Ionization Theory of Solar Corona.*

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(Received November 1, 1948.)

Abstract.

The theory of ionization in the corona is treated. The equilibrium of ionization in the corona is kept by the photoelectric recombination vrs. collisional ionization, if we assume the radiation temperature 6000, and a high electron temperature, as is suggested from the line widths and the continuous spectra. A new type of ionization formula is obtained and discussed in comparison with observations. Fe-ionization is treated in detail. The theory requires an electron temperature of the order 10^6 to account for the observed stage of ionization.

§ 1. *Introduction.* The identification of coronal emission lines has been made by B. Edlén.[1] According to him, emissions are forbidden lines, as is commonly seen in the tenuous atmospheres of peculiar stars. But, contrary to the general expectations, they were attributed to the metallic ions of extremely high orders. The ionization potential of these ions amounts to as high as several hundred electron volts, and it is, in fact, a remarkable phenomenon, how we can expect them in the atmosphere of the sun, whose effective temperature is only 6000 absolute degrees.

There is another peculiar conclusions, which necessarily follows from Edlén's identification: According to the observations, emission lines have a broad width,[2] which should be attributed to the thermal or to the turbulent motion of the corona. Observed widths correspond to the velocity of scores of km per sec. Assuming it to be due to the thermal motion of such a heavy ion as iron, the gas temperature of the corona must be of the order of some million degrees. Such a high temperature is also suggested from the observations of continuous spectra of the inner corona,[3] and it seems difficult to attribute to the observed width solely to the turbulence.

In every respect, the condition of the corona is far from the thermal equilibrium, and as is easily seen, it is difficult to give an interpretation by the Saha-like formula.

In the atmospheres of normal stars, the equilibrium of ionization is attained by the photo-electric processes, namely, photo-electric ionization vrs. photo-electric recombination. Consequently, the Saha-like expression is obtained for the equilibrium formula. But, in the case of corona, the condition is somewhat different: The temperature of the radiation, which flows out through the corona is low, and the electrom temperature is very high. The former is the so called effective temperature of the sun, 6000°, and the latter will be of the order of million degrees, according to the observations. In these extreme conditions, the radiation is incompetent to keep atoms in the stage of high ionization. The ionization will be caused by the electron collision. On the other hand, the recombination will be photo-electric, as in the normal atmospheres. The balancing of these two processes will result to a new type of ionization formula, which is different from Saha-like equations. The dependence of ionization condition on the temperature and density naturally differs from those of Saha.

§2. *Ionization formula.* We shall first formulate the condition for the general equilibrium, including the collisional effect. Let N_e be the number of electrons per unit volume, N_1 the number of atoms, or some kind of ion, and N_2 that of ion, or the ion of an order higher. Denote the coefficient of photo-electric capture and ionization

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*) *Contributions from the Institute of Astrophysics, Kyoto University, No. 1.* Translated from *Tenmon-Uchû-Butsurigaku Iho* (*Acta Astronomica Cosmophysicaque*) **2**, 29 (1943) published in Japanese.

by a_{21} and b_{12} respectively, and the coefficient of collisional ionization by β_{12} and that of capture due to the triple collision α_{21}. Then, the general formula expressing the equilibrium of ionization is

$$N_1 b_{12}(T_e) + N_1 N_\varepsilon \beta_{12}(T_\varepsilon) = N_2 N_\varepsilon a_{21}(T_\varepsilon) + N_2 N_\varepsilon^2 \alpha_{21}(T_\varepsilon), \quad (2,1)$$

where a_{21}, β_{12}, α_{21} are functions of election temperature T_ε, and b_{12}, of the effective temperature T_e. The nature of these dependence are designated in the above formula in the bracket. The above equation can be written as

$$\frac{N_2 N_\varepsilon}{N_1} = \frac{b_{12}(T_e) + N_\varepsilon \beta_{12}(T_\varepsilon)}{a_{21}(T_\varepsilon) + N_\varepsilon \alpha_{21}(T_\varepsilon)}. \quad (2,2)$$

The coefficients are related by the well known relations:

$$\left.\begin{aligned} b_{12}(T) &= G(T) a_{21}(T), \\ \beta_{12}(T) &= G(T) \alpha_{21}(T), \end{aligned}\right\} \quad (2,3)$$

where

$$G(T) = \frac{(2\pi m k T)^{3/2}}{h^3} e^{-\chi/kT}, \quad (2,4)$$

in which χ means the ionization potential and T the temperature of the system and other notations in $G(T)$ are as usual.

