Poynting–Robertson Drag and Orbital Resonance

R. GONCZI, CH. FROESCHLE, AND CL. FROESCHLE

*Laboratoire de Physique Théorique, Université de Nice, Equipe de Recherche Associée au CNRS Parc Valrose, 06034 Nice Cedex, France, and †Observatoire de Nice, B.P. 252, 06007 Nice Cedex, France

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Both the Poynting-Robertson drag and resonant orbits appear to be very important for the motion of small grains in the early solar system. While orbital resonances are very often stable and tend to force bodies into noncircular orbits, the Poynting-Robertson drag produces secular variations in the semimajor axis and tends to circularize the orbits. We study numerically the competition between the Poynting-Robertson drag and the gravitational interaction of grains with Jupiter near the 2/1 resonance. Computations are based on the plane-restricted problem. Numerical investigations show that the grains always cross the resonance region without any oscillation, except in the special case where the grains were initially inside the resonance. In both cases the variations of the osculating elements exhibit a drastic step, which can be explained by Greenberg’s and Schubart’s theories.

I. INTRODUCTION

It is well known that resonances (for instance, in the three-body Sun–Jupiter–asteroid problem) play a major role in celestial mechanics and very often tend to be stable, i.e., to maintain themselves against disturbances, while a body subjected to nongravitational forces will ultimately fall into the Sun. These two cases have been separately and extensively studied. The purpose of this paper is to study the behavior of a body near a resonance with Jupiter and subjected to nongravitational forces.

Poynting (1903) considered the effects of absorption and subsequent reemission of sunlight by small isolated particles in the solar system. Later Robertson (1937) modified the previous results; using a precise relativistic treatment, he established the equations of the motion in the solar reference frame to the first-order terms of the particle speed measured in light speed units. He found that the absorption and the subsequent reemission introduce a resisting force on the particle which is proportional to its velocity. The drag force exhibited in the equations results in a slow secular decrease of the semimajor axis $a$ and eccentricity $e$ of the orbit. Ultimately the body will fall into the Sun. Since, however, the time of fall will depend on $e$, Wyatt and Whipple (1950) generalized the calculations of Robertson for eccentric orbits and calculated the time of fall for different initial values of the semimajor axis $a$ and eccentricity $e$ for asteroidal particles. Burns et al. (1979), taking into account the radiation pressure efficiency factor and the asymmetry of the scattered radiation, consider the orbital consequences of the radiation pressure and Poynting–Robertson drag forces for heliocentric and planetocentric orbiting particles.

On the other hand, often motivated by the existence of Kirkwood gaps, many authors investigated the resonances. Though these gaps occur in the distribution of asteroids and not necessarily of small particles, they are relevant to the restricted three-body problem, since the asteroid’s masses are very small compared to the masses of the Sun and the planet.

A review of this problem is given by Greenberg and Scholl (1979). In 1975 Gold had already underlined the importance of the Poynting–Robertson drag for driving grains into resonant orbits and thereby per-
haps supplying material for satellite forma-

In order to explain the resonant structure of the asteroid belt, Greenberg (1978) con-

considered particles in orbital resonance with Jupiter in a dissipative medium. He found that for reasonable early solar system pa-

rameters, the gap near the 2/1 resonance with Jupiter is cleared on a time scale of a few thousand years. Greenberg suggested also that under the Poynting–Robertson drag, bodies not in resonance might tend to approach and become trapped into resonances according to different mechanisms.

In this paper, we propose to investigate numerically the competition between the Poynting–Robertson drag, pulling a particle in the resonance, and the gravitational in-

teraction of that particle with Jupiter near the 2/1 resonance. Our calculations are based on the plane-restricted three-body problem given by the Sun, Jupiter, and a planete-esimal. This object is assumed spherical, with radius $s$ and uniform density $\rho$. It absorbs all incident radiation from the Sun over a cross section $\pi s^2$ and reemits this radiation isotropically at the same rate. The computations are performed in the circular case (eccentricity of Jupiter $e_J = 0$) and in the elliptic case ($e_J \neq 0$) for some initial values of the semimajor axis $a$ and eccentricity $e$ by varying the initial value of the critical argument $\sigma$, which represents the angle between the longitude of the plan-

etesimal pericenter $\omega$ and the longitude of conjunction.

