CANS2D モジュール hdmlw

流体エンジン 改良Lax-Wendroff法＋人工粘性

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1　はじめに

このモジュールは、流体力学方程式・MHD 方程式を改良 Lax-Wendroff ＋人工粘性法で解くためのものです。

2　基礎方程式

以下で $\gamma =$ 定数 は比熱比、他の記号は通常の意味。

2.1　サブルーチン mlw_a；移流

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) = 0 \tag{1}$$

2.2　サブルーチン mlw_h；流体

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) = 0 \tag{2}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}(\rho V_x^2 + p) + \frac{\partial}{\partial y}(\rho V_x V_y) = 0 \tag{3}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}(\rho V_x V_y) + \frac{\partial}{\partial y}(\rho V_y^2 + p) = 0 \tag{4}$$

$$\frac{\partial}{\partial t}\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2\right) + \frac{\partial}{\partial x}\left[\left(\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2\right)V_x\right] + \frac{\partial}{\partial y}\left[\left(\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2\right)V_y\right] = 0 \tag{5}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{6}$$

$$V^2 = V_x^2 + V_y^2 \tag{7}$$

2.3　サブルーチン mlw_h_g；流体重力

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) = 0 \tag{8}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}(\rho V_x^2 + p) + \frac{\partial}{\partial y}(\rho V_x V_y) = \rho g_x \tag{9}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}(\rho V_x V_y) + \frac{\partial}{\partial y}(\rho V_y^2 + p) = \rho g_y \tag{10}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2\right) \quad &+ \quad \frac{\partial}{\partial x}\left[\left(\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2\right)V_x\right] \\ &+ \quad \frac{\partial}{\partial y}\left[\left(\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2\right)V_y\right] = \rho g_y V_x + \rho g_y V_y \end{aligned}\tag{11}$$

$$p = \frac{k_\mathrm{B}}{m}\rho T \tag{12}$$

$$V^2 = V_x^2 + V_y^2 \tag{13}$$

2.4　サブルーチン mlw_ht；等温流体

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) = 0 \tag{14}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}(\rho V_x^2 + p) + \frac{\partial}{\partial y}(\rho V_x V_y) = 0 \tag{15}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}(\rho V_x V_y) + \frac{\partial}{\partial y}(\rho V_y^2 + p) = 0 \tag{16}$$

$$p = \frac{k_\mathrm{B}}{m}\rho T \tag{17}$$

温度 T は既知の定数。

2.5　サブルーチン mlw_ht_g；等温流体重力

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) = 0 \tag{18}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}(\rho V_x^2 + p) + \frac{\partial}{\partial y}(\rho V_x V_y) = \rho g_x \tag{19}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}(\rho V_x V_y) + \frac{\partial}{\partial y}(\rho V_y^2 + p) = \rho g_y \tag{20}$$

$$p = \frac{k_\mathrm{B}}{m}\rho T \tag{21}$$

温度 T は既知の定数。

2.6 サブルーチン mlw_m；MHD

$$\frac{\partial}{\partial t}(\rho)+\frac{\partial}{\partial x}(\rho V_x)+\frac{\partial}{\partial y}(\rho V_y)=0 \tag{22}$$

$$\frac{\partial}{\partial t}(\rho V_x)+\frac{\partial}{\partial x}\left(\rho V_x^2+p+\frac{B^2}{8\pi}-\frac{B_x^2}{4\pi}\right)+\frac{\partial}{\partial y}\left(\rho V_x V_y-\frac{B_x B_y}{4\pi}\right)=0 \tag{23}$$

$$\frac{\partial}{\partial t}(\rho V_y)+\frac{\partial}{\partial x}\left(\rho V_x V_y-\frac{B_x B_y}{4\pi}\right)+\frac{\partial}{\partial y}\left(\rho V_y^2+p+\frac{B^2}{8\pi}-\frac{B_y^2}{4\pi}\right)=0 \tag{24}$$

