Radiation forces on small particles in the Solar System: A re-consideration

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ABSTRACT

We respond to Kláčka et al. (Kláčka, J., Petráš, J., Pár, P., Kómar, L. [2014]. Icarus, this issue, http://dx.doi.org/10.1016/j.icarus.2012.06.044.), who have criticized many previous derivations of the acceleration experienced by a spherical interplanetary particle owing to the Sun’s radiation. Much of their criticism arises from differences in semantics and notation as well as effects that are unimportant at Solar System speeds. Accordingly, in the appropriate limiting cases, most published expressions for the radiation forces, such as that found in Burns et al. (Burns, J.A., Lamy, P.L., Soter, S. [1979]. Icarus 40 1–48), are correct and duplicate the results of Kláčka et al. (Kláčka, J., Petráš, J., Pár, P., Kómar, L. [2014]. Icarus, this issue, http://dx.doi.org/10.1016/j.icarus.2012.06.044).

In the century since Poynting (1904) introduced his eponymous drag, a series of authors have used various approaches to derive expressions for the radiation forces that act on small particles in interplanetary space. The earliest versions of these equations disagreed in the numerical coefficients of several terms until Robertson (1937) clarified matters by investigating the relativistic motion of a perfectly absorbing, thermally equilibrated sphere. Much later, when surveying the topic, Burns et al. (1979), hereafter BLS, recalled some of the initial attempts to derive these subtle forces, and then presented two of their own derivations, a heuristic one based mostly upon classical (i.e., non-relativistic) physics and another employing special relativistic transformations; these two approaches arrived at the same force expression for slow speeds. In addition, BLS extended the efforts of previous researchers by computing the radiation forces that spherical grains of cosmically significant materials would experience when exposed to the Sun’s radiation flux. Before closing, BLS derived and explained many first-order dynamical effects of radiation forces for circumsolar and circumplanetary particles.

Recently, combining results developed over two decades across scores of publications on the topic by the senior author, Kláčka et al. (2014), hereafter KPPK (see references therein), summarize that author’s previous relativistic, covariant formulations to ascertain the nature of radiation forces. To compress their derivation, KPPK combined pieces of expressions from fifteen self-references that sometimes employed different notation, making the latest derivation more onerous to understand than it should be. We presume that KPPK’s goal in presenting a succinct derivation was to have a framework from which to highlight what they consider to be specific conceptual flaws or misstatements in the derivations of nine previous publications, including those by BLS. Earlier, Kláčka et al. (2009) had found fault with the manner that an additional thirty-three references treated the topic.

Owing to its brevity, differences in notation (across their various publications as well as with BLS), frequent reference to appendices for crucial material, semantics, and terminology, we have found many portions of KPPK’s derivation to be quite difficult to follow. Thus, we believe it is unproductive for us to now respond point-by-point to each of KPPK’s criticisms. Nonetheless, below we will compare their result to ours while also treating a few points of possible confusion. Throughout their paper, KPPK seem more concerned with how these somewhat-confounding effects are to be understood, rather than whether the correct expressions are employed appropriately when evaluating timescales for particular astrophysical applications. Thus, at this juncture it is relevant to note – as those authors themselves acknowledge both in their abstract and their text – that the final expressions for radiation forces found in BLS, as well as those given in most other treatments that KPPK find fault with, are correct. Accordingly, past conclusions...
about the orbital collapse times of circumstellar dust grains and debris disks (e.g., Wyatt and Whipple, 1950, BLS, Lamy and Perrin (1997) and Wyatt (2008)) are valid.

Let us begin by comparing the expressions of KPPK and BLS for the accelerations experienced by interplanetary particles. In each of these papers, the force sustained by a particle situated in a radiation beam is computed by subtracting the outgoing momentum flux from that in the incident beam, and calling upon the impulse-momentum theorem. Radiation’s interaction with a particle is traditionally described either as absorbed or scattered where, conventionally, scattered radiation includes Fresnel reflection and refraction plus diffraction. The efficiency coefficients $Q_{\text{abs}}$ and $Q_{\text{scat}}$, which provide fractional measures of these interactions, depend on the particle’s optical properties and on its non-dimensional size (see below), van de Hulst (1981) presents expressions for the absorption and scattering coefficients as the sums of infinite series of the Mie coefficients $a_n$ and $b_n$.