In Section II we briefly recall the theories of Greenberg and Schubart, which give a qualitative description of the motion of particles in resonance under purely gravitational forces. Section III is devoted to pre-

senting our model. In Section IV we present our numerical results and discuss them qualitatively using the Greenberg and Schubart models.

II. GREENBERG’S AND SCHUBART’S THEORIES FOR RESONANCE MECHANISM

As we will use results obtained by Green-

berg and Schubart in the resonance prob-

lem, we recall briefly these theories. For more details see Greenberg (1977), Green-

berg and Scholl (1979), and Schubart (1964, 1968).

1. Greenberg’s Theory

In the frame of the restricted coplanar three-body problem (Sun–Jupiter–asteroid) we consider the Jupiter–asteroid interaction in the 2/1 resonance. It will be assumed that Jupiter has a fixed circular orbit ($e_J = 0$) and that the orbital eccentricity $e$ of the asteroid is very small (i.e., the asteroidal angular velocity is nearly constant). Retaining only the secular terms and after combing the equations of variation of the orbital elements (cf. Danby, 1962), it is found that the behavior of the orbital motion of the asteroid in the 2/1 resonance is given by the equation

$$e \sin \sigma = C \sin(At + \sigma),$$

$$e \cos \sigma = C \cos(At + \delta)$$

+ $(m_J/m_\odot)(a/a_J)nF_3/A$, where

- the amplitude $C$ and the phase $\delta$ are the constants of integration,
- $a, a_J$ are the semimajor axes of the ast-

eroid and Jupiter, respectively,
- $A = 2n_J - n$ ($n_J$ and $n$ are Jupiter’s mean motion and the asteroidal mean motion),
- $F_3$ is a function of $(a/a_J)$, of order of unity,
- the critical argument $\sigma = 2l_J - l - \dot{\omega}$ represents the angle between the longitude of asteroidal pericenter $\omega$ and the longitude of conjunction $(2l_J - l)$; $l$ and $l_J$ are, respectively, the asteroidal and Jovian mean longitudes,
- $m_J$ and $m_\odot$ are, respectively, the mass of Jupiter and the solar mass.

In polar coordinates $(e, \sigma)$ the solution represented by Eqs. (1) and (2) is the sum of two vectors. The first one is called "forced" because it is the irreducible eccentricity forced by Jupiter. Its magnitude is
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given by

\[ \mathcal{C}_F = \left( \frac{m_2}{m_1} \right) \cdot (a/a_j)^n \frac{F_3}{A} \]  

(3)

and its direction is given by \( \sigma = 0 \). The second vector is the free eccentricity, as it is an arbitrary parameter of the motion. Its arbitrary magnitude is \( \mathcal{E}_F = C \) and its direction circulates at rate \( 2n_j - n \).

Note that the magnitude \( \mathcal{E}_F \) increases as the exact resonance is approached and formally becomes infinite for \( 2n_j - n = 0 \), although the analysis breaks down in that case. As defined in (3), \( \mathcal{E}_F \) is positive when \( 2n_j - n > 0 \) (i.e., when the asteroid is farther from the Sun than the exact resonance) and is negative when \( 2n_j - n < 0 \) (i.e., when the asteroid is closer to the Sun than the exact resonance). (Negative \( \mathcal{E}_F \) means that the forced \( e \) vector points toward \( \sigma = 180^\circ \).)

