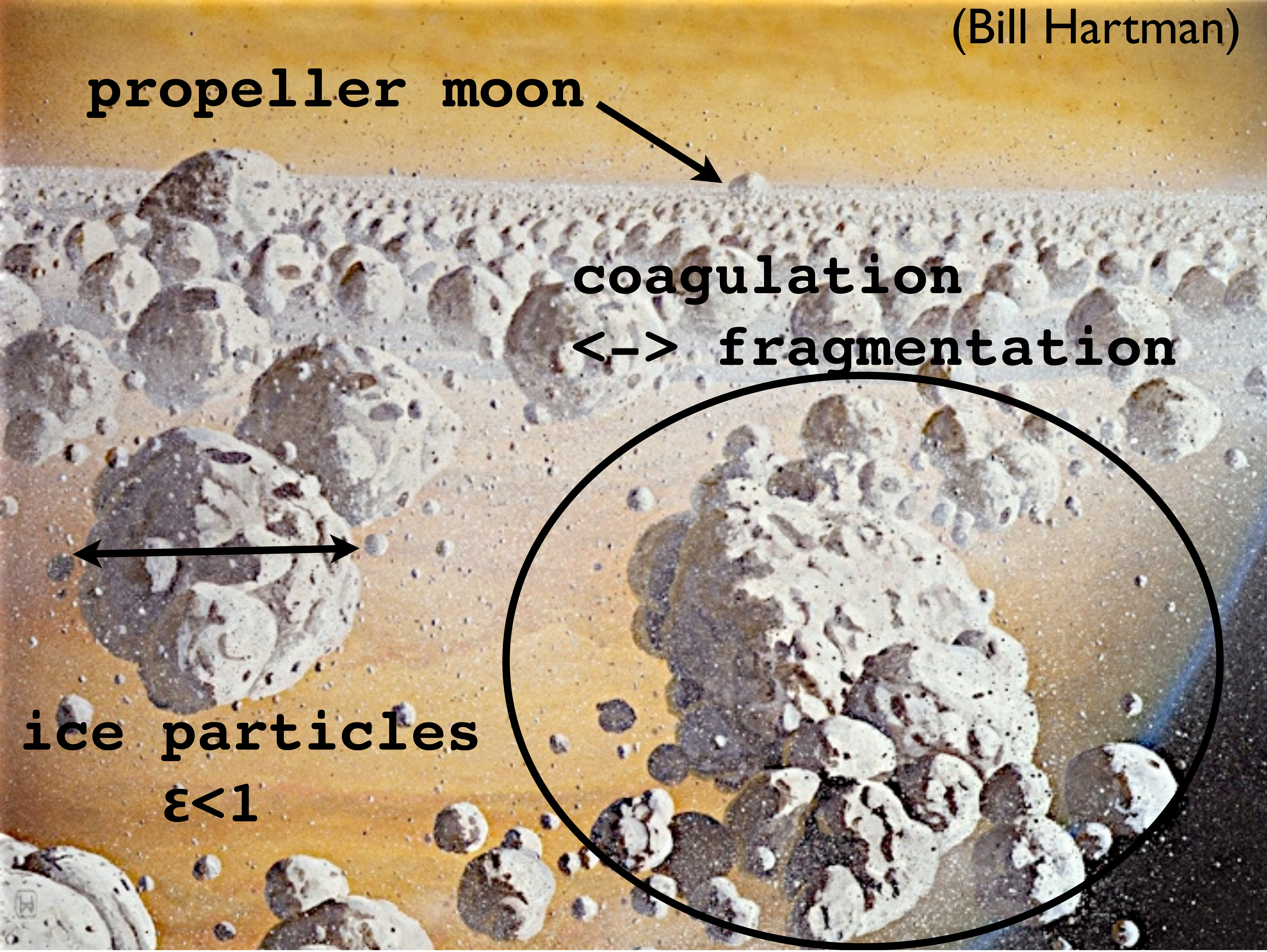
$\varepsilon<1$



(Bill Hartman)

coagulation
<-> fragmentation

ice particles
$\varepsilon<1$

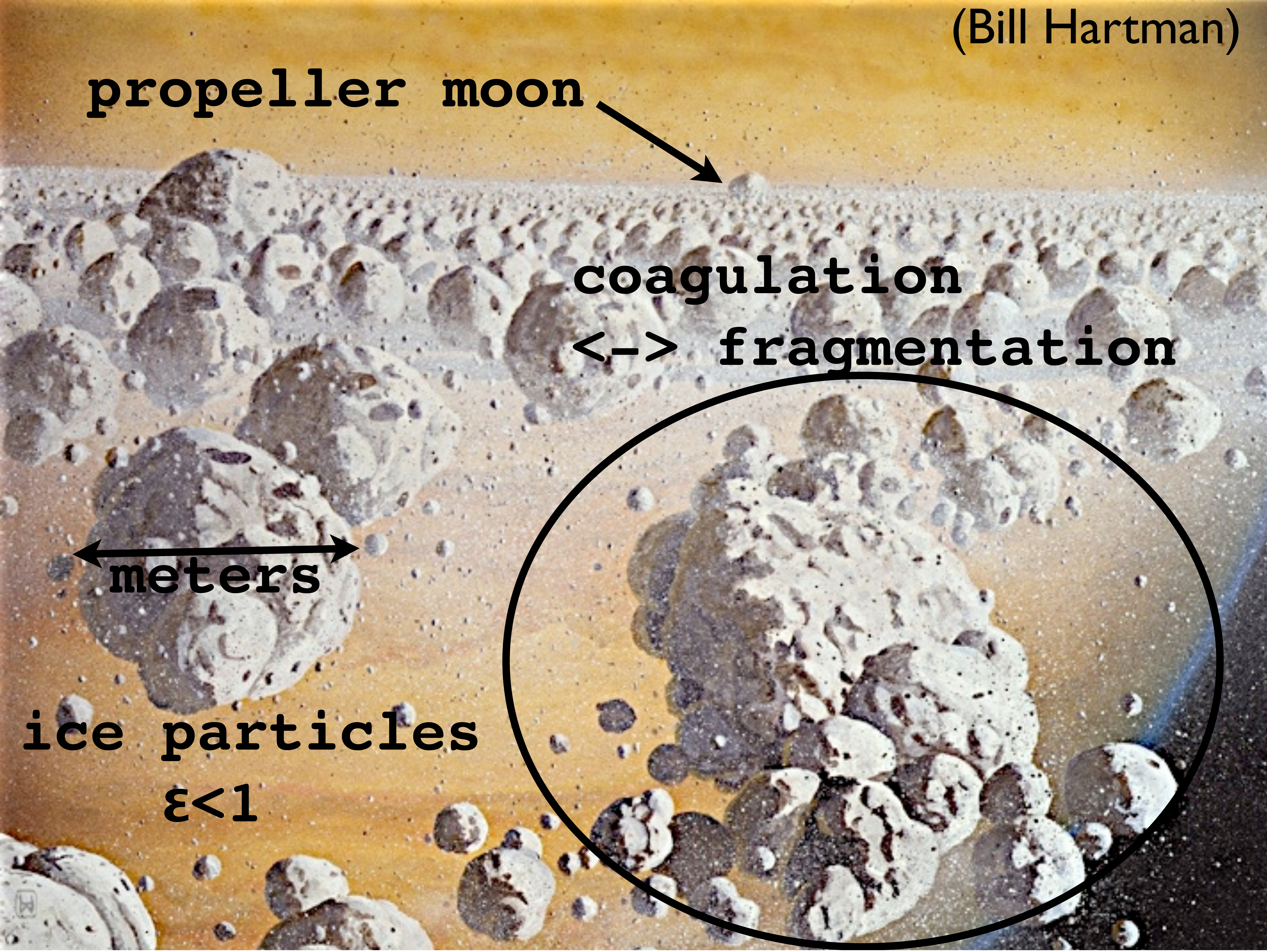


(Bill Hartman)

propeller moon

coagulation
<-> fragmentation

ice particles
$\varepsilon<1$



(Bill Hartman)