$$\frac{\partial}{\partial t}(B_x)+\frac{\partial}{\partial y}\left(E_z\right)=0 \tag{25}$$

$$\frac{\partial}{\partial t}(B_y)-\frac{\partial}{\partial x}\left(E_z\right)=0 \tag{26}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{p}{\gamma-1}+\frac{1}{2}\rho V^2+\frac{B^2}{8\pi}\right) \quad &+ \quad \frac{\partial}{\partial x}\left((\frac{\gamma}{\gamma-1}p+\frac{1}{2}\rho V^2)V_x-\frac{B_y E_z}{4\pi}\right)\\ &+ \quad \frac{\partial}{\partial y}\left((\frac{\gamma}{\gamma-1}p+\frac{1}{2}\rho V^2)V_y+\frac{B_x E_z}{4\pi}\right)=0\end{aligned} \tag{27}$$

$$E_z=-V_x B_y+V_y B_x \tag{28}$$

$$p=\frac{k_{\mathrm{B}}}{m}\rho T \tag{29}$$

$$B^2=B_x^2+B_y^2, \quad V^2=V_x^2+V_y^2 \tag{30}$$

2.7 サブルーチン mlw_m_g；MHD 重力

$$\frac{\partial}{\partial t}(\rho)+\frac{\partial}{\partial x}(\rho V_x)+\frac{\partial}{\partial y}(\rho V_y)=0 \tag{31}$$

$$\frac{\partial}{\partial t}(\rho V_x)+\frac{\partial}{\partial x}\left(\rho V_x^2+p+\frac{B^2}{8\pi}-\frac{B_x^2}{4\pi}\right)+\frac{\partial}{\partial y}\left(\rho V_x V_y-\frac{B_x B_y}{4\pi}\right)=\rho g_x \tag{32}$$

$$\frac{\partial}{\partial t}(\rho V_y)+\frac{\partial}{\partial x}\left(\rho V_x V_y-\frac{B_x B_y}{4\pi}\right)+\frac{\partial}{\partial y}\left(\rho V_y^2+p+\frac{B^2}{8\pi}-\frac{B_y^2}{4\pi}\right)=\rho g_y \tag{33}$$

$$\frac{\partial}{\partial t}(B_x)+\frac{\partial}{\partial y}\left(E_z\right)=0 \tag{34}$$

$$\frac{\partial}{\partial t}(B_y)-\frac{\partial}{\partial x}\left(E_z\right)=0 \tag{35}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{p}{\gamma-1}+\frac{1}{2}\rho V^2+\frac{B^2}{8\pi}\right) \quad &+ \quad \frac{\partial}{\partial x}\left((\frac{\gamma}{\gamma-1}p+\frac{1}{2}\rho V^2)V_x-\frac{B_y E_z}{4\pi}\right)\\ &+ \quad \frac{\partial}{\partial y}\left((\frac{\gamma}{\gamma-1}p+\frac{1}{2}\rho V^2)V_y+\frac{B_x E_z}{4\pi}\right)=\rho g_x V_x+\rho g_y V_y\end{aligned} \tag{36}$$

$$E_z = -V_x B_y + V_y B_x \tag{37}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{38}$$

$$B^2 = B_x^2 + B_y^2, \quad V^2 = V_x^2 + V_y^2 \tag{39}$$

2.8 サブルーチン mlw_m_e；MHD 抵抗

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) = 0 \tag{40}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}\left(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi}\right) + \frac{\partial}{\partial y}\left(\rho V_x V_y - \frac{B_x B_y}{4\pi}\right) = 0 \tag{41}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}\left(\rho V_x V_y - \frac{B_x B_y}{4\pi}\right) + \frac{\partial}{\partial y}\left(\rho V_y^2 + p + \frac{B^2}{8\pi} - \frac{B_y^2}{4\pi}\right) = 0 \tag{42}$$

$$\frac{\partial}{\partial t}(B_x) + \frac{\partial}{\partial y}\left(E_z\right) = 0 \tag{43}$$