When \( \mathcal{E}_F < |\mathcal{E}_F| \) the parameter \( \sigma \) can librate about 0° (if \( \mathcal{E}_F > 0 \), pericentric librator), or about 180° (if \( \mathcal{E}_F < 0 \), apocentric librator); \( \sigma \) circulates for \( \mathcal{E}_F > |\mathcal{E}_F| \).

Now if \( e \) is not sufficiently small to assume \( n \) nearly constant, it can be shown (Greenberg and Scholl, 1979) that the motion in \( e \) space continues at any instant to follow a circle, but the center keeps moving. The trajectory is distorted into a banana shape.

When the eccentricity \( e_j \) of Jupiter is taken into account, Greenberg and Franklin (1975) have shown that in addition to \( \sigma \), a new critical argument \( \Phi = \bar{\omega} - \bar{\omega}_j \) librates or circulates. These librations and circulations of \( \Phi \) produce a long-period variation in \( e \), in addition to the small variation found when \( e_j = 0 \).

2. Schubart’s Theory

The resonance mechanism described above applies in a strict sense only to special cases of small eccentricity. To study more general cases, Schubart (1964, 1968) has introduced an averaging method. Modifying Poincaré’s canonical equations (for the planar elliptic Sun–Jupiter–asteroid problem) he averages the Hamiltonian over the corresponding commensurability period. Using numerical models, Schubart plotted trajectories in polar coordinates \( (2S)^{1/2}, \sigma \), where

\[ S = (a/a_j)^{1/2} \left[ 1 - \left( \frac{1 - e^2}{2} \right)^{1/2} \right] \]

when \( e_j = 0 \) Schubart noted that the quantity

\[ K = (a/a_j)^{1/2} \left[ \frac{(p + q)}{p} \right] - (1 - e^2)^{1/2} \]

(4)

is a constant of integration (for the 2/1 resonance, \( p = q = 1 \)).

Figure 1 represents the trajectories for the 2/1 commensurability, with \( K \) constant. The curves differ only in the value of the remaining integral of the motion \( \dot{H} \), “Schubart’s average hamiltonian.”

3. Remarks

For \( e \) small, \( (2S)^{1/2} \propto e \). The trajectories in the immediate vicinity of points \( A \) and \( P \) in Fig. 1 are the extension of the solutions described by Eqs. (1) and (2). Trajectories about \( A \) are for “apocentric librators” (\( \sigma \) about 180°) and those about \( P \) are for “pericentric librators” (\( \sigma \) about 0°).

On Schubart’s diagram there appears a critical bifurcated trajectory (Fig. 1, darker line), which divides the space into three regions. If the origin belongs to the interior of the critical curve (region I), there will be circulators (inner circulators) as discussed before (\( \mathcal{E}_F > |\mathcal{E}_F| \) and \( 2n_j - n < 0 \)). Farthest from the origin is region III of outer circulators (\( \mathcal{E}_F > |\mathcal{E}_F| \), \( 2n_j - n > 0 \)). Between these two regions, there are banana-shaped librators, as predicted by Greenberg; for these trajectories \( e \) is no longer small and presents rather large variations (region II). Of course the Schubart picture is valid only in the circular averaged model. Only in this special case is the problem integrable (a second integral \( K \) exists besides the energy integral) and the critical point is called a homoclinic point in modern dynamics (Helleman, 1980). It is well known that integrable systems are not generic, i.e.,
a small perturbation can destroy the integrability and the separatrix or homoclinic orbit gives birth to a wild region where some kind of chaotic behavior occurs (Helleman, 1980). This peculiar behavior occurs near the separatrix and has been displayed by Scholl and Froeschle (1977) in the elliptic averaged case and by Bevilacqua et al. (1980) for the Titan–Hyperion case. Non-averaging or non-coplanarity, as well as ellipticity, will destroy the $K$ integral. However, the planar circular picture remains valid to some extent and is a good paradigm for understanding the phenomena.

Both theories will be used to explain our numerical results.

