Critical scaling for the jamming transition of granular materials

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Granular materials

Saturn ring

Sand

mustard seed

Ginkaku-ji temple
Sheared granular materials

Shear stress

Inhomogeneous flow

Gas
$(\Phi = 0.12)$

Homogeneous flow

Dense liquid
$(\Phi = 0.8)$

No flow

Amorphous solid
$(\Phi = 0.85)$
Jamming transition

Homogeneous flow

Transition point $\Phi_J$

Dense liquid ($\Phi = 0.8$)

Amorphous solid ($\Phi = 0.85$)
Jamming transition for athermal materials

- Foam
- Colloidal suspensions
- Josephson junction array
Model of granular materials

- $F_n = k \Delta - \eta \nu_n$
  - Elastic part
  - Dissipative part

- $\Delta = 1$ (Disk)
- $\Delta = 3/2$ (Sphere)

Tangential force

- Friction coefficient: $\mu$
- $F_t < \mu F_n$ (Coulomb’s friction)
- Frictionless: $\mu = 0$
- Frictional: $\mu > 0$
Critical properties

Frictionless case, $\Delta = 1$

Shear modulus

\[ G \sim (\Phi - \Phi_j)^{1/2} \]

Frictionless case, $\Delta = 1$

Pressure $P$

\[ P \sim (\Phi - \Phi_j) \]

Viscosity $\eta$

\[ \eta \sim (\Phi - \Phi_j)^{-3} \]
Rheological property

Shear stress $\sigma$

For $\Phi < \Phi_J$, $\sigma \propto \dot{\gamma}^2$ (liquid)

For $\Phi > \Phi_J$, $\sigma \approx \text{const}$ (solid)

For $\Phi \approx \Phi_J$, $\sigma \propto \dot{\gamma}^{\gamma}$

non-linear transport property

Frictionless case, $\Delta = 1$

$\sigma(\gamma, \Phi) / |\Phi - \Phi_J|^\beta$

$\dot{\gamma} |\Phi - \Phi_J|^{-\alpha}$

Hatano, 2008

$\Phi_J$, $\alpha$, $\beta$ : Critical exponents
Dynamics (constant shear rate)

\[ \Phi = 0.80 < \Phi_j \]

\[ \Phi = 0.85 > \Phi_j \]
Dynamics
(velocity fluctuation)

\[ \Phi = 0.80 < \Phi_j \]

\[ \Phi = 0.85 > \Phi_j \]
Characteristic features

The critical exponents depend on the type of the contact force.

\[ F_n = k \delta^\Delta \]

The critical exponents are independent of the dimension.

Mean field theory

\[ y_\gamma = \frac{2\Delta}{\Delta + 4} \]
Effect of Friction

Frictionless ($\mu = 0.0$)

Frictional ($\mu = 2.0$)

Hysteresis loop for frictional case
Effect of friction (pressure)

Frictionless ($\mu = 0.0$)
Continuous transition

Frictional ($\mu = 2.0$)
Discontinuous transition
Effect of friction (type of the transition)

Friction coefficient $\mu$

Continuous transition

Discontinuous transition

$\Delta P$
Phase diagram

\[ P \sim (\Phi - \Phi_S) \]

Area of the hysteresis loop

\[ \Phi_C, \Phi_S, \phi_L(\mu) \]
• **Jamming transition**: Athermal transition from liquid-like states to solid-like states.

• Critical exponents depend on the interaction.

• Continuous transition for frictionless case, discontinuous transition for frictional case.

• Hysteresis loop, many critical densities.

• Our result may provide a better understanding of dynamics and non-linear transport properties of dense matters.