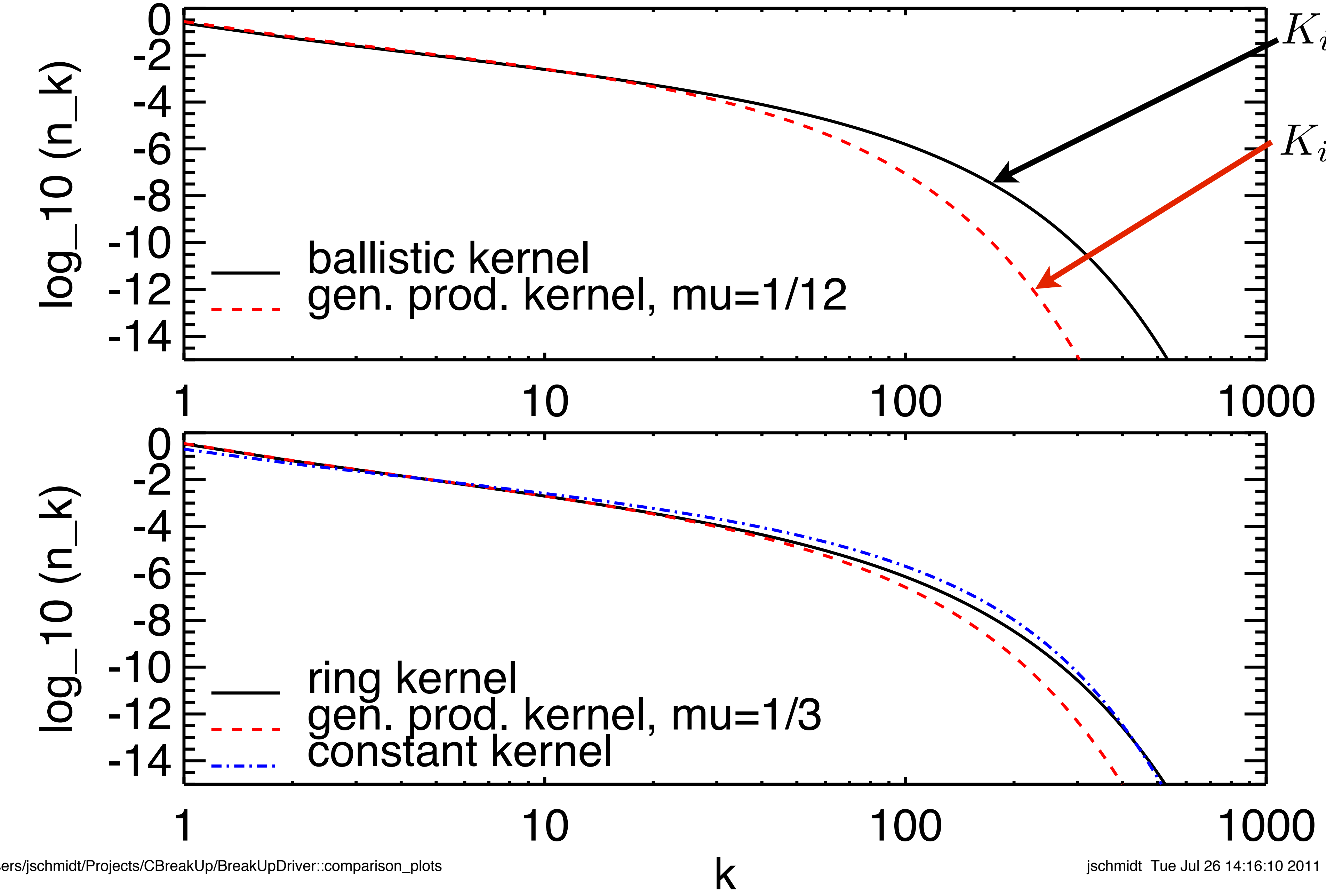
propeller moon

coagulation
<-> fragmentation

meters

ice particles
$\varepsilon<1$

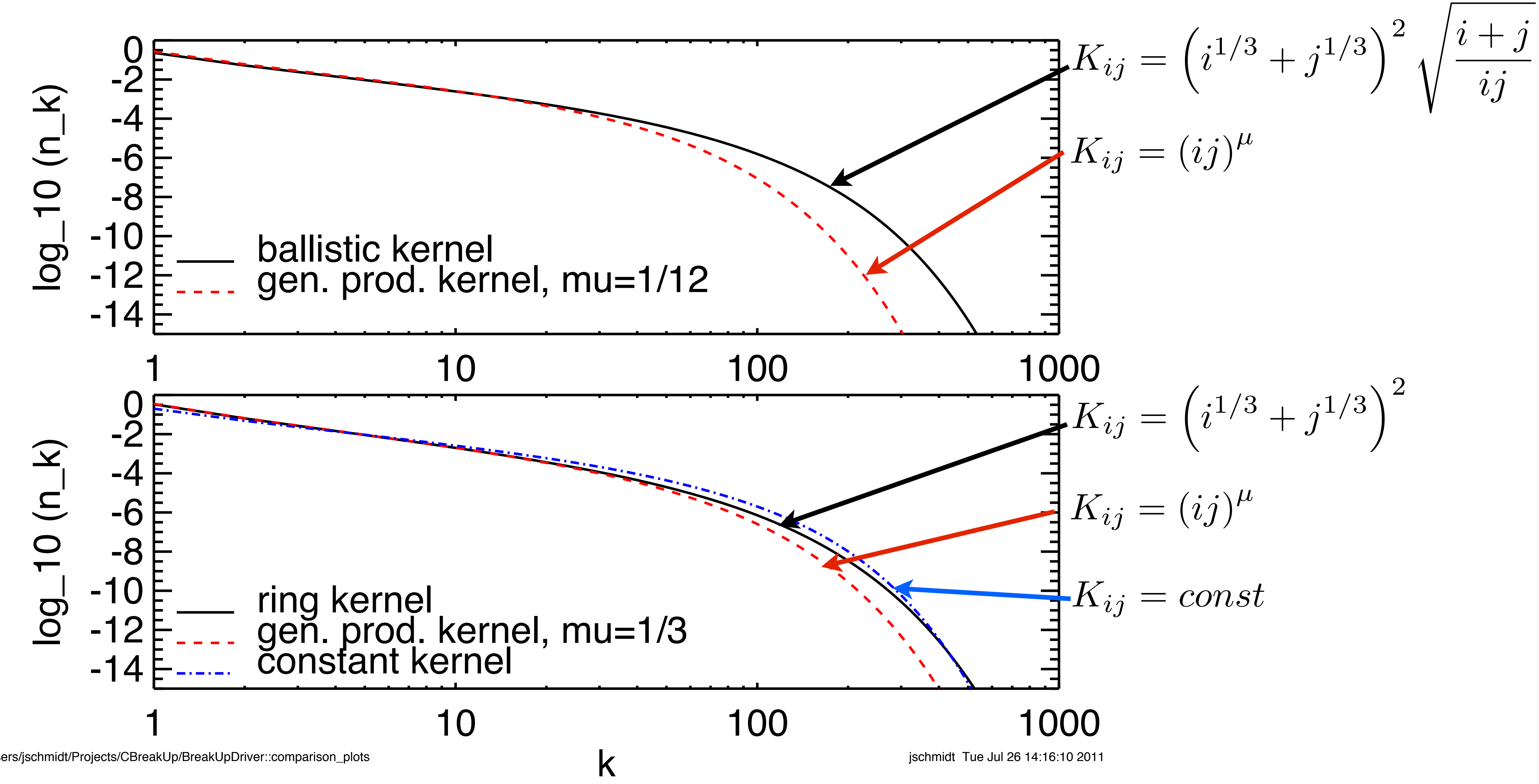
comparison of numerical solutions for various kernels