III. THE MODEL

We first briefly recall the main notations and equations of the planar three-body restricted problem in the presence of a non-gravitational force $f$. We consider a particle of negligible mass in the gravitational field of the Sun (mass $m_1$) and Jupiter (mass $m_2$), which are assumed to be point masses orbiting around their common center of mass $G$. The distances of the particle to these three points are, respectively, denoted by $r_1$, $r_2$, and $r$.

The unit of time is the year, the unit of mass the solar mass, and the unit of length the astronomical unit, so that the numerical value of the gravitational constant is $K = 4\pi^2$.

The asteroidal equation of motion then reads

$$\ddot{r} = -Km_1(\dot{r}_1/r_1^3) - Km_2(\dot{r}_2/r_2^3) + \ddot{f},$$

where $\ddot{f}$ is the non-gravitational force.

The numerical integration of this equa-
tion is performed by the Burlisch–Stoer (1966) method, and after each step we compute the elements of the osculating orbit with respect to the Sun.

We choose the origin of time when Jupiter is at its perihelion and set \( t_0 = 0 \). An orbit is entirely determined by the initial semimajor axis \( a_0 \), the eccentricity \( e_0 \), the perihelion longitude \( \omega_0 \), and the mean longitude \( l_0 \) of the particle. In the following we will set, for each orbit, \( a_0 = 3.36 \), \( e_0 = 0.14 \), and take as variable the critical angle \( \theta_0 = \omega_0 - l_0 \) (with \( p = q = 1 \) in the 2/1 resonance) which appears, through various numerical experiments, to be the relevant parameter of the problem. For the non-gravitational force, we consider a Poynting–Robertson drag of the form

\[
\begin{align*}
\dot{f} &= -\alpha \frac{v^2}{r^2},
\end{align*}
\]

where \( v \) is the particle velocity and \( \alpha \) depends on the radius \( s \) and the density \( \rho \) of the spherical object:

\[
\alpha = 2.5 \times 10^{11} s \rho \text{ (cm}^2/\text{sec)}.\]

The numerical value of \( \alpha \) was chosen such that it corresponds to \( s = 10^{-3} \text{ cm} \) and \( \rho = 2 \text{ g/cm}^3 \). In our system of units we get

\[
\alpha = 2 \times 10^{-5} \text{ AU}^2/\text{year}.
\]

Of course this peculiar value is very restricted, but we do not intend in this paper to give an extensive numerical study of this problem. However, preliminary results indicate that qualitatively similar phenomena occur for particles corresponding to other values of \( \alpha \), namely, \( \alpha/2 \) and \( 2\alpha \).

IV. RESULTS

Let us first recall some results from the two-body problem.

The secular perturbations of the semimajor axis and eccentricity for a particle in the solar gravitational field submitted to a Poynting–Robertson drag are given by (Wyatt and Whipple, 1950)

\[
\frac{da}{dt} = -\frac{a}{2a^2(1 - e^2)^{3/2}} \left( 2 + 3e^2 \right).
\]

A straightforward calculation shows that for small eccentricities we get

\[
a = a_0[1 - (2\alpha t/a_0^2)]
\]

and

\[
e = e_0[1 - \frac{3}{2}(\alpha t/a_0^2)],
\]

which means that \( a \) and \( e \) decrease linearly in time.

For the cases in which we are interested, the rate of change of \( e \) is insignificant: starting with \( a_0 = 3.36 \) and \( e_0 = 0.14 \), the variations of these quantities after \( 2 \times 10^4 \) years are, respectively, \( \Delta a = -0.24 \) and \( \Delta e = -0.014 \).

1. Plane-Restricted Three-Body Problem

In order to bring out the physical effects of the drag force, we now present some preliminary computations for the conservative case (\( \alpha = 0 \)); these computations also allowed us to check our program by comparing the results with those obtained with an averaging procedure by Schubart (1966) and Scholl and Froeschlé (1974, 1975).