For the hydrogen-like ions, the capture coefficient is given by

$$\left.\begin{aligned} a_{21}(T) &= \frac{2^9 \pi^5}{(6\pi)^{3/2}} \frac{e^{10}}{m^{1/2} c^3 h^3} \frac{Z^4}{(kT)^{3/2}} E\left(\frac{\chi}{kT}\right), \\ E(x) &\equiv e^x \int_x^\infty \frac{e^{-x}}{x} dx, \end{aligned}\right\} \quad (2,5)$$

where Ze is the nuclear charge of the ion. The exact form for various ions may be much more complicated. Here, we use the above expression as a rough approximation. The cross section for the collisional ionization is given by H. Bethe.[4] Putting the statistical factors to unity and integrating for the temperature T, we find

$$\beta_{12}(N) = \frac{4\pi e^4}{(2\pi m)^{1/2}} \frac{C_{nl}}{\chi} \frac{1}{(kT)^{1/2}} e^{-\chi/kT}, \quad (2,6)$$

where C_{nl} is a constant proper to the electronic shell. In the case of corona, considering Fe-ions, we tentatively take $C_{nl} = 0.15$. Now, we can derive the remaining coefficients from the above two by (2,3).

$$\left.\begin{aligned} b_{12}(T) &= \frac{2^9 \pi^5 m e^{10}}{3^{3/2} c^3 h^6} Z^4 F\left(\frac{\chi}{kT}\right), \\ F(x) &\equiv \int_x^\infty \frac{dx}{x(e^x - 1)}, \end{aligned}\right\} \quad (2,7)$$

$$\alpha_{21}(T) = \frac{e^4 h^3 C_{nl}}{\pi m^2 \chi} \frac{1}{(kT)^2}.$$

§ **3.** Neglecting collisional terms in the general formula (2,2), it reduces to the well known Saha equation. This case is also trivial. On the other hand, in the case of corona, ionization is due to the collision. And hence,

$$b_{12}(T_e) \ll N_\varepsilon \beta_{12}(T_\varepsilon).$$

Then, we get

$$\frac{N_2}{N_1} = \frac{\beta_{12}(T_\varepsilon)}{\alpha_{21}(T_\varepsilon)}, \quad (3,1)$$

or by (2,5) and (2,6),

$$\frac{N_2}{N_1} = \frac{3^{3/2}}{2^6 \pi^3} \frac{c^3 h^3}{e^6} \frac{C_{nl}}{Z^4} \frac{kT_\varepsilon}{\chi} e^{-\chi/kT_\varepsilon} \frac{1}{E\left(\frac{\chi}{kT_\varepsilon}\right)}. \quad (3,2)$$

The relative concentration N_2/N_1 is independent of election density N_ε, while in the usual Saha-like equation, it is inversely proportional to N_ε.

In connection with this, the observation of W. Grotrian[5] is very interesting. His observation leads to the conclusion that the decrement of the intensity of the coronal emission lines with the height from the sun's limb is practically the same for all lines and it is equal to that of the continuous spectrum. Now, according to the reduction of S. Baumbach,[6] the election density N_ε falls from 3×10^8 cm^{-3} to 4×10^7 cm^{-3}, as we proceed from the height 0′.8 to 5′ from the sun's limb, namely, the density drops to about one tenth at 5′ compared with the place 0′.8. Theoretically, the relative intensity of the emission lines 5303 (Fe XIV) and 6374 (Fe X) is derived by multiplying four times the ionization equation. Therefore, if we apply a Saha-like formula, $N_2/N_1 \propto 1/N_\varepsilon$, a density drop of one tenth predicts a change of ten thousandth to the relative concentration of Fe XIV/Fe X, and hence, to the relative intensity 5303/6374. Thus, we fail to give a simple explanation to the observed constancy of the intensity ratio. Possibility of interpretation in terms of the temperature change may be scarce, since every kind of lines keep constant ratios. New formula (3,2) is independent of density and just coincides with the observation.

The observed parallelism between the emission lines and the continuous spectrum will lead to an important conclusion. If the mechanism of the coronal emission is a capture spectra or an collisional excitation, as we know in the Wolf-Rayet stars or in the planetary nebula, the intensity of the emission lines depends on the gas density as follows:

$$\text{emission intensity} \propto N_1 N_\varepsilon \text{ or } N_2 N_\varepsilon . \qquad (3,3)$$

While, if the collisional excitation occurs with sufficient frequency, as in the case of slow novae, the population is adjusted to a Boltzmann distribution. Then, the intensity is determined by the transition probability and in this case,

$$\text{emission intensity} \propto N_1 . \qquad (3,4)$$

On the other hand, the continuous spectrum is the scattering by the free election and hence,

$$\text{intensity of continuous spectrum} \propto N_\varepsilon . \qquad (3,5)$$

In the first case, (3,3), the intensity is nearly proportional to the square of the density, while in the second, (3,4) and the third, (3,5) it is simply proportional to the density. Therefore, the observed parallelism between the line intensities and the continuous spectrum suggests the combination of (3,4) and (3,5), rather than (3,3) and (3,5). Namely, in the corona, the population to each energy levels is governed by the Boltzmann's distribution law.