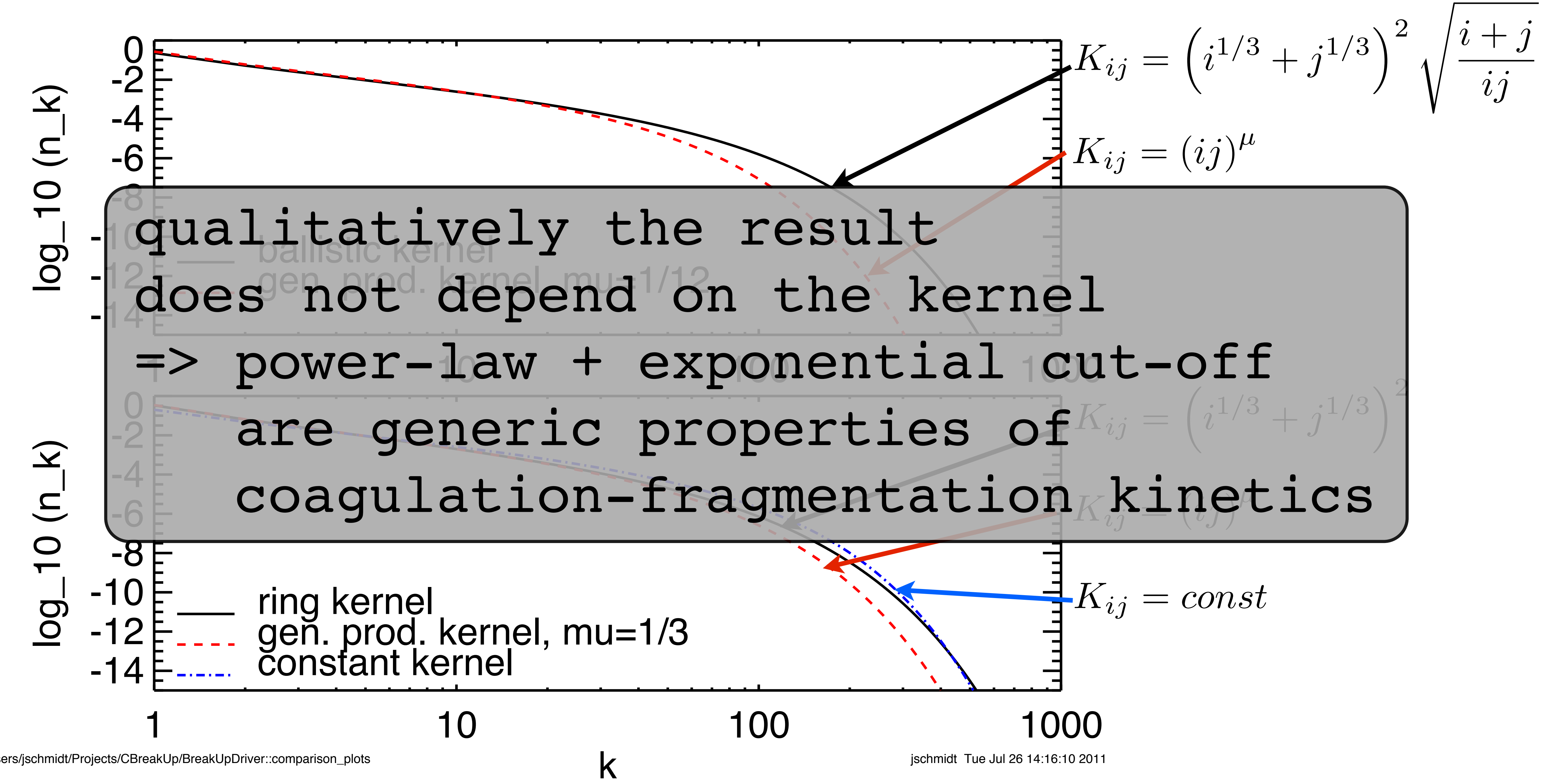
$$K_{ij} = \left(i^{1/3} + j^{1/3}\right)^2 \sqrt{\frac{i+j}{ij}}$$

$$K_{ij} = (ij)^{\mu}$$

comparison of numerical solutions for various kernels



comparison of numerical solutions for various kernels



qualitatively the result does not depend on the kernel

=> power-law + exponential cut-off are generic properties of coagulation-fragmentation kinetics

fragmentation into clusters with power law size distribution

fragmentation into clusters with power law size distribution

look at: $k \longrightarrow n'_j \propto j^{-\alpha} \quad \big(p(r) \propto r^{-\beta}, \quad \beta = 3\alpha - 2\big)$

fragmentation into clusters with power law size distribution

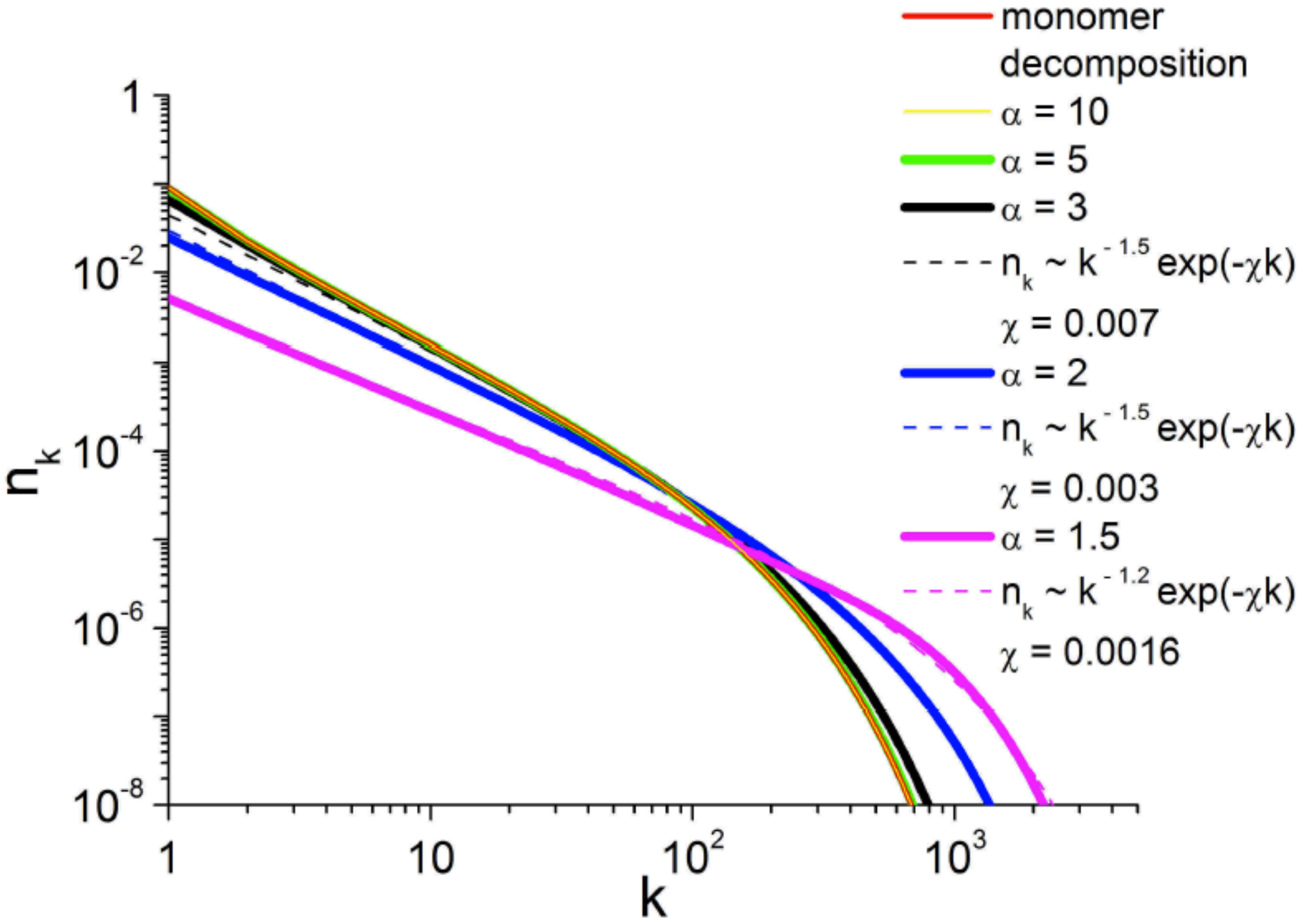
look at: $k \longrightarrow n'_j \propto j^{-\alpha} \quad \big(p(r) \propto r^{-\beta}, \quad \beta = 3\alpha - 2\big)$