$$\frac{\partial}{\partial t}(B_y) - \frac{\partial}{\partial x}\left(E_z\right) = 0 \tag{44}$$

$$\begin{aligned} \frac{\partial}{\partial t}\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right) &+ \frac{\partial}{\partial x}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_x - \frac{B_y E_z}{4\pi}\right) \\ &+ \frac{\partial}{\partial y}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_y + \frac{B_x E_z}{4\pi}\right) = 0 \end{aligned} \tag{45}$$

$$E_z = -V_x B_y + V_y B_x + \eta J_z \tag{46}$$

$$J_z = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \tag{47}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{48}$$

$$B^2 = B_x^2 + B_y^2, \quad V^2 = V_x^2 + V_y^2 \tag{49}$$

2.9 サブルーチン mlw_m3；3 成分 MHD

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) = 0 \tag{50}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}\left(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi}\right) + \frac{\partial}{\partial y}\left(\rho V_x V_y - \frac{B_x B_y}{4\pi}\right) = 0 \tag{51}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}\left(\rho V_x V_y - \frac{B_x B_y}{4\pi}\right) + \frac{\partial}{\partial y}\left(\rho V_y^2 + p + \frac{B^2}{8\pi} - \frac{B_y^2}{4\pi}\right) = 0 \tag{52}$$

$$\frac{\partial}{\partial t}(\rho V_z) + \frac{\partial}{\partial x}\left(\rho V_x V_z - \frac{B_x B_z}{4\pi}\right) + \frac{\partial}{\partial y}\left(\rho V_y V_z - \frac{B_y B_z}{4\pi}\right) = 0 \tag{53}$$

$$\frac{\partial}{\partial t}(B_x) + \frac{\partial}{\partial y}(E_z) = 0 \tag{54}$$

$$\frac{\partial}{\partial t}(B_y) - \frac{\partial}{\partial x}(E_z) = 0 \tag{55}$$

$$\frac{\partial}{\partial t}(B_z) + \frac{\partial}{\partial x}(E_y) - \frac{\partial}{\partial y}(E_x) = 0 \tag{56}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right) \quad &+ \quad \frac{\partial}{\partial x}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_x + \frac{B_z E_y - B_y E_z}{4\pi}\right) \\ &+ \quad \frac{\partial}{\partial y}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_y + \frac{B_x E_z - B_z E_x}{4\pi}\right) = 0\end{aligned} \tag{57}$$

$$E_x = -V_y B_z + V_z B_y, \quad E_y = -V_z B_x + V_x B_z, \quad E_z = -V_x B_y + V_y B_x \tag{58}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{59}$$

$$B^2 = B_x^2 + B_y^2 + B_z^2, \quad V^2 = V_x^2 + V_y^2 + V_z^2 \tag{60}$$

2.10 サブルーチン mlw_m3_e；3 成分 MHD 抵抗

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) = 0 \tag{61}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}\left(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi}\right) + \frac{\partial}{\partial y}\left(\rho V_x V_y - \frac{B_x B_y}{4\pi}\right) = 0 \tag{62}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}\left(\rho V_x V_y - \frac{B_x B_y}{4\pi}\right) + \frac{\partial}{\partial y}\left(\rho V_y^2 + p + \frac{B^2}{8\pi} - \frac{B_y^2}{4\pi}\right) = 0 \tag{63}$$

$$\frac{\partial}{\partial t}(\rho V_z) + \frac{\partial}{\partial x}\left(\rho V_x V_z - \frac{B_x B_z}{4\pi}\right) + \frac{\partial}{\partial y}\left(\rho V_y V_z - \frac{B_y B_z}{4\pi}\right) = 0 \tag{64}$$

$$\frac{\partial}{\partial t}(B_x) + \frac{\partial}{\partial y}(E_z) = 0 \tag{65}$$