§4. *Ionization of iron.* In order to investigate the ionization condition of highly ionized ions in a more concrete form, we shall consider the series of *Fe*-ions. The observed ions, identified by Edlén, are *Fe* X, *Fe* XI, *Fe* XIII and *Fe* XIV. The election configuration from *Fe* IX to *Fe* XIV belongs to the $3p^n$-electrons. Their ionization potential increases from 220 eV to 355 eV. Assuming appropriate crude figures for the ionization potentials, relative concentrations have been calculated by the formula (3,2). For the case, $T_\varepsilon = 10{,}000$, the photoionization cannot be neglected, and the general formula (2,2) is used with the density $N_\varepsilon = 10^8 \text{cm}^{-3}$ and the effective temperature $T_e = 6000$. The results are shown in Fig. 1. The ordinate presents the logarithms of the relative abundance, in units of the concentration of the ion of maximum abundance, and the abscissa, the series of *Fe*-ions in its order. Numericals in the figures are electron temperatures corresponding to each curve.

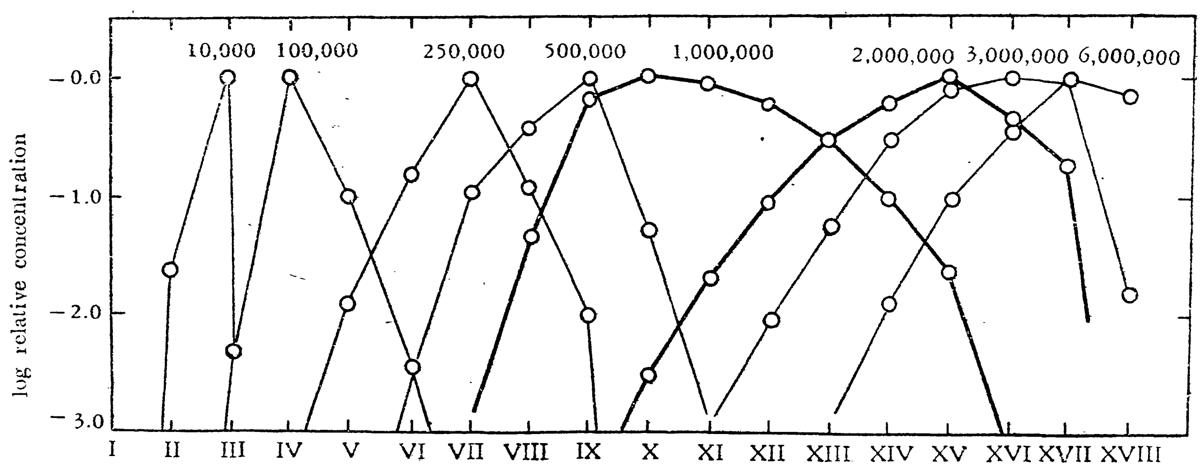

Fig. 1 Ionization of Iron in the Corona

For $T_\varepsilon = 500{,}000$, the concentration of *Fe* XIII and *Fe* XIV seems to be too small to observe 3388 (*Fe* XIII) and 5303 (*Fe* XIV) simultaneously with 6374 (*Fe* X), since their transition probabilities do not differ much. Moreover, the concentration of *Fe* VII is still large and many forbidden emissions belonging to this ion may be expected to be found in the corona. On the other hand, for the temperature as high as $T_\varepsilon = 3{,}000{,}000$, the concentration of *Fe* X and *Fe* XI becomes too small, and may contradict to the visibility of emissions, 6374 (*Fe* X) and 3987 (*Fe* XI). From these considerations, it may be concluded that the appearance of the forbidden emission lines suggests from one million to two million degrees for the electron temperature of the corona. This is in accord with those derived from the line width and the continuous spectrum. By the way, it may be mentioned that the formal application of the Saha's formula requires the temperature of a hundred thousand degrees or so.

§ 5. (i) The relative intensity is very sensitive to the change of temperature. Therefore, if there is any local fluctuation in the electron temperature, the intensities of emission lines will show a variation from place to place. For example, the famous emission line 6374 (*Fe* X) appears stronger at the place, where the temperature is comparatively low, while 5303 (*Fe* XIV), for the high temperature zone. When the coronal streamers have different temperatures, the images of corona observed by the monochromatic lights 6374 and 5303 will not show a similar appearance. On the contrary, they may be a reversal in each other, just like a negative and a positive of the photographic plate. We can find a good example, in the sketches of the eastern limb, drawn by Emma T. Williams from the spectrograph taken at the eclipse of 1930 Oct. 21.[7] Another example will be found in the observations of W. Grotrian at the eclipse 1929 May 9.[3]

(ii) In the corona, there appears none of the lines familiar in the spectra of the chromosphere. According to the above theory of ionization, every kind of elements must be ionized strongly, and this will change the spectroscopic appearance of the corona radically from those of the chromosphere. Even the hydrogen escapes from our detection, though it may be the most abundant element in the corona.

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