up to now: $k \longrightarrow \underbrace{1 + 1 + \cdots + 1}_{k \text{ times}}$

(monomer decomposition)

fragmentation into clusters with power law size distribution

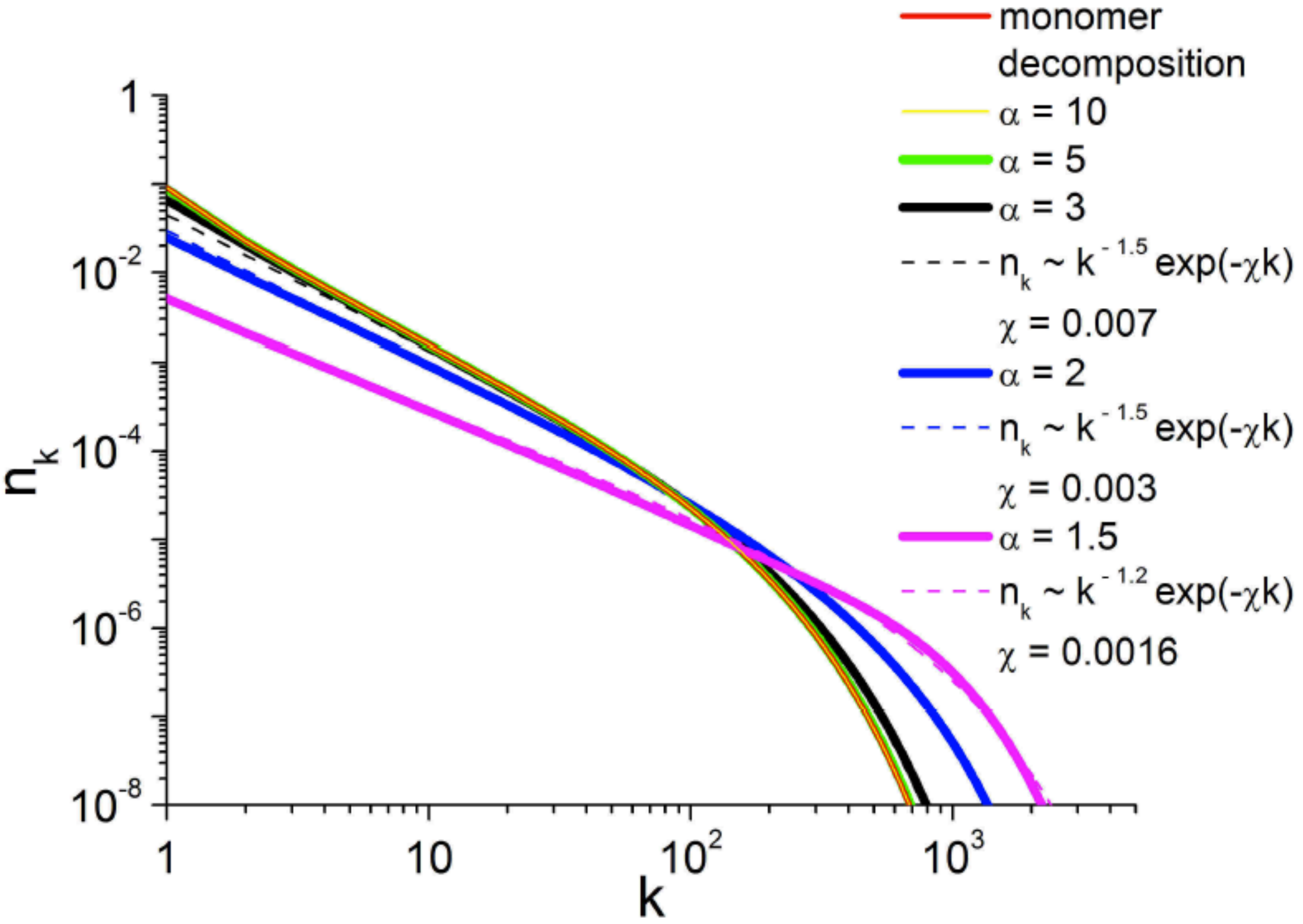
look at: $k \longrightarrow n'_j \propto j^{-\alpha} \quad \left(p(r) \propto r^{-\beta}, \quad \beta = 3\alpha - 2\right)$



fragmentation into clusters with power law size distribution

look at: $k \longrightarrow n'_j \propto j^{-\alpha} \quad \left(p(r) \propto r^{-\beta}, \quad \beta = 3\alpha - 2\right)$

α≥2:
approach
monomer
decomposition

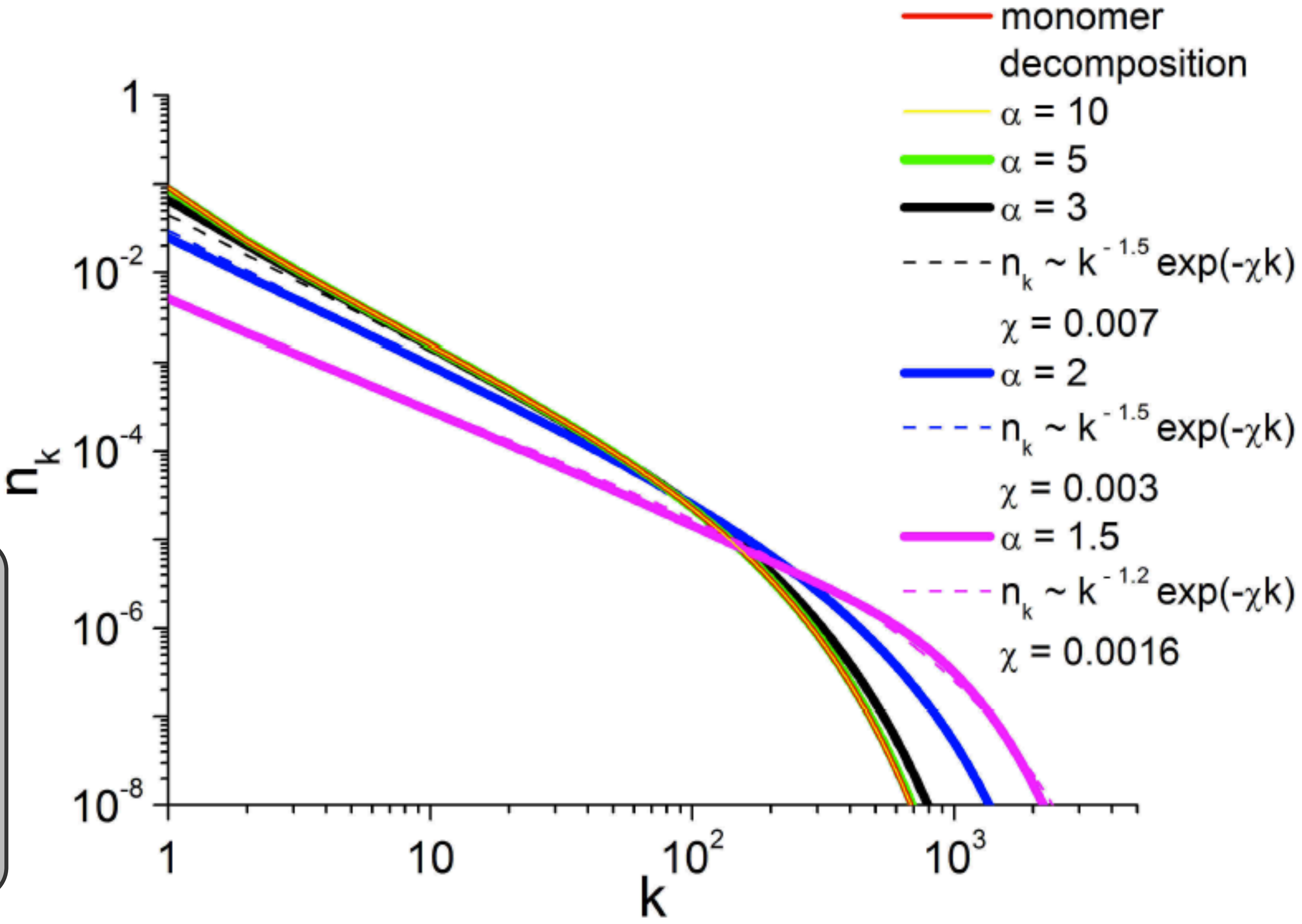


fragmentation into clusters with power law size distribution

look at: $k \longrightarrow n'_j \propto j^{-\alpha} \quad \left(p(r) \propto r^{-\beta}, \quad \beta = 3\alpha - 2\right)$