$$\frac{\partial}{\partial t}(B_y) - \frac{\partial}{\partial x}(E_z) = 0 \tag{66}$$

$$\frac{\partial}{\partial t}(B_z) + \frac{\partial}{\partial x}(E_y) - \frac{\partial}{\partial y}(E_x) = 0 \tag{67}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right) \quad &+ \quad \frac{\partial}{\partial x}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_x + \frac{B_z E_y - B_y E_z}{4\pi}\right) \\ &+ \quad \frac{\partial}{\partial y}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_y + \frac{B_x E_z - B_z E_x}{4\pi}\right) = 0\end{aligned} \tag{68}$$

$$E_x = -V_y B_z + V_z B_y + \eta J_x, \quad E_y = -V_z B_x + V_x B_z + \eta J_y, \quad E_z = -V_x B_y + V_y B_x + \eta J_z \tag{69}$$

$$J_x = \frac{\partial B_z}{\partial y}, \quad J_y = -\frac{\partial B_z}{\partial x}, \quad J_z = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \tag{70}$$

$$p = \frac{k_{\mathrm{B}}}{m} \rho T \tag{71}$$

$$B^2 = B_x^2 + B_y^2 + B_z^2, \quad V^2 = V_x^2 + V_y^2 + V_z^2 \tag{72}$$

2.11 サブルーチン mlw_m3_g；3 成分 MHD 重力

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) = 0 \tag{73}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}\left(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi}\right) + \frac{\partial}{\partial y}\left(\rho V_x V_y - \frac{B_x B_y}{4\pi}\right) = \rho g_x \tag{74}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}\left(\rho V_x V_y - \frac{B_x B_y}{4\pi}\right) + \frac{\partial}{\partial y}\left(\rho V_y^2 + p + \frac{B^2}{8\pi} - \frac{B_y^2}{4\pi}\right) = \rho g_y \tag{75}$$

$$\frac{\partial}{\partial t}(\rho V_z) + \frac{\partial}{\partial x}\left(\rho V_x V_z - \frac{B_x B_z}{4\pi}\right) + \frac{\partial}{\partial y}\left(\rho V_y V_z - \frac{B_y B_z}{4\pi}\right) = \rho g_z \tag{76}$$

$$\frac{\partial}{\partial t}(B_x) + \frac{\partial}{\partial y}(E_z) = 0 \tag{77}$$

$$\frac{\partial}{\partial t}(B_y) - \frac{\partial}{\partial x}(E_z) = 0 \tag{78}$$

$$\frac{\partial}{\partial t}(B_z) + \frac{\partial}{\partial x}(E_y) - \frac{\partial}{\partial y}(E_x) = 0 \tag{79}$$

$$\begin{aligned} & \frac{\partial}{\partial t}\left(\frac{p}{\gamma - 1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right) + \frac{\partial}{\partial x}\left((\frac{\gamma}{\gamma - 1}p + \frac{1}{2}\rho V^2)V_x + \frac{B_z E_y - B_y E_z}{4\pi}\right) \\ + \quad & \frac{\partial}{\partial y}\left((\frac{\gamma}{\gamma - 1}p + \frac{1}{2}\rho V^2)V_y + \frac{B_x E_z - B_z E_x}{4\pi}\right) = \rho g_x V_x + \rho g_y V_y + \rho g_z V_z \end{aligned} \tag{80}$$

$$E_x = -V_y B_z + V_z B_y, \quad E_y = -V_z B_x + V_x B_z, \quad E_z = -V_x B_y + V_y B_x \tag{81}$$