After some minor manipulation, KPPK’s Eq. (15) for the particle’s acceleration can be written, in covariant form, as

$$\frac{du'^i}{dt} = w^i\left(\frac{SA}{mc}\right)Q_{pr}\left(b^i - \frac{u'^i}{c}\right),$$  \hspace{1cm} (1)$$

where the particle’s four-velocity $u'$ has components $(\gamma c, \gamma v)$ with $v$ denoting the particle’s velocity, $c$ is the speed of light and the Lorentz factor $\gamma = \sqrt{1 - v^2/c^2}$; $\tau$ is the proper time measured in the particle’s rest frame; $w = \gamma(1 - v \cdot e/c)$ with $e$ the unit vector that points from the Sun to the particle; $S$ is the energy flux density in the radiation beam; $A'$ is the geometrical cross-section of an assumed spherical particle of radius $s$; $m$ is the particle’s rest mass; in the Sun’s rest frame, the four-vector $b^i$ has components $(1/e)/w$; primes ($'$) designate quantities measured in the particle’s rest frame; and finally $Q_{pr}$ is the traditional radiation pressure efficiency factor (van de Hulst, 1981); however, see the further discussion of radiation pressure coefficients in the paragraph below our Eq. (5). The latter is a dimensionless constant that is proportional to the forward momentum removed when an incident light beam encounters a particle and not subsequently returned to the forward momentum in the scattered light. BLS evaluates these momenta by incorporating laboratory-derived optical constants as functions of wavelength and then integrating the results from individual Mie calculations taken across the solar spectrum.

For $v/c \ll 1$ (as is valid in many astrophysical circumstances, specifically for particles orbiting the Sun), Eq. (1) may be simplified (cf. Eq. (21) in KPPK) to

$$\frac{dv}{dt} = \left(\frac{SA}{mc}\right)Q_{pr}\left(1 - \frac{v}{c}\right)e - \frac{v}{c}.$$  \hspace{1cm} (2)$$

We now compare this to BLS’s result, including only terms of order $v/c$. Their Eq. (5), which also serves as a representative of the expressions of most of the papers that KPPK criticize, presents this acceleration as

$$\frac{dv}{dt} = \left(\frac{SA}{mc}\right)Q_{pr}\left(1 - \frac{r}{c}\right)e - \frac{v}{c},$$  \hspace{1cm} (3)$$

where an over-dot denotes the time derivative $d/dt$ (e.g., $\dot{r} = dr/dt$); $r$ is the radial distance from the central body to the particle. BLS occasionally fail to specify the reference system in which the quantities $A$ and $Q_{pr}$ are measured. For equations like those above that are valid to first order in $v/c$, this difference is immaterial because physical quantities (e.g., lengths) would be measured to be the same to order $v/c$ in the two reference frames. Thus, for solar-system cases, we have that Eq. (2) taken from KPPK is identical to Eq. (3) from BLS.

We write $v = \dot{r}e_r + \dot{\theta}e_\theta$, so as to distinguish the radial and tangential velocity components, $\theta$ being the particle’s longitudinal position from some arbitrary reference line. Then Eq. (3) (cf. BLS’s Eq. 6)

$$\frac{dv}{dt} = \left(\frac{SA}{mc}\right)Q_{pr}\left(1 - \frac{2\dot{r}}{c}\right)e_r - \frac{\dot{r}}{c}e_\theta.$$  \hspace{1cm} (4)$$