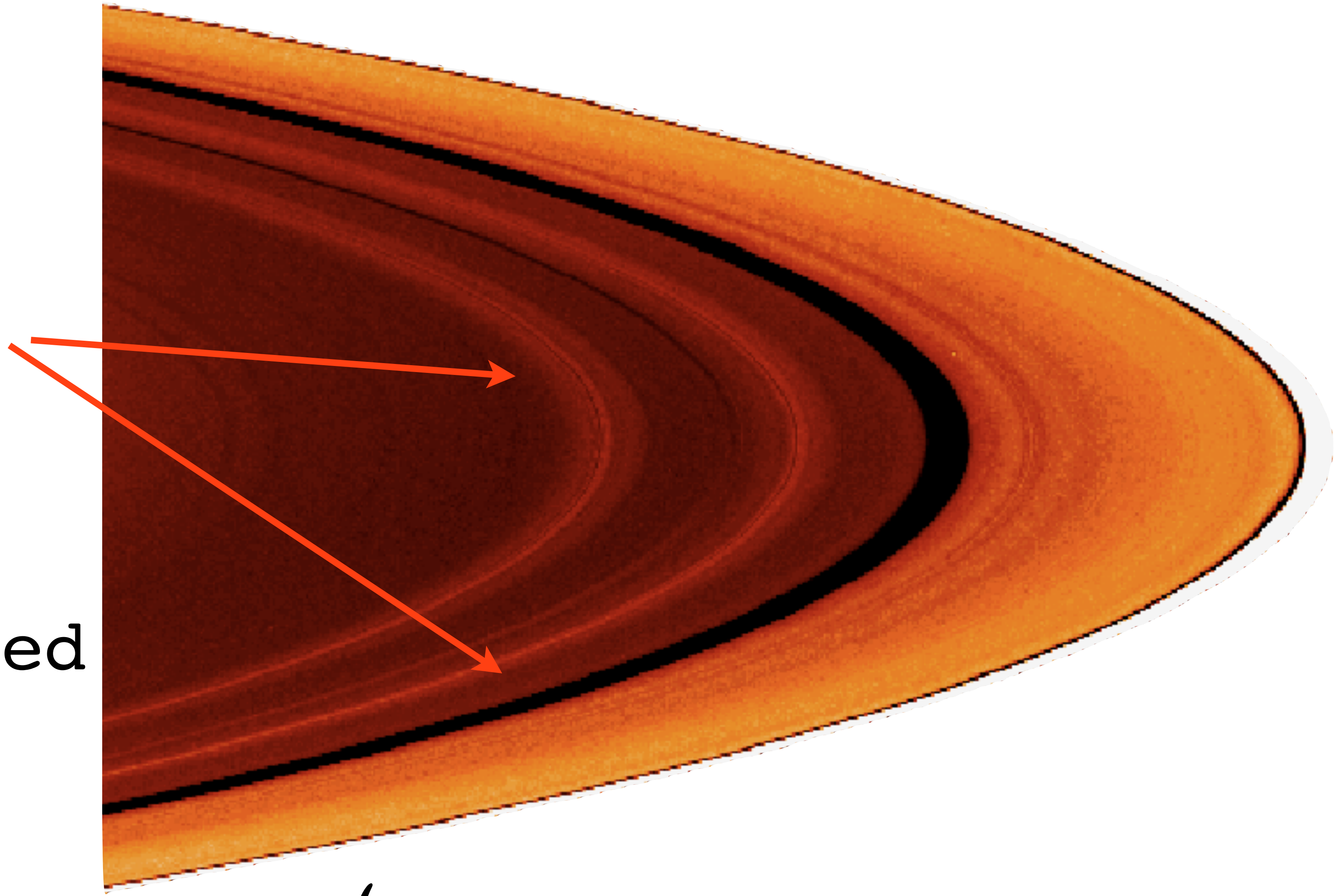
α≥2:
approach
monomer
decomposition

α<2:
decay flattens
final
distribution



local changes in the size distribution, in response to perturbations?

Response to perturbations: local changes in the size distribution?



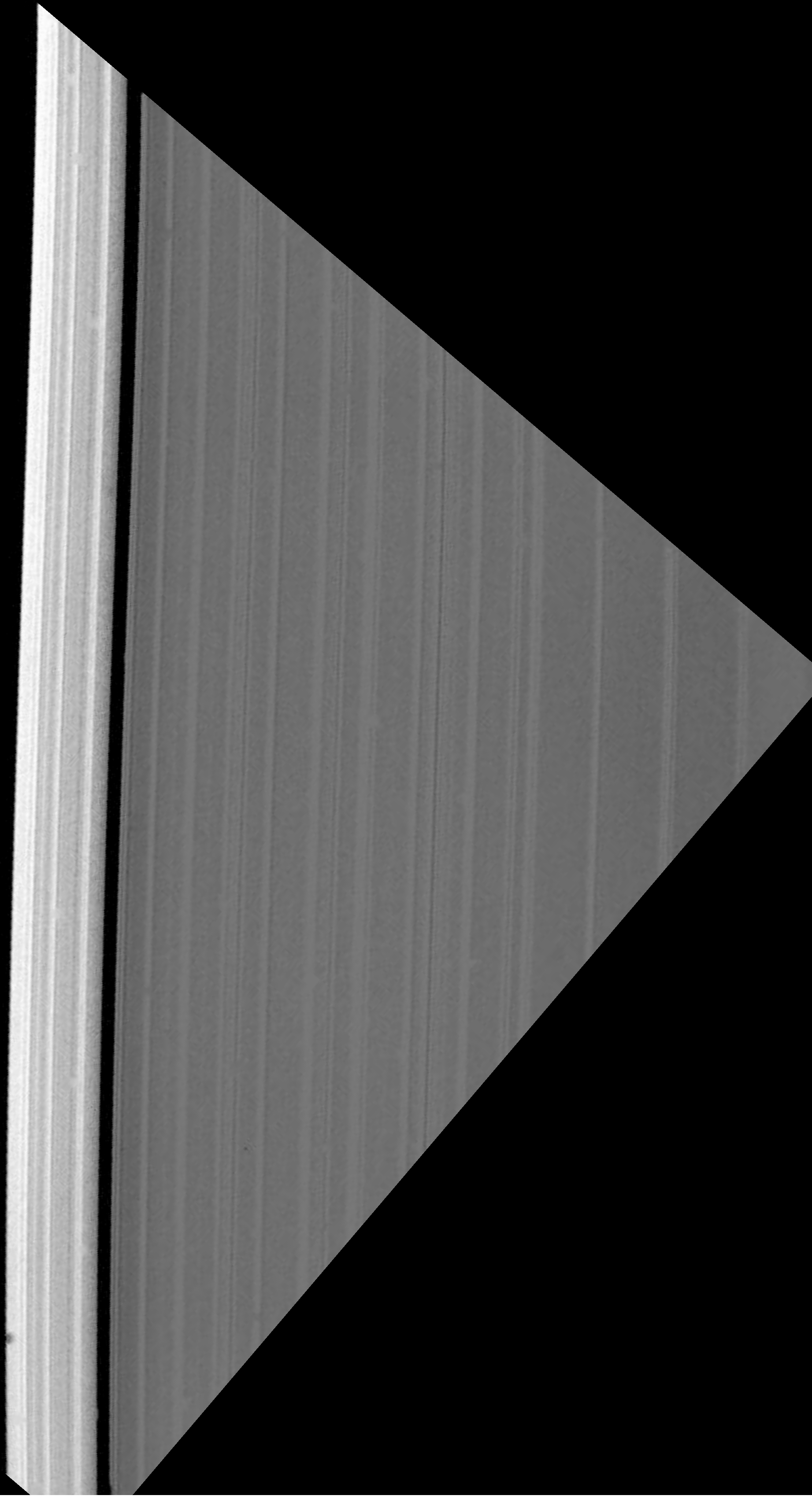
`Halos' of density waves in B:
diffusion of small particles
released in perturbed regions?