$$p = \frac{k_{\mathrm{B}}}{m} \rho T \tag{82}$$

$$B^2 = B_x^2 + B_y^2 + B_z^2, \quad V^2 = V_x^2 + V_y^2 + V_z^2 \tag{83}$$

2.12 サブルーチン mlw_m3_t；3 成分 MHD 潮汐&Coriolis 力

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial z}(\rho V_z) = 0 \tag{84}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}\left(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_r^2}{4\pi}\right) + \frac{\partial}{\partial z}\left(\rho V_x V_z - \frac{B_r B_z}{4\pi}\right) = 2q_0\Omega_0^2 x\rho + 2\Omega_0\rho V_y \tag{85}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}\left(\rho V_x V_y - \frac{B_r B_y}{4\pi}\right) + \frac{\partial}{\partial z}\left(\rho V_z V_y - \frac{B_z B_y}{4\pi}\right) = -2\Omega_0\rho V_x \tag{86}$$

$$\frac{\partial}{\partial t}(\rho V_z) + \frac{\partial}{\partial x}\left(\rho V_x V_z - \frac{B_r B_z}{4\pi}\right) + \frac{\partial}{\partial z}\left(\rho V_z^2 + p + \frac{B^2}{8\pi} - \frac{B_z^2}{4\pi}\right) = 0 \tag{87}$$

$$\frac{\partial}{\partial t}(B_x) - \frac{\partial}{\partial z}(E_y) = 0 \tag{88}$$

$$\frac{\partial}{\partial t}(B_z) + \frac{\partial}{\partial x}(E_y) = 0 \tag{89}$$

$$\frac{\partial}{\partial t}(B_y) - \frac{\partial}{\partial x}(E_z) + \frac{\partial}{\partial z}(E_x) = 0 \tag{90}$$

$$\begin{aligned}
& \frac{\partial}{\partial t}\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right) + \frac{\partial}{\partial x}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_x + \frac{B_z E_y - B_y E_z}{4\pi}\right) \\
+ & \frac{\partial}{\partial z}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_z + \frac{B_y E_x - B_x E_y}{4\pi}\right) \\
= & 2q_0\Omega_0^2 x\rho V_x
\end{aligned} \tag{91}$$

$$E_x = -V_y B_z + V_z B_y, \quad E_y = -V_z B_x + V_x B_z, \quad E_z = -V_x B_y + V_y B_x \tag{92}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{93}$$

$$B^2 = B_x^2 + B_y^2 + B_z^2, \quad V^2 = V_x^2 + V_y^2 + V_z^2 \tag{94}$$

$\Omega_0 =$ 定数 で z 軸まわりをまわる回転基準系。パラメータ $q_0 \equiv -d(\ln\Omega)/d(\ln R)$ は、（局所化近似前の）平衡状態での回転速度分布 $\Omega(R)$ を表す外部パラメータ。Kepler 回転では $q_0 = 3/2$、定速度回転では $q_0 = 1$。

2.13 サブルーチン mlw_h_c；流体・円柱座標軸対称

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial r}(\rho V_r) + \frac{\partial}{\partial z}(\rho V_z) = -\frac{1}{r}(\rho V_r) \tag{95}$$

$$\frac{\partial}{\partial t}(\rho V_r) + \frac{\partial}{\partial r}\left(\rho V_r^2 + p\right) + \frac{\partial}{\partial z}\left(\rho V_r V_z\right) = -\frac{1}{r}(\rho V_r^2) \tag{96}$$

$$\frac{\partial}{\partial t}(\rho V_z) + \frac{\partial}{\partial r}\left(\rho V_r V_z\right) + \frac{\partial}{\partial z}\left(\rho V_z^2 + p\right) = -\frac{1}{r}\left(\rho V_r V_z\right) \tag{97}$$

$$\begin{aligned} &\frac{\partial}{\partial t}\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2\right) + \frac{\partial}{\partial r}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_r\right) \\ +\quad &\frac{\partial}{\partial z}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_z\right) = -\frac{1}{r}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_r\right) \end{aligned} \tag{98}$$

$$p = \frac{k_\mathrm{B}}{m}\rho T \tag{99}$$

$$V^2 = V_r^2 + V_z^2 \tag{100}$$

2.14 サブルーチン mlw_h_cg；流体重力・円柱座標軸対称

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial r}(\rho V_r) + \frac{\partial}{\partial z}(\rho V_z) = -\frac{1}{r}(\rho V_r) \tag{101}$$