As usual, an orbit will be described by means of osculating elements which refer to a Keplerian orbit, the perturbing parameters being both the mass of Jupiter and the drag force. Similarly, following a procedure used by Scholl and Froeschlé (1977) to describe the orbits within the resonances, we will also represent them in the Schubart plot: in the circular conservative case (\( e_f = 0 \) and \( \alpha = 0 \)), an orbit is depicted by a closed curve in the two-dimensional phase space. In the nonconservative problem, the two isolating integrals \( K \) and \( H \) disappear, but are used as the third and fourth dimension in an extended "phase space." This new phase space consists of superimposed sheets which are two-dimensional phase spaces of the conservative circular problem for different values of \( K \) and \( H \). The orbits are therefore represented by "osculating
curves" of the circular conservative problem.

Let us consider the circular case \((e_j = 0)\).

When the eccentricity of the planetesimal is small, we have seen, in Section II, that it is proportional to the radius vector of Schubart's plot. Hence for a small fixed value \(e_0 = 0.14\) we see (Fig. 1) that depending on \(\sigma_0\) there are only two possibilities for the orbit: it lies either in the region of inner circulators (starting for instance at point \(M_1\)) or in that of pericentric librators (point \(M_2\)). Indeed in the many different orbits we have numerically studied by varying the initial \(\sigma_0\), only two types of behavior arose:

1. Figure 2 shows the osculating elements for an orbit starting with initial conditions \(a_0 = 3.36, e_0 = 0.14, \omega_0 = l_0 = \pi/3, \sigma_0 = -2\pi/3\). It is of the first type (inner circulator): \(a\) and \(e\) remain nearly constant with only some little perturbations due to Jupiter's mass.

2. A librator-type orbit, starting with \(a_0 = 3.36, e_0 = 0.14, \omega_0 = l_0 = \pi/10, \sigma_0 = -2\pi/10\), is plotted in Fig. 3 and we note, as expected, that \(a\) oscillates indefinitely around the center 3.28 of the resonance while \(e\) has large-amplitude variations.

These two orbits are also shown on the Schubart plot (Fig. 4). Note that these curves are quite similar to those obtained by Schubart (1964, 1966) after averaging. As we have not used this averaging procedure, the problem is no more integrable and our curves are a little thicker.

Concerning now the elliptic problem, Greenberg and Franklin (1975) have shown that Jupiter's eccentricity induces an extra-long-period term in the variation of \(e\). Figures 5b and 6b clearly display this feature.

2. Circular-Restricted Three-Body Problem with Drag

Introducing the Poynting–Robertson drag force, we first investigate the circular case.

As in Section IV. 1, for fixed initial values \(a_0 = 3.36\) and \(e_0 = 0.14\) we perform several computations with different values of \(\sigma_0\). Again the computations show that the orbits belong to two classes which have drastically different behaviors: if an orbit starts in the region of inner circulators, it crosses the resonance in less than 500 years without any oscillation (Fig. 7); otherwise (Fig. 9) the particle remains inside the resonance for a while. In both cases the curves describing the variations of the osculating elements consist of two distinct parts separated by a drastic step which corresponds, in the Schubart plot, to the crossing of the critical curve. This is clearly illustrated in Fig. 8, where one can follow the quick passage from inner to outer circulator, and in Fig. 10, which shows the evolution from pericentric libratot to outer circulator. In both cases, the change of regime occurs when the orbit reaches the critical point, as already suggested by Figs. 7c and 9c.

In the case represented in Figs. 7 and 8, the quick crossing of the resonance is explained by the fact that in the \((e, \sigma)\) plane, regions I and III (Fig. 1) are connected through the single point \(Q\), so that the orbits may go directly from one region to the other without passing in region II.