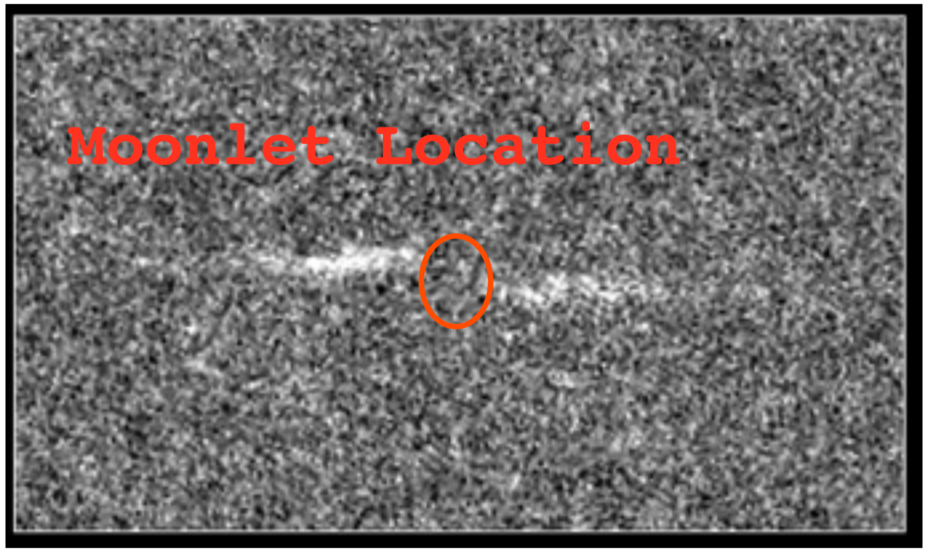
(Dones et al. 1993,
Nicholson et al. 2008)

reduced amplitude of brightness asymmetry in outer A ring.

-> No or weak
self-gravity wakes?

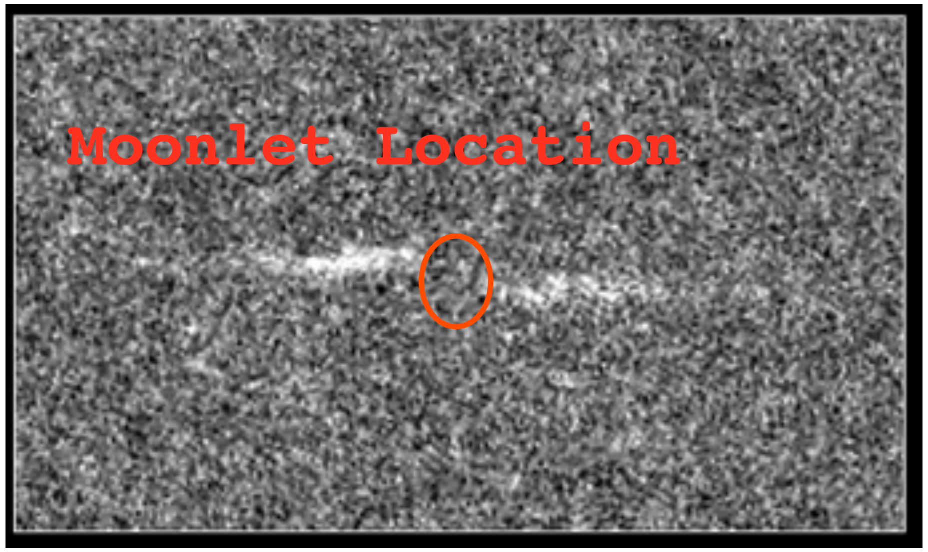
-> Or: Numerous resonances
with moons perturb the
ring matter and
locally change the
size distribution,
change wake properties
or reduce wake contrast?