$$\frac{\partial}{\partial t}(\rho V_r) + \frac{\partial}{\partial r}\left(\rho V_r^2 + p\right) + \frac{\partial}{\partial z}\left(\rho V_r V_z\right) = -\frac{1}{r}(\rho V_r^2) + \rho g_r \tag{102}$$

$$\frac{\partial}{\partial t}(\rho V_z) + \frac{\partial}{\partial r}\left(\rho V_r V_z\right) + \frac{\partial}{\partial z}\left(\rho V_z^2 + p\right) = -\frac{1}{r}\left(\rho V_r V_z\right) + \rho g_z \tag{103}$$

$$\begin{aligned} &\frac{\partial}{\partial t}\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2\right) + \frac{\partial}{\partial r}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_r\right) \\ +\quad &\frac{\partial}{\partial z}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_z\right) = -\frac{1}{r}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_r\right) + \rho g_r V_r + \rho g_z V_z \end{aligned} \tag{104}$$

$$p = \frac{k_\mathrm{B}}{m}\rho T \tag{105}$$

$$V^2 = V_r^2 + V_z^2 \tag{106}$$

2.15 サブルーチン mlw_ht_cg；等温流体重力・円柱座標軸対称

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial r}(\rho V_r) + \frac{\partial}{\partial z}(\rho V_z) = -\frac{1}{r}(\rho V_r) \tag{107}$$

$$\frac{\partial}{\partial t}(\rho V_r) + \frac{\partial}{\partial r}\left(\rho V_r^2 + p\right) + \frac{\partial}{\partial z}\left(\rho V_r V_z\right) = -\frac{1}{r}(\rho V_r^2) + \rho g_r \tag{108}$$

$$\frac{\partial}{\partial t}(\rho V_z) + \frac{\partial}{\partial r}\left(\rho V_r V_z\right) + \frac{\partial}{\partial z}\left(\rho V_z^2 + p\right) = -\frac{1}{r}\left(\rho V_r V_z\right) + \rho g_z \tag{109}$$

$$p = \frac{k_\mathrm{B}}{m}\rho T \tag{110}$$

2.16 サブルーチン mlw_m_cg；MHD 重力・円柱座標軸対称

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial r}(\rho V_r) + \frac{\partial}{\partial z}(\rho V_z) = -\frac{1}{r}(\rho V_r) \tag{111}$$

$$\frac{\partial}{\partial t}(\rho V_r) + \frac{\partial}{\partial r}\left(\rho V_r^2 + p + \frac{B^2}{8\pi} - \frac{B_r^2}{4\pi}\right) + \frac{\partial}{\partial z}\left(\rho V_r V_z - \frac{B_r B_z}{4\pi}\right) = -\frac{1}{r}(\rho V_r^2 - \frac{B_r^2}{4\pi}) + \rho g_r \tag{112}$$

$$\frac{\partial}{\partial t}(\rho V_z) + \frac{\partial}{\partial r}\left(\rho V_r V_z - \frac{B_r B_z}{4\pi}\right) + \frac{\partial}{\partial z}\left(\rho V_z^2 + p + \frac{B^2}{8\pi} - \frac{B_z^2}{4\pi}\right) = -\frac{1}{r}\left(\rho V_r V_z - \frac{B_r B_z}{4\pi}\right) + \rho g_z \tag{113}$$

$$\frac{\partial}{\partial t}(B_r) - \frac{\partial}{\partial z}(E_\phi) = 0 \tag{114}$$

$$\frac{\partial}{\partial t}(B_z) + \frac{\partial}{\partial r}(E_\phi) = -\frac{1}{r}E_\phi \tag{115}$$