In the other case, the behavior of the eccentricity can also be interpreted in terms of evolution in the Schubart plot: when the particle is temporarily trapped in the resonance, the value of \(e\) on coming out of this region (Fig. 10c) is nearly the same as its maximum value during the period of libration (Fig. 10a). This fact is easily checked in Fig. 9b. Contrast this behavior with the other case, in which the crossing of the critical curve from inner to outer circulators (Fig. 8) involves the sharp increase of \(e\) exhibited in Fig. 7b.

As a consequence of this increase, the semimajor axis shows a sharp decrease (Fig. 7a); this is due to Jacobi's integral:

\[
C = 1/a - [a(1 - e^2)]^{1/2}.
\]

This quantity is constant in the conservative case, but in the presence of the dissipative force \(f\), it will change, during the pas-
Fig. 2. Variations of (a) semimajor axis and (b) eccentricity of an orbit in the circular case without drag. The initial conditions are $a_0 = 3.36$, $e_0 = 0.14$, $\sigma_0 = -2\pi/3$. 
Fig. 3. Same curves as in Fig. 2, for initial conditions \( a_0 = 3.36, e_0 = 0.14, \sigma_0 = -0.2\pi \).
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Fig. 4. Schubart’s plot corresponding to the orbits of Fig. 2 (inner part) and Fig. 3 (banana shape).

sage through the resonance, by a quantity

\[
C' = \int_{t_1}^{t_2} \dot{V} \cdot \ddot{V} \ dt
\]

\[
= \alpha \int_{t_1}^{t_2} \frac{\dot{V}}{r^2} \cdot \ddot{V} \ dt < \int_{t_1}^{t_2} \frac{V}{r^2} V_t \ dt,
\]

where \( V_t \) is the particle velocity relative to the rotating frame, \( t_1 \) is the time when the particle enters into the resonance, and \( \Delta t = t_2 - t_1 \) is the duration (=400 years) of the jump. Near the commensurability, the integrand is nearly constant so that

\[
C' < \alpha (VV_d/r^2) \quad \Delta t = \alpha [V^2/(2r^2)] \Delta t.
\]

If we compare \( C \) and \( C' \) we find that the relative variation \( C'/C \) is only \( 3 \times 10^{-4} \), so that during the short time of the crossing, \( C \) can still be considered as a constant.

In order to evaluate the frequencies of cases 1 and 2 we have performed a statistical study of 100 particles starting far away from the 2/1 resonance (\( a_0 = 3.45, e_0 = 0.14 \), and 100 random values of \( \sigma_0 \)). All these particles are found to reach the resonance region as inner circulators. This is not surprising since, for a particle orbiting on the external side of the resonance, \( \frac{d\theta}{dt} \) is known to be positive (see, for example, Greenberg and Scholl, 1979), so that in the Schubart plot the orbit is an inner circulator.

Then pericentric librators (case 2) occur only if the particle is initially inside the resonance.

3. Elliptic-Restricted Three-Body Problem with Drag

The ellipticity of Jupiter’s trajectory (\( e_j = 0.049 \)) introduces two specific features to the behavior, although most characteristic points of the circular problem remain valid. First, outside the resonance, the variation of \( e \) is no longer linear (Fig. 11b) but shows the same oscillating behavior as in the elliptic case without drag (Fig. 5b). It can be shown (Morando, 1981) that the frequency of these oscillations decreases with \( a \) (of course, all other Jovian parameters are constant). Figure 11b confirms this property.

Second, when the particle is trapped (Fig. 12) it remains in the resonance for a time which is not the same as in the circular case. It may be longer (as in the case described in Fig. 12) or shorter (for some other initial conditions or other values of \( \alpha \)). However, it seems to be difficult to give an explanation of this quantitative property through Schubart’s curves.

Finally, the typical behavior of an orbit passing successively through several resonances in about \( 6 \times 10^4 \) years is shown in Fig. 13. It can be seen that the features described above for the 2/1 resonance are qualitatively similar in the other commensurabilities.