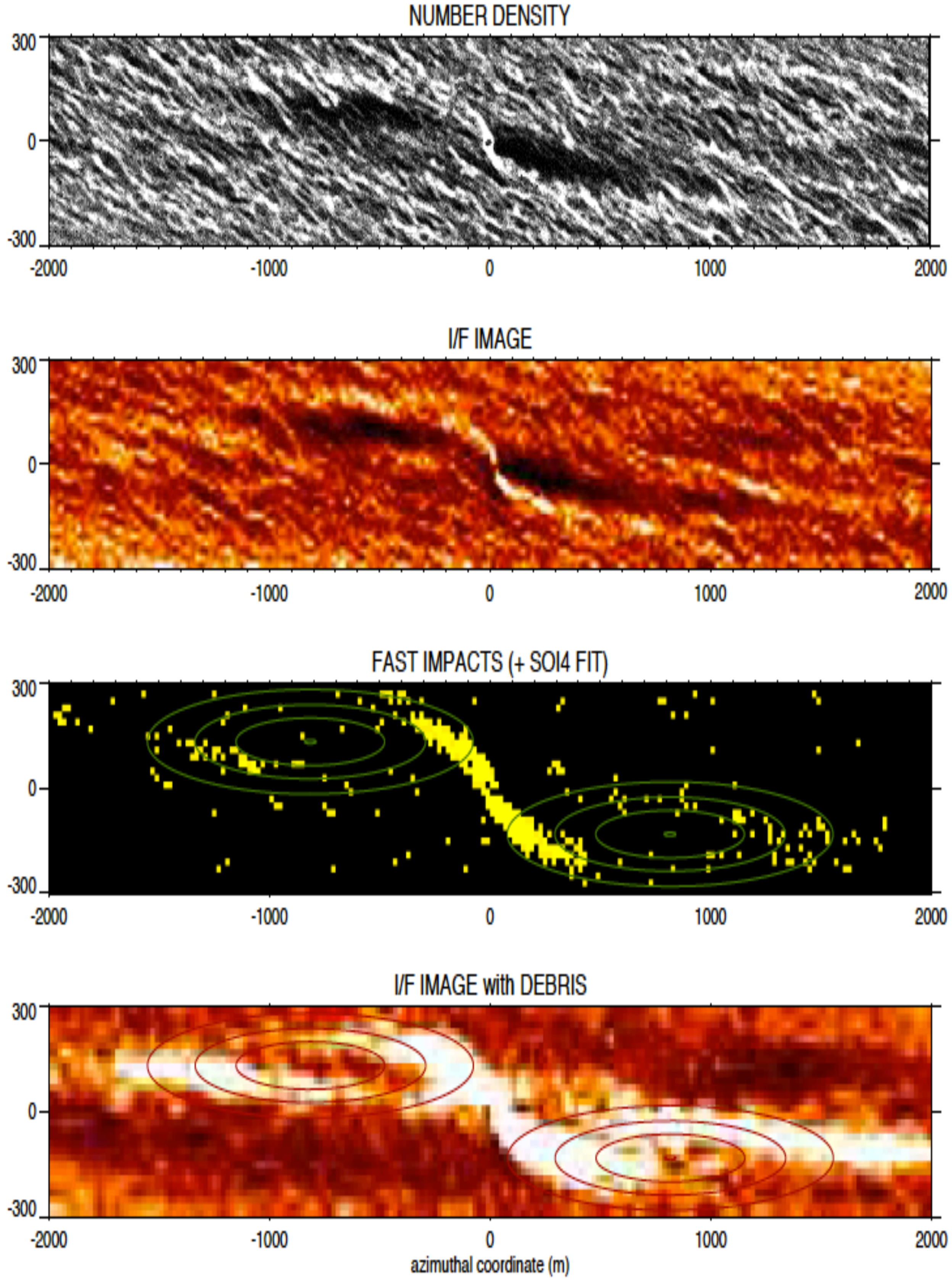
propellers

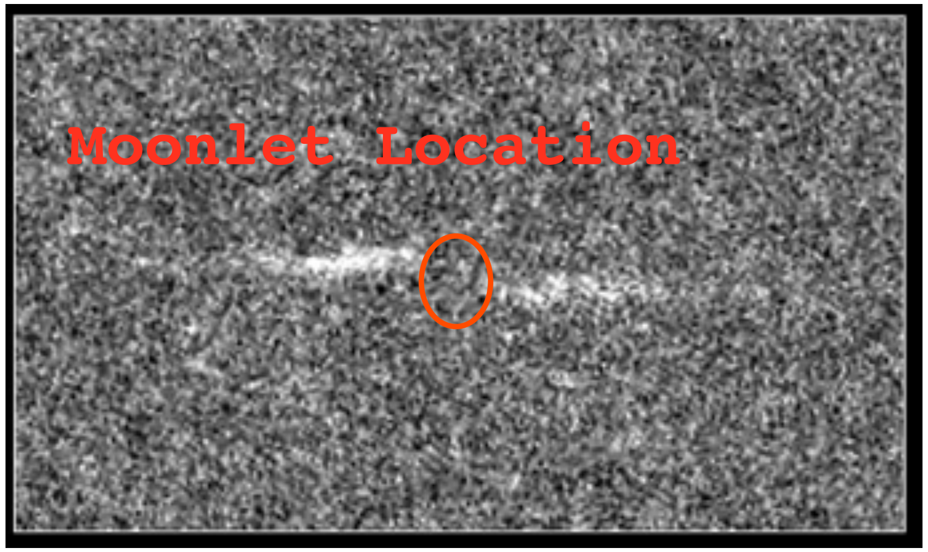
(Tiscareno et al., 2006)



propellers

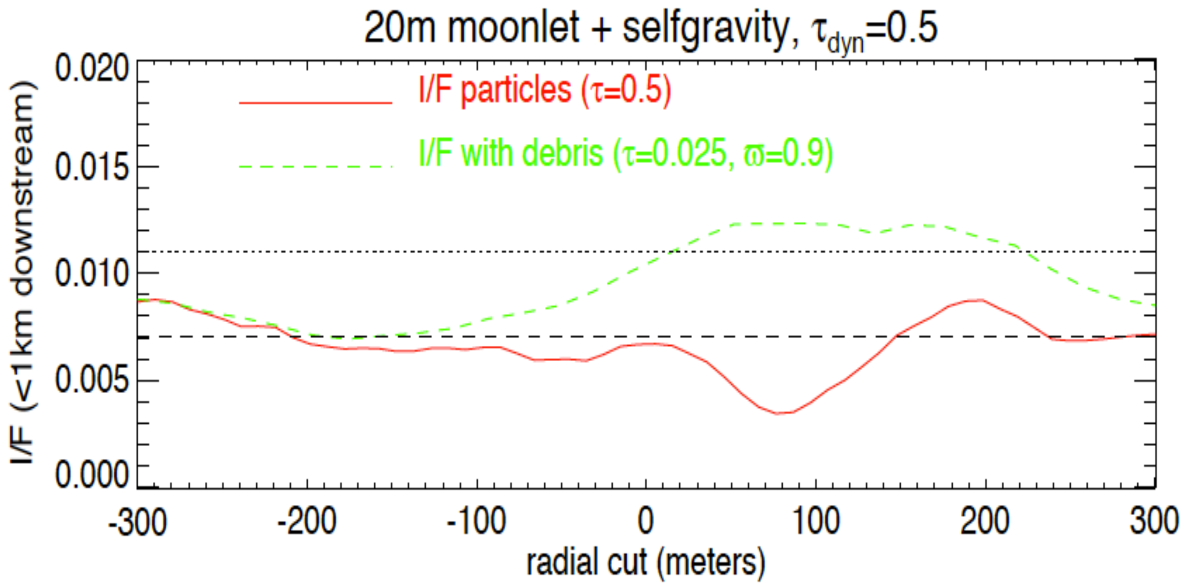
(Tiscareno et al., 2006)





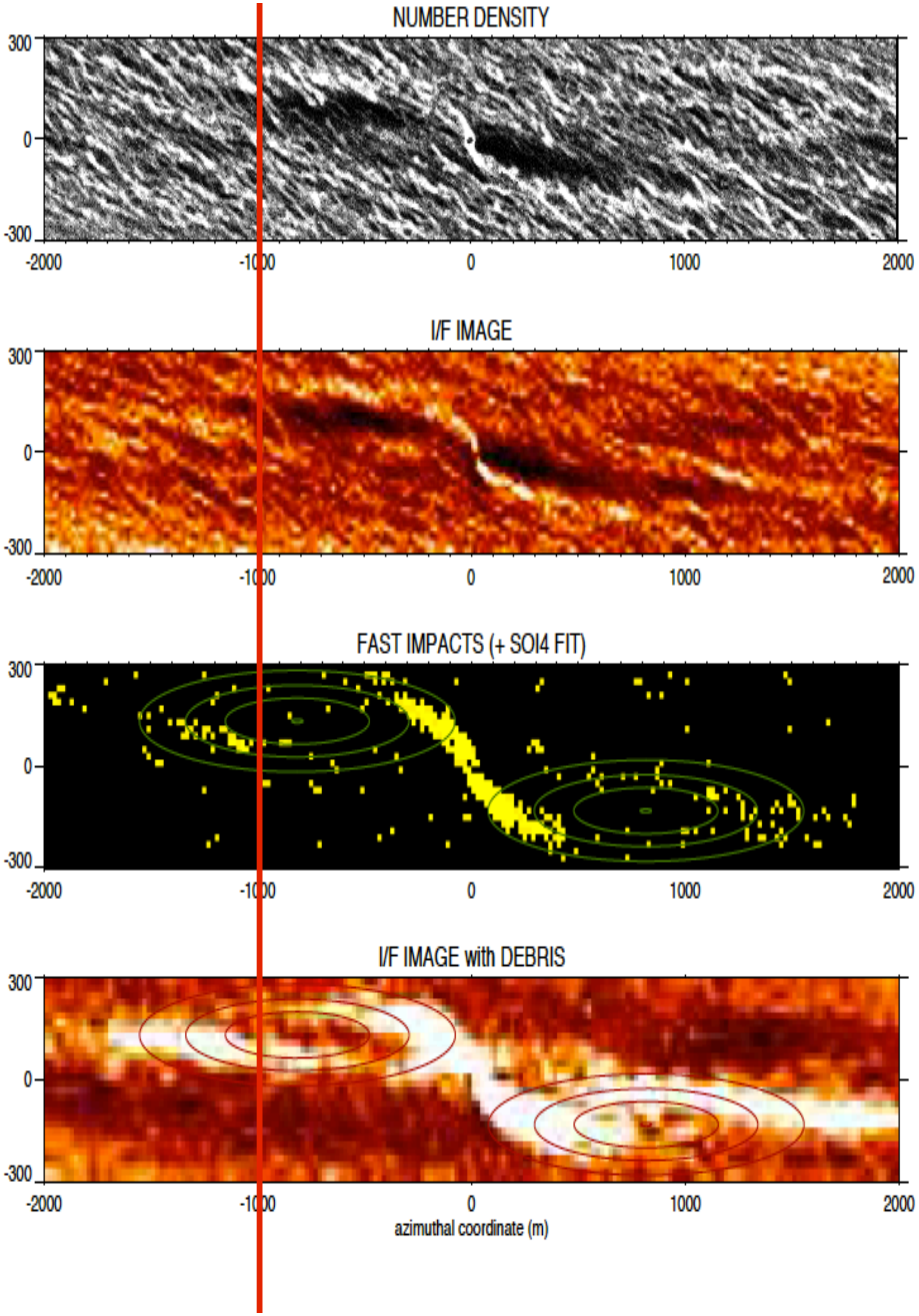
propellers

(Tiscareno et al., 2006)



H. Salo

(see Sremcevic et al., 2007)