Usually we refer to the velocity-dependent portion of Eq. (4) as the Poynting–Robertson drag and the constant radial term as the radiation pressure, but such nomenclature is not universally employed. Starting from this expression, it is straightforward to compute the orbital collapse time. Because $\dot{r}$ has a constant sign, whereas $\dot{r}$ oscillates over the orbital period (so that its integral around a full orbit vanishes approximately), the longitudinal drag force usually dominates the particle’s orbital energy loss. It thus sets the orbital lifetimes of grains that orbit about radiation sources (Burns, 1976). This argument is even stronger for low-eccentricity orbits because then $\dot{r} \gg \dot{r}$, again showing that longitudinal forces account for most energy loss. Hence, owing to Poynting–Robertson drag, orbits of Solar System grains collapse in

$$T_{\text{PR}} = 7.0 \times 10^6 s \rho R^2/Q_{pr} = 400 R^2/\beta,$$

in years, where here $s$ and the particle mass density $\rho$ are in cgs units; $R$ is the heliocentric distance in AU and $\beta$ is the ratio of the solar radiation force to the Sun’s gravitational attraction (cf. BLS’ Eq. 50; note that Robertson (1937); Wyatt and Whipple (1950) previously derived this for perfectly absorbing grains).

We were initially confused by several fundamental KPPK expressions involving various optical factors; so to help others, we clarify them here. For example, the $Q_{\text{pr}}$ that we have written in Eq. (1) actually was presented by KPPK as $Q_{pr}/Q$ in their Eq. (15), with $Q$ subsequently defined as $Q_{pr}/Q_{\text{ext}}$. As can be seen in the terms just written, most of KPPK’s optical factors carry over-bars; such bars are used to indicate quantities that are averaged over the stellar spectrum. However, in KPPK’s Appendix B, where the radiation pressure is derived for the scattering of a photon stream, just the final expression contains such a bar and its meaning is not stated there. This is important because results from Appendix B (“Ratio of the pressure term to the extinction term”) are carried forward into Appendix A (“Derivation of four-momentum change of a spherical dust particle caused by incoming and outgoing radiation”) and ultimately appear in Section 2, where the particle’s equation of motion (15) (i.e., Eq. (1) above) is derived. However, the development in KPPK’s Appendix B assumes a perfectly reflecting particle, whereas in the general case when $Q_{\text{abs}} = 0$, the coefficient $Q_{\text{ext}}$ on the left-hand side of KPPK’s (B.8) would be replaced by $Q_{\text{ext}}$. As a final illustration, the appearance of $Q_{pr}/Q_{\text{ext}}$ in many of KPPK’s basic expressions (e.g., the example given at this paragraph’s start, and also their Eqs. (1), (15), (A.4), (A.26), (B.8) and (B.9)) is in marked contrast with familiar results, and so many of KPPK’s equations look odd. However, in all cases, this term is multiplied by $Q_{\text{ext}}$, whether explicitly or by a term (such as the incoming momentum) that is proportional to $Q_{\text{ext}}$. Hence, these terms always reduce to the expected $Q_{pr}$, as also found by BLS and other researchers. BLS employ $Q_{pr}$ to represent both the radiation pressure coefficient for a single wavelength as well as across the solar spectrum.

Because Eqs. (2) and (3) above are linear in $Q_{pr}$, we will now elaborate on $Q_{pr}$’s properties, so that a reader can place our results in an astronomical context. In accord with van de Hulst (1981), BLS define

$$Q_{pr} = Q_{\text{ext}} - Q_{\text{abs}}(\cos \theta),$$  \hspace{1cm} (6)$$

where the extinction coefficient $Q_{\text{ext}} = Q_{\text{abs}} + Q_{\text{ext}}$ pertains to the entire energy removed (i.e., absorbed, reflected, refracted and
from the beam, whereas the scattering coefficient \( Q_{\text{sca}} \) and the asymmetry factor \( \cos \theta \) incorporate all of the radiation except that absorbed. Each of these terms is directly calculated from the Mie series expansions. Since Mie theory involves a rigorous solution to Maxwell’s equations, BLS’s results for these quantities and the associated \( \beta \) factor are above reproach.

The character of the interaction that a spherical particle undergoes with radiation depends significantly on that particle’s non-dimensional size parameter \( x = 2\pi s/\lambda \), where \( \lambda \) is the light beam’s wavelength (van de Hulst, 1981). Very small grains \( (x \ll 1) \) respond as Rayleigh scatterers for which only a term (or two) of the series expansions is sufficient to accurately compute the efficiency factors (Section 10.3 of van de Hulst, 1981). Intermediate-sized objects \( (x \sim 1) \) require extensive Mie series. Large bodies \( (x \gg 1) \) obey geometric optics, in which the diffracted beam is very narrow and is essentially indistinguishable from the incoming radiation.