$$\begin{aligned} & \frac{\partial}{\partial t}\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right) \\ + & \frac{\partial}{\partial r}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_r + \frac{B_z E_\phi}{4\pi}\right) \\ + & \frac{\partial}{\partial z}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_z + \frac{-B_r E_\phi}{4\pi}\right) \\ = & -\frac{1}{r}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_r + \frac{B_z E_\phi}{4\pi}\right) + \rho g_r V_r + \rho g_z V_z \end{aligned} \tag{116}$$

$$E_\phi = -V_z B_r + V_r B_z \tag{117}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{118}$$

$$B^2 = B_r^2 + B_z^2, \quad V^2 = V_r^2 + V_z^2 \tag{119}$$

2.17 サブルーチン mlw_m3_cg；3 成分 MHD 重力・円柱座標軸対称

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial r}(\rho V_r) + \frac{\partial}{\partial z}(\rho V_z) = -\frac{1}{r}(\rho V_r) \tag{120}$$

$$\frac{\partial}{\partial t}(\rho V_r) + \frac{\partial}{\partial r}\left(\rho V_r^2 + p + \frac{B^2}{8\pi} - \frac{B_r^2}{4\pi}\right) + \frac{\partial}{\partial z}\left(\rho V_r V_z - \frac{B_r B_z}{4\pi}\right) = -\frac{1}{r}(\rho V_r^2 - \frac{B_r^2}{4\pi}) + \frac{1}{r}(\rho V_\phi^2 - \frac{B_\phi^2}{4\pi}) + \rho g_r \tag{121}$$

$$\frac{\partial}{\partial t}(\rho V_\phi) + \frac{\partial}{\partial r}\left(\rho V_r V_\phi - \frac{B_r B_\phi}{4\pi}\right) + \frac{\partial}{\partial z}\left(\rho V_z V_\phi - \frac{B_z B_\phi}{4\pi}\right) = -\frac{1}{r}\left(\rho V_r V_\phi - \frac{B_r B_\phi}{4\pi}\right) \tag{122}$$

$$\frac{\partial}{\partial t}(\rho V_z) + \frac{\partial}{\partial r}\left(\rho V_r V_z - \frac{B_r B_z}{4\pi}\right) + \frac{\partial}{\partial z}\left(\rho V_z^2 + p + \frac{B^2}{8\pi} - \frac{B_z^2}{4\pi}\right) = -\frac{1}{r}\left(\rho V_r V_z - \frac{B_r B_z}{4\pi}\right) + \rho g_z \tag{123}$$

$$\frac{\partial}{\partial t}(B_r) - \frac{\partial}{\partial z}(E_\phi) = 0 \tag{124}$$

$$\frac{\partial}{\partial t}(B_z) + \frac{\partial}{\partial r}(E_\phi) = -\frac{1}{r}E_\phi \tag{125}$$

$$\frac{\partial}{\partial t}(B_\phi) - \frac{\partial}{\partial r}(E_z) + \frac{\partial}{\partial z}(E_r) = 0 \tag{126}$$

$$\begin{aligned}
& \frac{\partial}{\partial t}\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right) + \frac{\partial}{\partial r}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_r + \frac{B_z E_\phi - B_\phi E_z}{4\pi}\right) \\
+ & \quad \frac{\partial}{\partial z}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_z + \frac{B_\phi E_r - B_r E_\phi}{4\pi}\right) \\
= & \quad \rho g_r V_r + \rho g_z V_z - \frac{1}{r}\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_r + \frac{B_z E_\phi - B_\phi E_z}{4\pi}\right)
\end{aligned} \tag{127}$$

$$E_r = -V_\phi B_z + V_z B_\phi, \quad E_\phi = -V_z B_r + V_r B_z, \quad E_z = -V_r B_\phi + V_\phi B_r \tag{128}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{129}$$

$$B^2 = B_r^2 + B_\phi^2 + B_z^2, \quad V^2 = V_r^2 + V_\phi^2 + V_z^2 \tag{130}$$

2.18 サブルーチン mlw_m3_sg；3 成分 MHD 重力・球座標軸対称