V. CONCLUSION

We knew that even in the presence of Jupiter, the Poynting–Robertson drag tends in general to reduce the value of the orbital semimajor axis, and to circularize the orbits. Therefore a particle orbiting far away from a resonance will necessarily reach this region. Our results show that at this stage
Fig. 5. Same curves and initial conditions as in Fig. 2, but for the elliptic problem.
Fig. 6. Same curves and initial conditions as in Fig. 3, but for the elliptic problem.
Fig. 7. Variations of (a) semimajor axis, (b) eccentricity, and (c) critical angle $\sigma$, for an orbit in the circular case with resisting force. The initial conditions are the same as in Figs. 2 and 4: $a_0 = 3.36$, $e_0 = 0.14$, $\sigma_0 = -2\pi/3$. In order to see more clearly the behavior of $\sigma$ during the change of regime, the time scale of (c) is not the same as in (a) and (9b).
Fig. 7—Continued.
Fig. 8. Schubart's plot corresponding to the orbit of Fig. 7, for different time spans. (a) $0 < t < 1000$: inner circulator, $\sigma$ circulates anticlockwise. (b) $1000 < t < 1250$: evolution from inner to outer circulator, $\sigma$ changes direction. (c) $1250 < t < 1900$: outer circulator, $\sigma$ circulates clockwise. (d) $0 < t < 1900$: display of the whole orbit.
Fig. 9. Same curves as in Fig. 7 but with the initial conditions of the orbit described in Figs. 3 and 5: $a_0 = 3.36$, $e_0 = 0.14$, $\sigma_0 = -0.2\pi$. 
Fig. 9—Continued.
Fig. 10. Schubart's plot corresponding to the orbit of Fig. 8 for different time spans. (a) $0 < t < 3400$: The orbit initially follows the right edge of the banana (single arrows). As time increases, it moves slowly toward the left edge while the two ends of the banana drew nearer and nearer to each other (double arrows). (b) $3400 < t < 3600$: $\sigma$ changes sense for the last time. (c) $3600 < t < 6000$: outer circulator. (d) $0 < t < 6000$: display of the whole orbit.
Fig. 11. Variations of (a) semimajor axis and (b) eccentricity for an orbit in the elliptic case with resisting force and same initial conditions as in Figs. 2 and 7: $a_0 = 3.36$, $e_0 = 0.14$, $\sigma_0 = -2\pi/3$. 
Fig. 12. Same curves as in Fig. 11 but for the same initial conditions as in Figs. 3 and 8: \(a_0 = 3.36, e_0 = 0.14, \sigma_0 = -0.2\pi\).
FIG. 13. Orbital elements for a particle passing successively through several resonances.
the semimajor axis decreases with a drastic step while the eccentricity is sharply lifted up. The particle then proceeds on its way toward the Sun.

On the other hand, if a particle is initially inside the resonance, it remains there oscillating for a long time until the drag causes it to be ejected toward the Sun.

These two behaviors can be interpreted in terms of Schubart’s and Greenberg’s theories: in both cases, the orbit is submitted to a change of regime which occurs when it reaches the critical point, either from the region of pericentric librators to that of outer circulators, or from the region of inner to outer circulators.

As can be seen in Fig. 13, a particle under the influence of the Poynting–Robertson drag will cross rapidly all the resonances. If in a given resonance there was no preexisting large body, this mechanism can explain the depletion of this resonance. Now if in a resonance there were particles as librators, the Poynting–Robertson drag could supply material coming continuously into the resonance, and snowball large particles which rapidly are no longer sensible to the Poynting–Robertson drag as they become too large; these bodies remain as librators within the resonance. Such a descriptive mechanism could tell in favor of the Hilda group.

However, we are fully aware that our study is far from exhaustive. We have begun to perform a systematic exploration of the crossing time of the different resonances and the variations of this crossing time with the size and density of particles which might explain why some resonances are empty and others contain librators. Other kinds of drags, such as tides and rotating gas, coupled with resonance, would be of interest to explore.

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