When BLS calculated the solar radiation forces experienced by spheres of various sizes and different compositions, they relied solely on Mie expansions, which are valid for all particle sizes, and so they implicitly incorporated diffraction. Two BLS plots (their Figs. 7a and 7b) provide \( \beta \) for particles of various compositions across all three size regimes. Because these diagrams are often reproduced in textbooks and review articles (e.g., Mignard, 1992, p. 436; de Pater and Lissauer, 2010, p. 46), it is valuable to recall their character. Rayleigh particles scatter radiation inefficiently and thus radiation forces are small; \( \beta \) for Mie particles depends significantly on their size and composition (cf. Figs. 28, 54 and 56 of van de Hulst, 1981); and for large particles, where the diffracted signal does not contribute measurably to the radiation pressure, \( Q_{\text{sca}} \) is fixed by the particle’s albedo, and then only the particle’s size and density determine \( \beta \).

When describing their results, KPPK (see their Section 4) are consistent that the right-hand side of Eq. (1) be called the Poynting–Robertson effect, rather than a radiation force, and they are equally firm (cf. the last two paragraphs of their Section 4) as to which parts of these acceleration expressions are to be designated as the radiation pressure and which should be termed the Poynting–Robertson effect. We assert that such distinctions in terminology are unnecessary; see our comment following Eq. (4). Many authors have employed other definitions; indeed Poynting (because of an error) himself had the incorrect numerical coefficient on “his” term. Some of KPPK’s criticism of other publications might have been muted had they been less dogmatic about the use of this terminology as well as about other semantic differences that they have with historical papers.

KPPK further maintain that the physics behind radiation forces is non-classical in its nature. We counter that physical processes occur without regard to whether they are designated as classical or non-classical. If expressions, even approximate ones, can be derived and understood in classical terms, we believe that it is instructive to do so (and, in most teaching situations, one should do so). KPPK apparently share this viewpoint: the description given in their Section 6 (“Simple Explanation of the P-R Effect”) is entirely compatible with BLS’s interpretation.

We acknowledge at least one error that KPPK have identified in our 1979 presentation. In BLS’s Eq. (1), the energy flux density that is received by a moving particle should have been expressed as \( \epsilon = S(1 – 2\eta/c) \), as BLS later conclude in the relativistic derivation of their Eq. (17). Previous covariant treatments (Bertotti et al., 2003, p. 496; Mignard, 1992, p. 438) directly determine that the coefficient of the \( \eta \) term is 2. But we point out that the discrepancy between the above-mentioned equation in BLS is not due to the effect being inextricably relativistic; rather, BLS’s Eq. (1) erroneously neglected to include the purely classical fact that an outward-moving surface collects fewer photons per second.

In conclusion, here we have reconsidered the radiation forces experienced by solar-system bodies while being frustrated when trying to understand KPPK’s harsh criticism of BLS. We have often found it difficult to follow their treatment because of KPPK’s brevity, semantic differences, inconsistent notation and rigid terminology as well as their unconventional handling of radiation pressure coefficients. We identify a few errors in both papers; fortunately, they are usually unimportant in solar-system circumstances.

Despite its century-long history, the topic of radiation forces continues to attract considerable interest today (e.g., Rubincam, 2013). This is appropriate because, among other reasons, such forces (along with the related drag owing to stellar winds that both BLS and KPPK discuss) compete with self-collisions to set the lifetimes of circumstellar grains, thus defining the timescales for debris disks surrounding young stars (Wyatt, 2008).

Hence it is a considerable relief to learn that BLS’s expressions for Poynting–Robertson lifetimes (cf. Eq. 5), which have found extensive application over many decades, remain valid. Furthermore, we confirm BLS’s estimates for radiation forces on various-size particles having cosmically significant compositions.

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