Summary

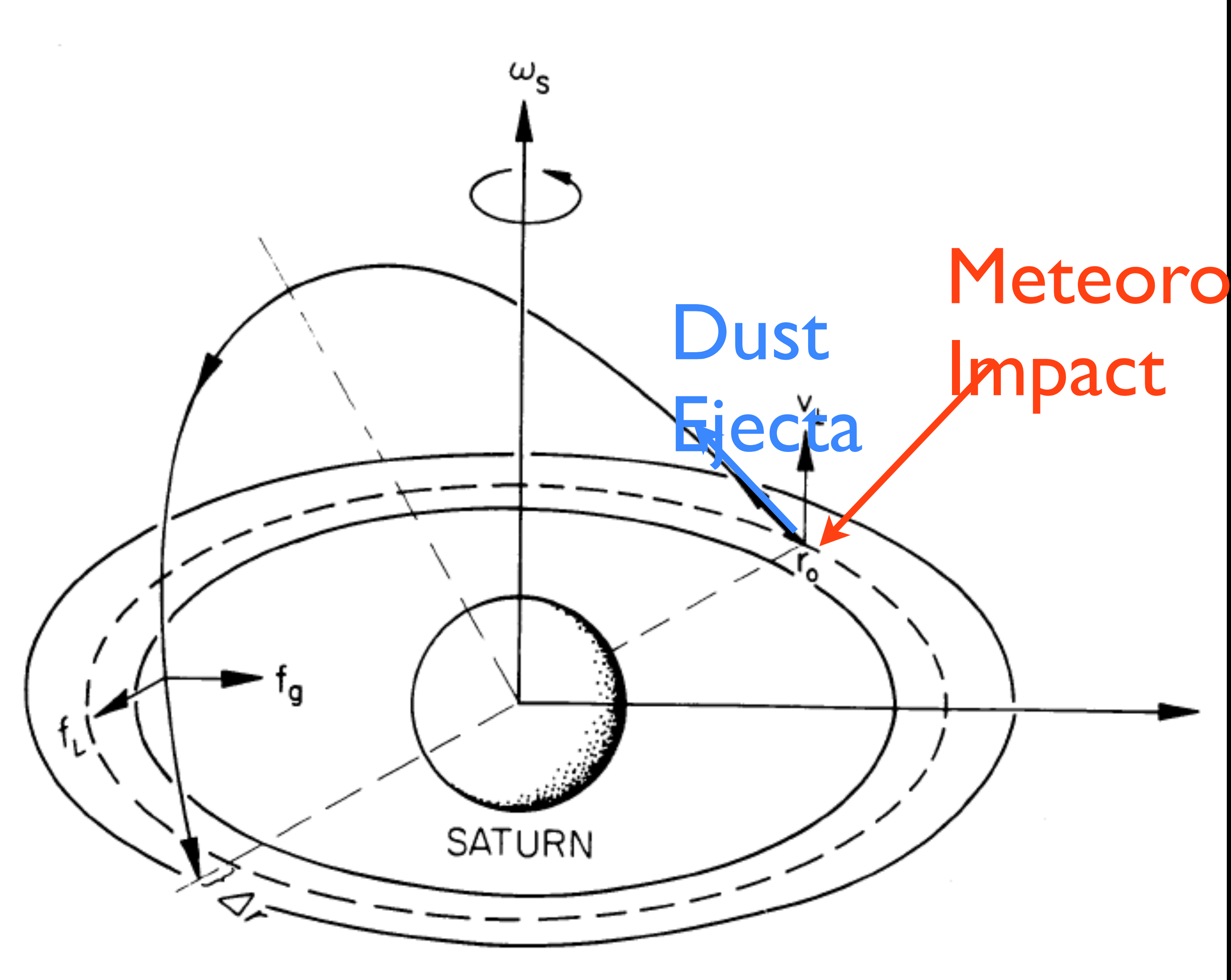
- new kinetic model:
  coagulation <-> fragmentation
  all ring particles are transient clusters

- small frequency of sticky/disruptive collisions:
  continuous size-distribution establishes with power-law part and exponential cut-off

- strong simplifications/neglects:
  so far we find that result is generic property of coagulation/fragmentation kinetics

Instabilities

Transport instabilities



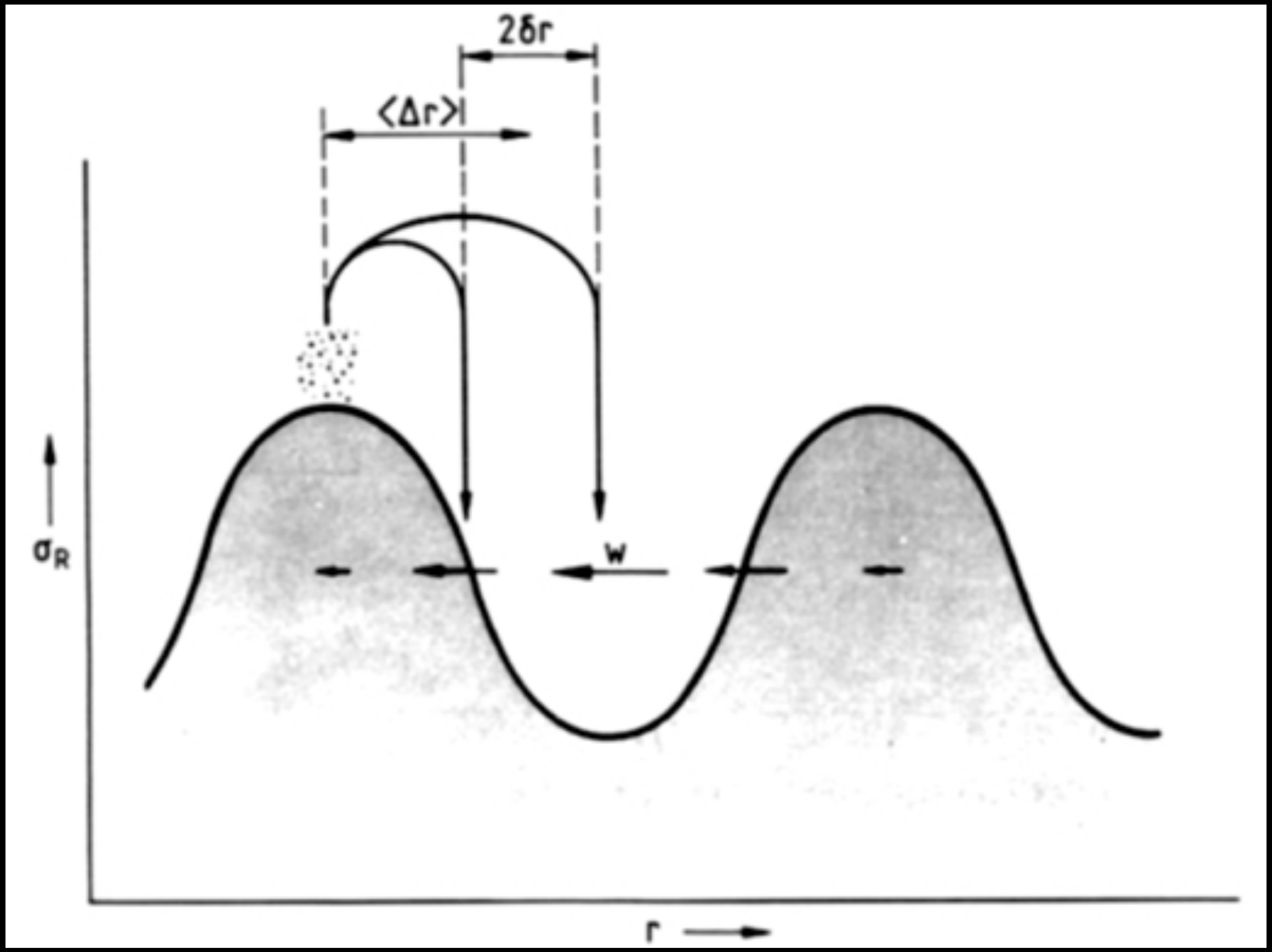
Dust Ejecta

Meteoroid Impact

From Shan&Goertz, 1990

Instabilities

Transport instabilities



Goertz & Morfill, 1988

Surface undulations on the order of characteristic hopping distances will amplify

Instabilities

Transport instabilities

Ballistic Transport Instability

-> radial transport of mass by ejecta
-> typical scales ~ 50-100km
-> ramps interior to A and B rings
-> variations in ring density/brightness
-> works best at intermediate optical depth

Ip83,84
Lissauer84,
Durisen,Durisen&Cuzzi

Electromagnetic Transport Instability

-> small (micron-sized) ejecta get charged in/after impact
-> get accelareted/decelarated by planetary magnetic field: momentum transfer to rings
-> typical scales ~50-100km

Goertz&Morfill88,
Shan&Goertz91